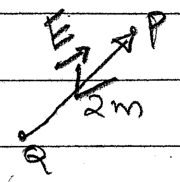
Week 4 — SOLUTIONS

4-1) $q\vec{E} = \vec{F}_E$. A charge feels a force in the presence of an $\vec{E}$ field. Therefore if the measuring device shows a nonzero value, there must be $\vec{E}$.
Assume the device can measure both magnitude & direction
$\vec{E} = \frac{\vec{F}}{q}$ [Compare with your measurement of the gravitational field of Earth when you measure weight]

4.2

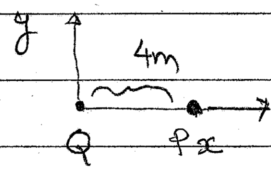
i) $\vec{E} = \frac{K_e Q}{r^2}\hat{r}$



$\Rightarrow$ $-100\,\frac{N}{C}\,\cancel{\hat{r}} = 9\times10^9\,\frac{N\,m^2}{C^2} \times \frac{Q}{(2\,m)^2}\,\hat{r}$

$\Rightarrow$ $Q = \frac{-100\times4}{9\times10^9}\,C \approx -4.44\times10^{-8}\,C$.

ii) Using Q from i)



$\vec{E} = \frac{k_e Q}{r^2}\hat{r} = \frac{9\times10^9\times(-4.44\times10^{-8})}{4^2}\hat{x}$

$\simeq -25\ N/C\ \hat{x}$

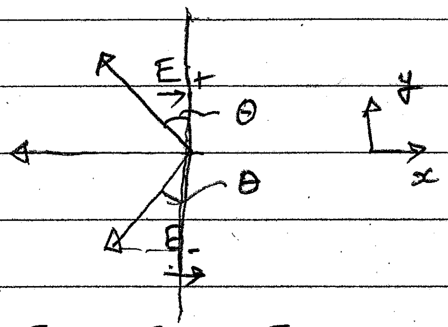
Alternate way, since the charge is -ve the electric field will be directed towards Q $\therefore$ the direction at P is $\underline{-\hat{x}}$

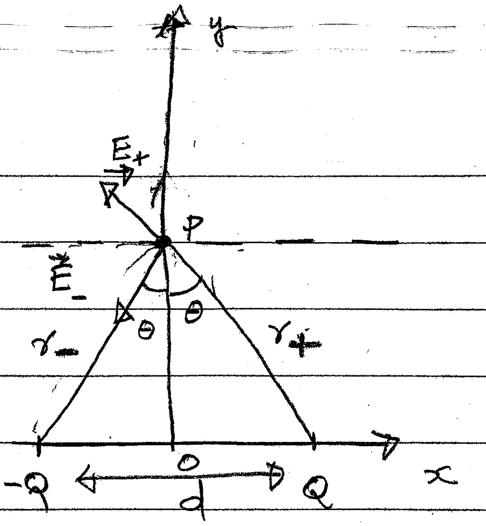
Consider the ratio of magnitude of $\vec{E}$, denoting $\vec{E}$ in i) & ii) by $\vec{E}_1$ & $\vec{E}_2$.

$$\frac{E_1}{E_2} = \frac{\frac{K_e Q}{r_1^2}}{\frac{K_e Q}{r_2^2}} = \frac{r_2^2}{r_1^2} = \frac{4^2}{2^2} = 4 \Rightarrow E_2 = \frac{E_1}{4} = 25\,N/C$$

Hence $\vec{E}_2 = -25\,N/C\,\hat{x}$

4-3)





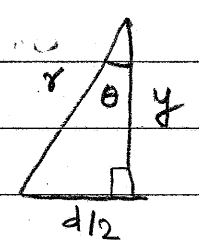
$\vec{E} = \vec{E}_+ + \vec{E}_-$

From symm. $r_+ = r_- = r$

$E_x = E_{+x} + E_{-x} \Rightarrow E_x = -\frac{k_e Q}{r^2}\sin\theta - \frac{k_e Q}{r^2}\sin\theta = -2\frac{k_e Q}{r^2}\sin\theta$

$E_y = E_{+y} + E_{-y} = \frac{k_e Q}{r^2}\cos\theta - \frac{k_e Q}{r^2}\cos\theta = 0$

$\therefore \vec{E} = -\frac{2 k_e Q}{r^2}\sin\theta\,\hat{x}$



$y^2 + \frac{d^2}{4} = r^2$ From Pythagoras thm

$\sin\theta = \frac{d/2}{r} = \frac{d/2}{\sqrt{y^2 + d^2/4}}$

In terms of d & y

$\vec{E} = -\frac{2 k_e Q}{(y^2 + d^2/4)} \times \frac{d/2}{(y^2+d^2/4)^{1/2}}\hat{x} = -\frac{k_e Q d}{\left(y^2 + \frac{d^2}{4}\right)^{3/2}}\hat{x}$

Since $y \gg d$ $\quad y^2 + d^2/4 \approx y^2$

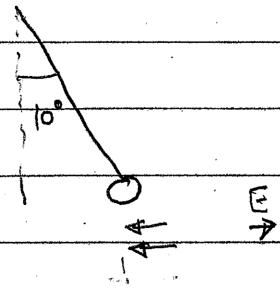
$\therefore \vec{E} = -\frac{k_e Q d\,\hat{x}}{y^3} = -\frac{k_e \vec{P}}{y^3}$ where $\vec{P} = Qd\,\hat{x}$ [Dipole moment]

$\left[k_e = \frac{1}{4\pi\varepsilon_0}\right]$

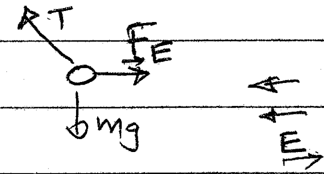
$\vec{p} = 10\times 10^{-6} \times .01\hat{x} = 10^{-7}\ \mathrm{Cm}\ \hat{x}$

4

4.4) i) Electric field lines are directed from +ve to -ve charge. q moved in the opposite ~~direction~~ direction of the field, hence q is -ve.

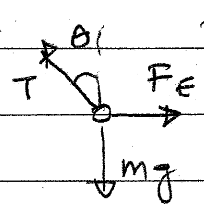


For $\equiv$m total Force is zero! it is clear $\vec{F_E}$ is opposite to $\vec{E}$



since $\vec{F_E} = q\vec{E}$ $\quad$ q is -ve.

ii) $\theta = 10^\circ$



$\Sigma F = 0$

$\Rightarrow F_E - T\sin\theta = 0$ & $T\cos\theta - mg = 0$

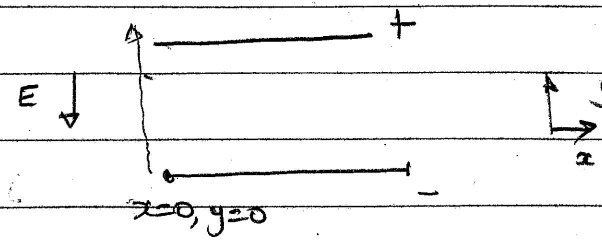
$\Rightarrow \dfrac{F_E}{mg} = \dfrac{\cancel{T}\sin\theta}{\cancel{T}\cos\theta} = \tan\theta$

$\Rightarrow |q|E = mg\tan\theta$

$\Rightarrow |q| = \dfrac{0.001\times 9.8\tan 10^\circ}{100} = 1.73\times 10^{-5}\ \mathrm{C}$

$\therefore\ q = -1.73\times 10^{-5}\ \mathrm{C}$ //

4.5) i) $\vec{F_e} = q_e \vec{E}$



$m_e \vec{a_e} = q_e \vec{E} \Rightarrow \vec{a}$

$$\vec{a_e} = \frac{q_e \vec{E}}{m_e} = \frac{-1.6\times10^{-19} \times (-50)\,\hat{y}}{9.1\times10^{-31}} = 8.8\times10^{12}\ m/s^2\ \hat{y}$$

ii) Since the acceleration is in the $\hat{y}$ direction, V_x is const

The time taken to travel from $x=0$, $x=l$

$$\Delta t = \frac{l}{V_x} = \frac{0.15}{10^7} = 1.5\times10^{-8}\ s$$

$$V_y = V_{y_0} + a\Delta t = 0 + 8.8\times10^{12} \times 1.5\times10^{-8} = 1.32\times10^{5}\ m/s$$

$$\vec{V} = 10^7\ m/s\ \hat{x} + 1.32\times10^{5}\ m/s\ \hat{y}$$

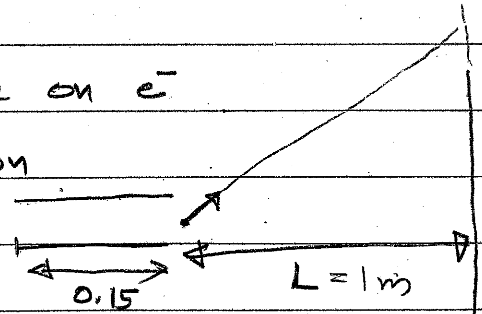
iii) At $x=l$, we need to calculate y.

$$y = \cancel{y_0} + \cancel{V_{y_0}}\Delta t + \frac{1}{2}a\Delta t^2 = \frac{1}{2}\times 8.8\times10^{12}\times(1.5\times10^{-8})^2$$
$$= 9.9\times10^{-4}\ m$$

$$\vec{r} = 0.15\,m\,\hat{x} + 9.9\times10^{-4}\,m\,\hat{y}$$

iv) After $x=l$, there is no force on e^-

At $x=l$ it starts from the position given in iii)



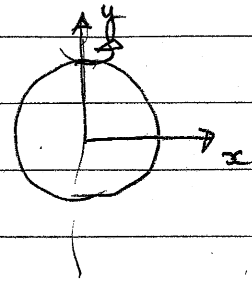
Time taken for ~~reaching~~ the screen from $x=l$ $\quad \Delta t = \frac{L}{V_x} = \frac{1}{10^7} = 10^{-7}\ s$

(No acceleration)

$$y = y_0 + V_{y_0}\Delta t = 9.9\times10^{-4} + 1.32\times10^{5}\times10^{-7} = 1.42\times10^{-2}\ m$$

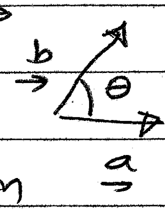
Final position $\vec{r} = 1.15\,m\,\hat{x} + 1.42\times10^{-2}\,m\,\hat{y}$

4.6) $\Phi_E = \sum \vec{E} \cdot \Delta \vec{A}$



For any two vector $\vec{a}$ & $\vec{b}$

$\vec{a} \cdot \vec{b} = a b \cos\theta$



The max value of $\vec{a} \cdot \vec{b}$ is when

$\theta = 0°$ & it is zero when $\theta = 90°$

For this problem since the disk is a flat surface, the normal vector($\hat{n}$) perpendicular to the disk is the same throughout the disk. $\vec{A} = \pi R^2 \hat{n}$

$\Phi_E = \vec{E} \cdot \hat{n} \; \pi R^2$

The maximum value of Φ_E is when $\hat{n} \parallel \vec{E}$

Max $\Phi_E = \pi R^2 |\vec{E}| |\hat{n}| = \pi R^2 E$ { $\hat{n}$ is unit vector $\therefore |\hat{n}| = 1$ }

$= \pi \times 1^2 \times 60$

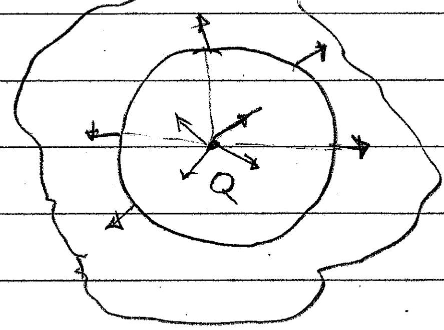
$= 60\pi \ \frac{Nm^2}{C}$ $\hat{n}$ & $\vec{E}$ are parallel

Min Φ_E :- $\theta = 180°$

Min $\Phi_E = -60\pi \ \frac{N\text{-}m^2}{C}$ $\hat{n}$ & $\vec{E}$ are ANTI-PARALLEL

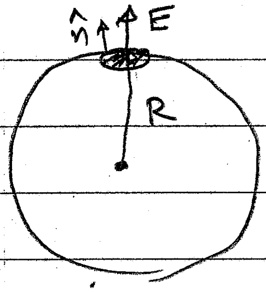
N.B: ... the absolute ...

in ...

4.7) 

The flux through a closed surface around a charge configuration is the same for all surfaces if they enclose the same amount of charge.

The easiest way to calculate the flux for a pt. charge is by considering a sphere of radius R centered at the charge.



The electric field is $\perp$ to the sphere.
Hence at each pt. of the sphere $\hat{n} \parallel \vec{E}$.

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^2}\hat{r}$$ { on the surface of sphere.

$$\Phi_E = \Sigma \vec{E}\cdot\vec{\Delta A} = \Sigma E \Delta A \cos\theta = \Sigma E \Delta A$$
$$= E \Sigma \Delta A = EA$$

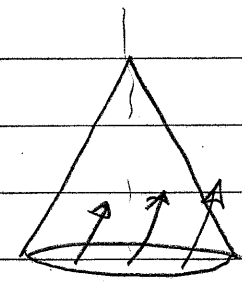
surface area of sphere $A = 4\pi R^2$

$$\therefore \Phi_E = EA = \frac{1}{\cancel{4\pi}\epsilon_0}\frac{Q}{\cancel{R^2}} \times \cancel{4\pi R^2} = \frac{Q}{\epsilon_0}$$ independent of R

4.8)

i) $\vec{E}$ starts and ends at charges

$\therefore$ charges acts like sinks/sources of $\vec{E}$. Since $\vec{E}$ in the cone doesn't start or terminate inside the cone, there can be no charge (source/sink) inside the cone

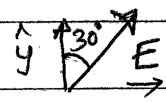


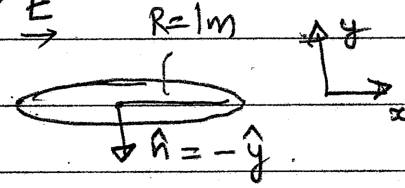
ii) From Gauss's law

$$\sum \Phi_E = \frac{Q}{\epsilon_0} = 0 \quad (\text{ Flux through the whole cone })$$

But $\underbrace{\sum \Phi_E}_{=0}$ ~~the~~ $= \Phi_E^B$ (through the base) $+ \Phi_E^T$ (through the curved surface)

$$\Rightarrow \therefore \Phi_E^B = -\Phi_E^T$$





It's easier to calulate Φ_E^B

$$\Phi_E^B = \sum \vec{E} \cdot \Delta \vec{A} = \vec{E} \cdot \pi R^2 (-\hat{y})$$

Using the fact the $\vec{E}$ makes $\theta = 30°$ with $\hat{y}$

$$\Phi_E^B = -\pi R^2 \vec{E} \cdot \hat{y} = -\pi R^2 |\vec{E}| |\hat{y}| \cos 30° \qquad \{ \hat{y} \text{ is unit vector } |\hat{y}| = 1 \}$$

$$= -\pi R^2 E \cos 30°$$

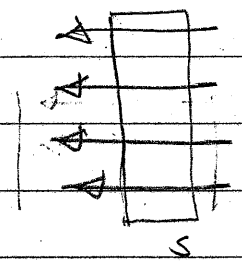
$$= -\pi \times 30 \times \frac{\sqrt{3}}{2} \ \frac{N}{C} m^2$$

$$\therefore \Phi_E^T = -\Phi_E^B = 15\sqrt{3}\pi \ \frac{N}{C} m^2 \quad //$$

4.9) Using Gauss' law

i) $$\sum \Delta \Phi_E = \frac{\sum Q}{\varepsilon_0} = \frac{10\mu C + 20\mu C + 30\mu C - 60\mu C}{\epsilon_0} = 0$$

ii) $\vec{E}$ at any point on S can only be determined if we know the ~~exact~~ location of all the charges and the precise coordinates of every point on S. The $\vec{E}$ at any pt. need not be zero if the total flux is zero through S. Ex:- Uniform field, The flux through surface S = 0 but $\vec{E} \neq 0$



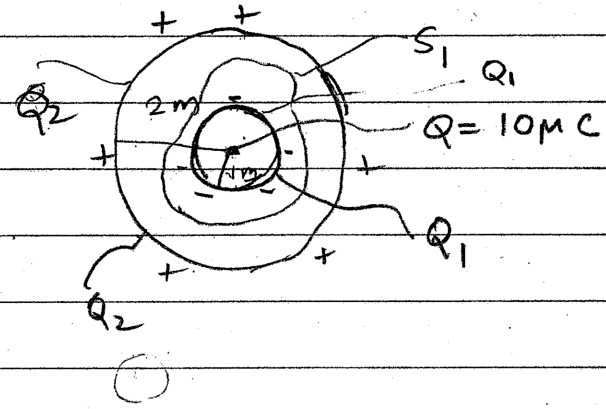
UNDER STATIC CONDITIONS

4-10) The electric field inside a conductor is zero

Consider a surface (S_1) between $r = 1\,m$ & $r = 2\,m$

since $\vec{E} = 0$ at S_1

$$\Phi_{S_1} = \Sigma \vec{E} \cdot \vec{\Delta A} = 0$$



but $\Phi_{S_1} = \frac{\Sigma Q}{\varepsilon_0} = 0 \Rightarrow Q_1 + Q = 0 \Rightarrow Q_1 = -Q = -10\mu C$

charge on the sphere at $r = 1\,m$

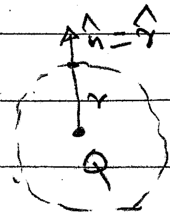
The conducting sphere is neutral $\therefore$ sum of charges at $r = 1\,m$ & $r = 2\,m$ should be zero

$\therefore Q_2 + Q_1 = 0 \Rightarrow Q_2 = -Q_1 = 10\mu C$.

The problem is spherically symmetric, there $\vec{E}$ will be in radial direction.

$r < 1\,m$ case, consider a Gaussian surface which is spherical and has radius r,

$\Phi_E = \frac{Q}{\epsilon_0}$



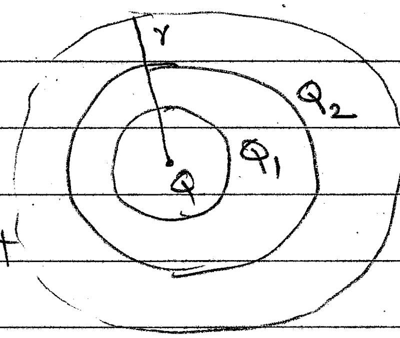
$\vec{E} \parallel \hat{n}$

$\therefore \vec{E} \cdot \vec{\Delta A} = \vec{E}\, \Delta A$

$\Rightarrow E\, 4\pi r^2 = \frac{Q}{\epsilon_0}$

$\therefore \vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$ { Coloumb's law for single charge }

$r > 2m$



$\Phi_E = \frac{Q+Q_1+Q_2}{\epsilon_0} = \frac{Q+Q-Q}{\epsilon_0} = \frac{Q}{\epsilon_0}$

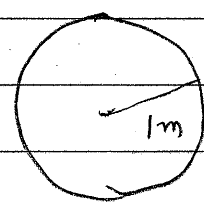
Again using the above argument

$\Phi_E = E\, 4\pi r^2 = \frac{Q}{\epsilon_0}$

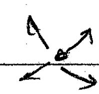
$\Rightarrow \vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$

If you donot want charge inside to move

4.11 Electric field inside a conductor is zero. If the charge is inside the sphere, there will be a non zero $\vec{E}$ around it.

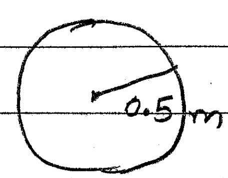


Hence the charge should reside on the surface of the sphere at $r = 1m$. The charge gets uniformly distributed on the surface.



4-12 i) $\vec{E} = 0$ inside the conductor

$\therefore \quad \vec{F}_E = q\vec{E} = 0$



ii) A spherically charged conductor with charge Q has the same electric field for $r > R$ as a pt. charge at the centre of sphere with charge Q.

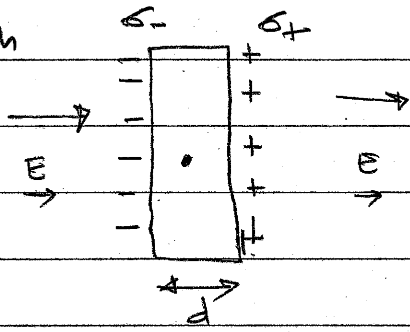
$r > R \qquad 0.51 > 0.5$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\vec{F}_E = \frac{qQ}{4\pi\epsilon_0 r^2}\hat{r} = \frac{9\times10^9 \times 10^{-6} \times 100\times10^{-6}}{(0.51)^2} \text{ N}$$

$$= 3.46 \text{ N}$$

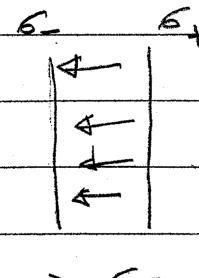
4-13) Since $\vec{E}$ is the direction in which +ve charge moves, +ve charges will get accumulated on the right hand side & −ve charges on the left hand side.



The charge densities can be calculated using the fact the field inside the conductor is zero. The charge

Inside conductor $\vec{E}_{tot} = \vec{E} + \vec{E}'$ (due to induced charges)

Using from the text (Ex 8)



$$\vec{E}' = -\frac{\sigma}{\varepsilon_0}\hat{x}$$

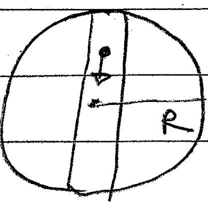
$$\therefore \vec{E}_{tot} = 100\frac{N}{C}\hat{x} - \frac{\sigma}{\varepsilon_0}\hat{x} = 0 \Rightarrow \sigma = 100\times\varepsilon_0 \text{ C/m}^2$$

$$= 9\times10^{-12}\times100 \text{ C/m}^2$$

$$= 9\times10^{-10} \text{ C/m}^2$$

N.B :- $|\sigma_-| = |\sigma_+|$ $\because$ conductor is neutral

4.14) From Ex 5 :- in the text



$r<R$ $\quad \vec{E} = \frac{\rho}{3\epsilon_0} r\hat{r}$

Force on q $\quad \vec{F}_E = q\vec{E} = -\frac{|q|\rho}{3\epsilon_0} r\hat{r}$.

Since q is -ve force on q is radially inwards.
From the force eq. the force is opp. to displacement & is proportional to the displacement.
Hence the motion will be harmonic

$\omega = \sqrt{\frac{|q|\rho}{3\epsilon_0 m}}$ $\quad \left\{ \text{Compare with } \vec{F} = -kx\hat{x} \quad k = \frac{|q|\rho}{3\epsilon_0} \right\}$

4.15 Conservative force has the property the ~~the~~ work done is independent of the path and only depends on the ~~stinal~~ points of the path.
Example force of gravity near the surface of earth $\vec{F} = -mg\hat{y}$. The work done in moving a mass to height h
$W = mgh$ is independent on how you move it to height h.