Formulae for Week 5

coulomb force is conservative — work done independent of path

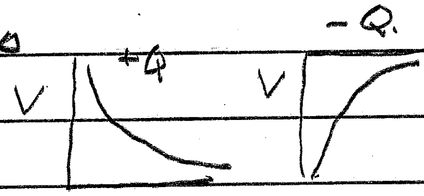
change of Potential energy $\quad \Delta U_E = -\vec{F_E} \cdot \vec{\Delta S}$

Change of Electric potential

$$\Delta V = -\frac{\vec{F_E} \cdot \vec{\Delta S}}{q} = -\vec{E} \cdot \vec{\Delta S}$$

Potential due to ±ive Q at r = 0

$$V = \pm \frac{k_e Q}{r}.$$



Potential Energy of Q_1, Q_2

$$U_E = \frac{k_e Q_1 Q_2}{r}.$$

Conservation of Energy

$$K_f + P_f(sp) + P_f(g) + U_E(f) = K_i + P_i(sp) + P_i(g) + U_E(i)$$

Capacitor $\quad C = \frac{Q}{V}$

Vac or Air between plates $\quad C_0 = \frac{\varepsilon_0 A}{d}.$

Dielectric between plates $\quad C_k = \frac{k \varepsilon_0 A}{d}.$

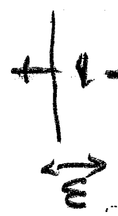
Many Capacitors: Series Q is common, V's add

$$\frac{1}{C_S} = \Sigma \frac{1}{C_i}$$

parallel, V is common, Q's add

$$C_P = \Sigma C_i$$

Battery Generates Coulombs $\vec{E}$ using chemical Energy: Output is potential difference between plates



Called Emf. Emf measures work done in transporting a charge of one unit so its unit is also Volt.

SOLUTIONS - WEEK 5

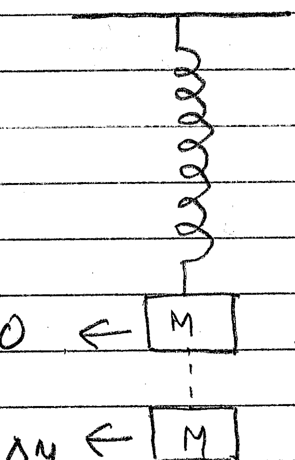
5-1 POTENTIAL ENERGY is the work stored in a system when it is assembled in the presence of a conservative force because in a conservative force the work done is independent of the path and determined only by the end points.

5-2 $$\Delta U = -\vec{F}_E \cdot \vec{\Delta S}$$

The negative sign comes from the fact that in order to assemble a system of charges and hence in the presence of $\vec{F}_E$ we must apply a force which is equal BUT OPPOSITE to $\vec{F}_E$ so that during assembly the charges will not accelerate and acquire Kinetic Energy.

5.3

This problem we discussed in Test Questions.



$y=0 \leftarrow$ Initial position

$y=\Delta y \leftarrow$ Position of maximum displacement before it starts up.

In the initial position, $(y=0)$ the block is at rest, so,

Kinetic Energy $KE = 0$

the Spring is unstretched, and the gravitational potential is set to zero at $y=0$, So

Potential Energy $P_{sp} + P_g = 0$

$\therefore$ At $y=0$, <u>Total Energy $E_1 = 0$</u>

In the final position $(y = \Delta y)$

Potential energy Stored in the Spring

$$PE_{spring} = \frac{1}{2} K (\Delta y)^2$$

Gravitational potential Energy

$$PE_G = mg.\Delta y$$

Kinetic Energy $KE = 0$
(Since the mass comes to rest)

$\therefore$ At $y = \Delta y$

Total Energy $E_2 = \frac{1}{2}K(\Delta y)^2 + mg\Delta y$

From conservation of energy

$$E_1 = E_2 = 0$$

$$\therefore \frac{1}{2}K(\Delta y)^2 + mg\Delta y = 0$$

or, $\Delta y \left\{ \frac{K}{2}.\Delta y + mg \right\} = 0$

$\therefore$ which gives us two solutions

(i) $\Delta y = 0$ (which is the initial position)

(ii) $\frac{K}{2}.\Delta y + mg = 0$

or, $\Delta y = -\frac{2mg}{K}$

which is the final position.

$\Delta y = -\dfrac{2mg}{K}$ is the maximum displacement, hence the block will oscillate between

$$y = 0 \text{ and } y = -\frac{2mg}{K}.$$

[Note: when the block comes back to $y = 0$, its Potential Energy $= 0$

since, $KE + PE = 0$ (Conservation of Energy)

$\Rightarrow$ $KE = 0$ also

$\therefore$ the block will come to a rest when reaching $y = 0$.

So, it will oscillate between

$y = 0$ and $y = -2mg/K$]

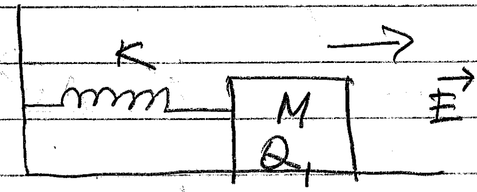
THE INTERESTING FACT IS

THAT THE MOTION OCCURS WHILE

THE TOTAL ENERGY IS <u>ZERO</u> AT

ALL TIMES

5.4



$x = 0$

(i) The block will start returning when it comes to a rest momentarily, say at a distance 'x'.

Now, at $x = 0$,
Since the spring is unstretched and the Electrostatic potential is set to zero,
Potential Energy $PE = 0$

Also, the block is at rest at $x = 0$
$\therefore$ Kinetic energy $KE = 0$

$\therefore$ at $x = 0$, total Energy $E_1 = 0$

When the block reaches the distance 'x' before returning, it again comes to rest momentarily

$\therefore$ at 'x', $KE = 0$
Potential Energy due to the Stretched Spring $PE = \frac{1}{2} K x^2$

b

Electrostatic potential Energy at x :

$$PE_{Electrostatic} = -\vec{F}\cdot\vec{\Delta S}$$

$$= -Q\vec{E}\cdot\vec{x}$$

$$= -QEx$$

$\therefore$, at position 'x',

Total Energy $E_2 = \frac{1}{2}Kx^2 - QEx$

From Conservation of energy

$$\frac{1}{2}Kx^2 - QEx = 0$$

or, $x\left\{\frac{1}{2}Kx - QE\right\} = 0$

it has 2 solutions

(i) $x = 0$, (initial position)

(ii) $\frac{1}{2}Kx - QE = 0$

or, $x = \frac{2QE}{K}$ (final position)

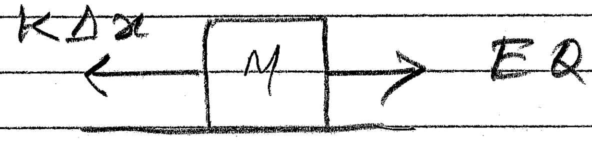
$\therefore$, the block moves by
$x = 2QE/K$

before returning.

5.4 (ii) When the mass is in equilibrium, no net force acts on it

Say $x = \Delta x$ when this happens.

The forces acting on the mass at $x = \Delta x$ are shown below



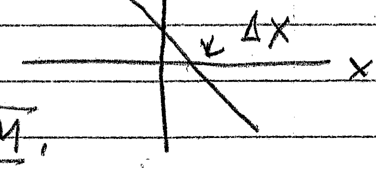
Since the net force acting is zero, we have

$$K\Delta x = EQ$$

$$\therefore \Delta x = EQ/K.$$

(iii) Force acting on the block is

$$F = -Kx + QE \leftarrow \text{CONSTANT}$$

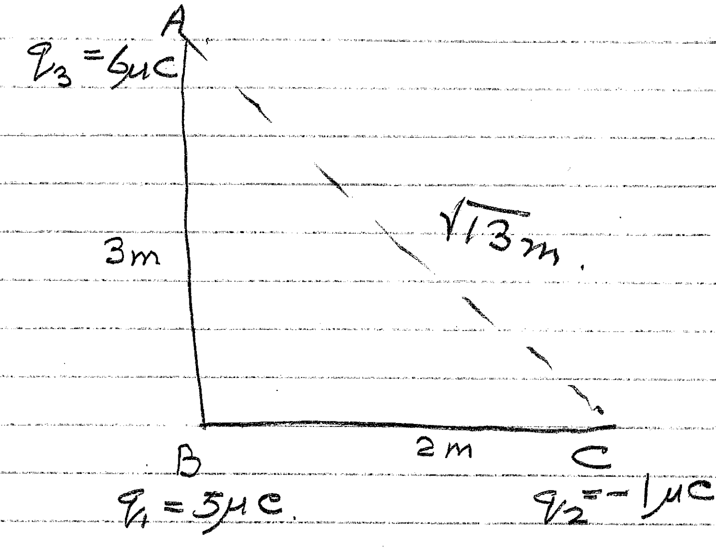


$$\therefore \omega = \sqrt{K/M}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{K}}.$$

T is not influenced by the $\vec{E}$ field.

5.5



For Two charges

Potential Energy

is

$$U_{1,2} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}.$$

For our System

$$U_{1,2} = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{BC}$$

$$= -\frac{9\times10^9 \times 5\times1\times10^{-12}}{2} = -0.0225\,J$$

$$U_{1,3} = \frac{9\times10^9 \times 6\times5\times10^{-12}}{3} = 0.09\,J$$

$$U_{2,3} = -\frac{9\times10^9 \times 6\times1\times10^{-12}}{\sqrt{13}} = -0.015\,J.$$

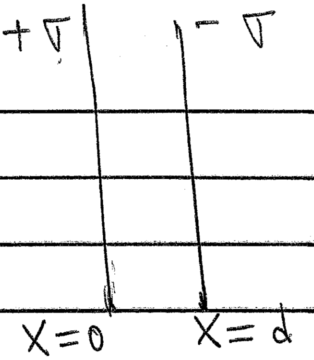
Total

$$U = U_{1,2} + U_{1,3} + U_{2,3}.$$

$$= (-0.0225 + 0.09 - 0.015)\,J$$

$$= 0.0525\,J$$

5.6



Large plates of 4m x 4m carry charges of $\pm 10\,\mu C$ with $d = 1$ mm.

i) By definition

$$\sigma = \frac{Q}{A}$$

$$\Longrightarrow \pm\sigma = \pm\frac{10\,\mu C}{16\,m^2} = \pm\frac{5}{8}\frac{\mu C}{m^2}$$

(ii) $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{x}$, $\varepsilon_0 = \frac{1}{4\pi k_e}$

$$\Longrightarrow \vec{E} = \frac{5}{8}\,4\pi k_e\,\frac{\mu C}{m^2}\,\hat{x}$$

$$= \frac{5}{2} \times \pi \times 9 \times 10^9\,\frac{N \cdot m^2}{C^2} \times \frac{10^{-6}\,C}{m^2}\,\hat{x}$$

$$\vec{E} \simeq 70685.8\ N/C\ \hat{x}$$

(iii) $V = |Ed| = \left|\frac{\sigma}{\varepsilon_0}d\right| = 70685{,}8 \times 10^{-3}\,\frac{N}{C}\,m$

$V = 70{,}7$[illegible] Volts

(iv) $W = \vec{F}\cdot\Delta\vec{x}$ $\vec{F} = q\vec{E}$

- Electrons charge e: $-1{,}6 \times 10^{-19}$ C

$W = |\vec{F}_e| d$

$|\vec{F}_e| = 70685{,}8 \times 1{,}6 \times 10^{-19}$ N

$= 1{,}130 \times 10^{-14}$ N

$$\Longrightarrow W = |\vec{F}_e| d = 1{,}1309 \times 10^{-14}\,N \times 10^{-3}\,m$$

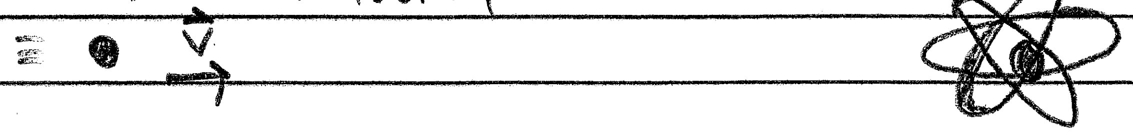
$$= 1{,}13 \times 10^{-17}\,J$$

Alternately $W = eV = 1.6 \times 10^{-19} \times 70.7 = 1.13 \times 10^{-17}$ J

5.7 Conservation of energy

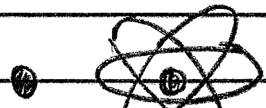
Initial time — α-particle comes from "∞" with velocity

Au. Atom



$\vec{v}$

Final time



$\vec{v} = 0$

due to repulsive coulomb force the α-particle reaches a closest point of approach before turning around.

Now at ∞ P_e is $\emptyset$ ($P_{ei} = 0$). Thus conservation of energy yields

$$K_i + P_{ei} = K_f + P_{ef}$$

$$\frac{1}{2} m V^2 + \emptyset = \emptyset + K_e \frac{Q Q'}{r}$$

$$\Rightarrow \frac{1}{2}\, 6{,}4 \times 10^{-27} \times (10^7)^2 \, kg \frac{m^2}{s^2} = \frac{K_e (2e)(79)e}{r}$$

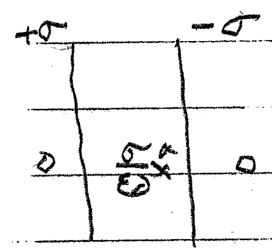
$$3{,}2 \times 10^{-13} \, Kg \frac{m^2}{s^2} = 9 \times 10^9 \, \frac{N m^2}{C^2} \times \frac{158}{r} e^2$$

$$\Rightarrow r = \frac{9 \times 10^9 \, N \times 158 \times (1{,}6 \times 10^{-19} \, C) s^2}{3{,}2 \times 10^{-13} \, Kg}$$

$$\boxed{r = 1{,}147 \times 10^{-13} \, m}$$ distance of closest approach

5-8 A capacitor (C) is a device for storing an $\underline{E}$-field. Usually it consists of two metal plates separated by a small distance, there is either air (~ vacuum) or an insulator between the plates so no charge ever flows inside a capacitor. Indeed, you have charge sheets with $\pm\sigma$ which traps an $\underline{E}$-field

$$\underline{E} = \frac{\sigma}{\varepsilon_0}\hat{x} \qquad \Delta V = \frac{\sigma}{\varepsilon_0} d$$



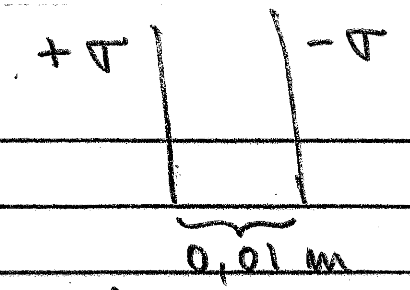
between them. If you apply a potential difference C will give you how much charge appears on the plates

$$Q = C\,V$$

to sustain the field.

A battery: Generates a coulomb $\underline{E}$ using chemical energy. Again two metal plates are immersed in an electrolyte and the chemical reactions cause plus charges to accumulate on one plate and negatives on the other thereby creating an $\underline{E}$-field. When connected in a circuit charges (ions) flow freely between the plates.

5.9



Area = 0,5 m × 0,5 m

(a) $C = \dfrac{Q}{V}$ $\qquad Q = \sigma A$

$V = \left| \dfrac{\sigma d}{\varepsilon_0} \right|$

$\Rightarrow C = \dfrac{\varepsilon_0 A}{d} = \dfrac{1}{4\pi k_e} \dfrac{A}{d}$

$= \dfrac{1}{4\pi \times 9 \times 10^{9} \ \frac{N \cancel{m^2}}{C^2}} \times \dfrac{25 \times \cancel{10^{-2}} \ \cancel{m^2}}{\cancel{10^{-2}} \ m}$

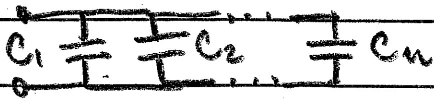
$C = 2,21 \times 10^{-10}$ F
(Farad)

(b) $C = K \dfrac{\varepsilon_0 A}{d} = 2 \times \dfrac{\varepsilon_0 A}{d} \simeq 4,42 \times 10^{-10}$ F

dielectric constant

5.10 In parallel we know that

$C_{eq} = C_1 + C_2 + \ldots + C_n$



In our case

$\Rightarrow$ $C_{eq} = n\,(1{,}0\ \mu F)$

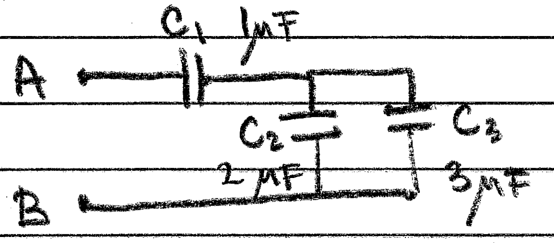
$\uparrow$ number of capacitors

Now $C_{eq} = \dfrac{Q}{V} = \dfrac{0{,}1\ C}{300\ V}$

$\Rightarrow n = \dfrac{0{,}1\ C}{300\ V \times \mu F} = \dfrac{0{,}1}{300 \times 10^{-6}} \dfrac{\cancel{C}}{\cancel{V}} \dfrac{\cancel{V}}{\cancel{C}}$

$n \simeq 333$

5.11



Let us first consider the block in the right side. There we have two capacitors in parallel. Then

$C_{eq} = C_2 + C_3 = 5\ \mu F$

Now we consider the 'remaining' system of two capacitors in series. In this case we know that the new equivalent capacitance C_{eq}' is given by $\dfrac{1}{C_{eq}'} = \dfrac{1}{C_1} + \dfrac{1}{C_{eq}} = \dfrac{1}{1\mu F} + \dfrac{1}{5\,\mu F}$

$C_{eq}' = \dfrac{5}{6}\ \mu F$. This is the equivalent capacitance across AB.

For capacitors in series we know that

$Q_1 = Q_2 = Q_3 = Q_T = \ldots$

Using $V_{AB} = 12\ V = \dfrac{Q_T}{C_T}$, $C_T = C_{eq}$

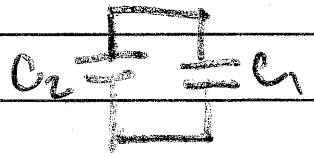
$\Rightarrow \quad Q_T = V_{AB} C_T$

$= 12 \times \dfrac{5}{6}\ V \mu F$

$Q_T = 10 \times 10^{-6}\ C = 10^{-5}\ C$,

this is the charge at C_1.

Now the voltage for the subsystem V', is such that



$V_{AB} = V_{C_1} + V' = \dfrac{Q_T}{C_1} + V'$

$\Rightarrow \quad V' = 12 - \dfrac{10^{-5}}{10^{-6}}\ V = 2$

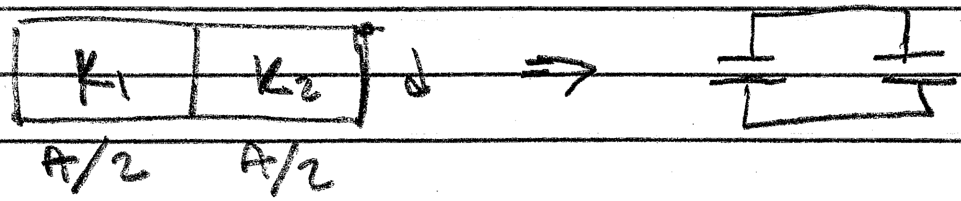
So, the voltage across the subsystem is 2 Volts. With this,

$Q_2 = C_2 V' = 2 \times 2\ \mu F\ V = 4 \times 10^{-6}\ C$

$Q_3 = C_3 V' = 3 \times 2\ \mu F V = 6 \times 10^{-6}\ C$

15

5.12



V common on
Q's add

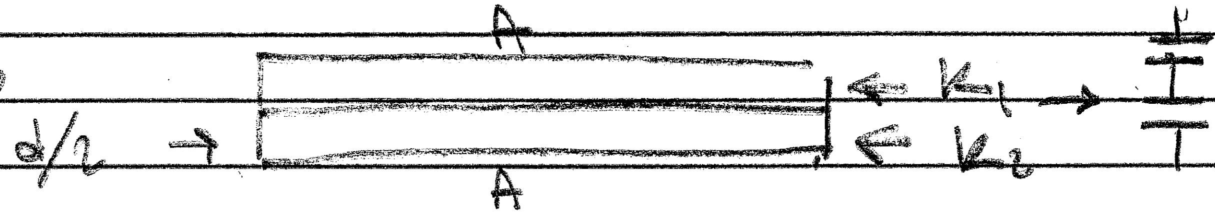
For capacitor in parallel,

$$C_{eq} = C_1 + C_2$$

$$= \frac{K_1 \varepsilon_0}{d}\frac{A}{2} + \frac{K_2 \varepsilon_0}{d}\frac{A}{2}$$

$$\Rightarrow \quad C_{eq} = \frac{\varepsilon_0 A}{2d}(K_1 + K_2)$$

5.13



Q COMMON
V's add.

For capacitors in series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$\Rightarrow$ Now $C_1 = \frac{2K_1 \varepsilon_0 A}{d}$, $C_2 = \frac{2K_2 \varepsilon_0 A}{d}$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2\varepsilon_0 A}\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

$$= \frac{d}{2\varepsilon_0 A}\left(\frac{K_1 + K_2}{K_1 K_2}\right)$$

So $$C_{eq} = \frac{2\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$$