PHYS 122 WEEK 2

SOLUTIONS

Problem 2-1 The period T of a pendulum is given by $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\ell/g}$

Knowing the period (one second) and the value of g at the Earth's surface ($9.8\ m/s^2$), we can calculate:

$$T = 2\pi\sqrt{\ell/g}$$
$$\frac{1}{2\pi}(1\ \text{second}) = \sqrt{\ell/(9.8\ m/s^2)}$$
$$\left(\frac{1}{2\pi}\right)^2(1\ s)^2 = \ell/9.8\ m/s^2$$
$$.0253\ s^2 = \ell/9.8\ m/s^2$$
$$(9.8\ \tfrac{m}{s^2})(.0253\ s^2) = \ell$$
$$\boxed{\ell = 0.248\ m}$$

Since T is proportionate to $\sqrt{1/g}$, if we change g to one sixth its former value, T will be changed by a factor of $\sqrt{\frac{1}{1/6}} = \sqrt{6} \simeq 2.45$: we see that making g smaller will make T larger.

Plugging in the weaker lunar gravity into the equation $T = 2\pi(\ell/g)$, without changing the length, gives us a period of:

$$T = 2\pi\sqrt{\frac{0.248m}{1.63\ m/s^2}}$$
$$T = 2\pi\sqrt{0.152 s^2}$$
$$T = 0.390 s$$

Since period is controlled by ℓ/g, and we decreased g to 1/6 its former value, we would need to do the same to ℓ to get $T = 1s$.

We can check this, though: $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$1s = 2\pi\sqrt{\ell/1.63\ m/s^2}$$
$$\left(\frac{1}{2\pi}\right)^2 s^2 = \ell/1.63\ m/s^2$$
$$(.0253 s^2)(1.63\ \tfrac{m}{s^2}) = \ell$$
$$\boxed{\ell = 0.0412\ m}$$

This is roughly one sixth the previous value, supporting this argument above.

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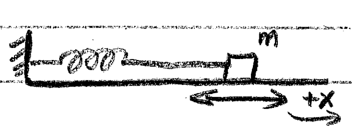
$\boxed{\text{Problem 2-2}}$



i) Common sense tells us that kinetic energy will be at a maximum where the mass is moving fastest. Thinking about this, we might conclude that the highest speed is reached when no force acts on the mass - if a force is speeding the mass up, it has not yet reached top speed; if a force is slowing the mass down, it must have been at top speed some time ago, before the slowing-down began.

This brings us back to a point you've most likely heard in class: speed of a harmonic oscillator is highest at $\boxed{x=0}$! (Here $F = kx = 0$ for $x=0$).

ii) Potential energy is at a maximum when the spring is fully compressed or fully stretched: when the mass is as far from $x=0$ as possible.

Look at the equation: $x = (0.05\,m)\cos(\omega t)$.

It's mathematically impossible for cosine of anything to be bigger than one, or less than -1. Therefore, the biggest x can be is $+0.05\,m$, and the smallest it can be is $-0.05\,m$

Therefore, it is at these maximum and minimum displacements of $\pm 0.05\,m$ that we see the greatest potential energy.

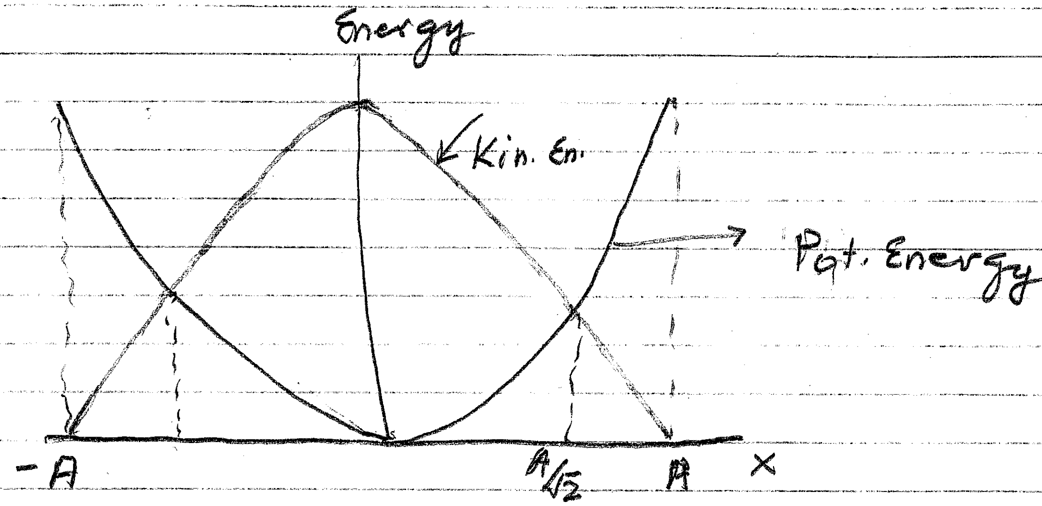
iii) Total energy is $E = \overbrace{\tfrac{1}{2}mv^2}^{\text{kinetic}} + \overbrace{\tfrac{1}{2}kx^2}^{\text{spring potential}}$. Total energy is conserved. At $x = x_{max} = 0.05\,m$, all the energy is potential, so we can say $E = \frac{1}{2}k(0.05\,m)^2$. Now, if kinetic and potential energy are equal at some point, $\frac{1}{2}mv^2 = \frac{1}{2}kx^2 \Rightarrow E = 2(\frac{1}{2}kx^2)$.

So at this point, we can put the second and third equations together: $2(\frac{1}{2}kx^2) = \frac{1}{2}k(0.05\,m)^2$

$$2x^2 = (0.05\,m)^2 \Rightarrow \boxed{x = \pm 0.0353\,m}$$

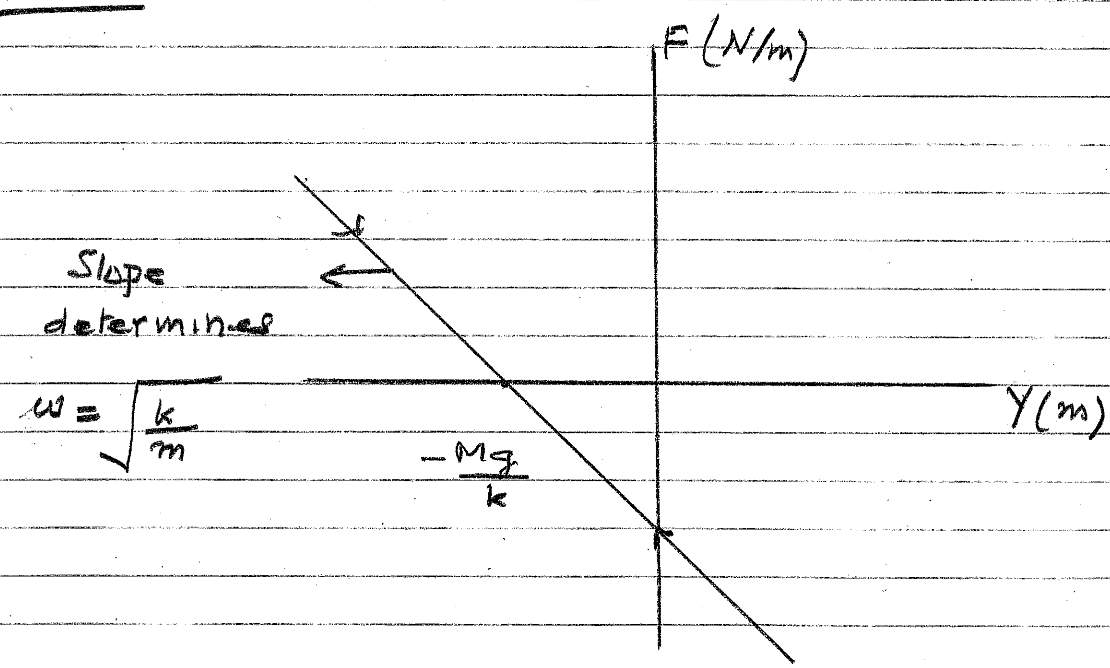
Prob 2-2

Picture.



$$\frac{1}{2} k A^2 = \frac{1}{2} k x^2 + \frac{1}{2} M V^2$$

Prob 2-3. Picture



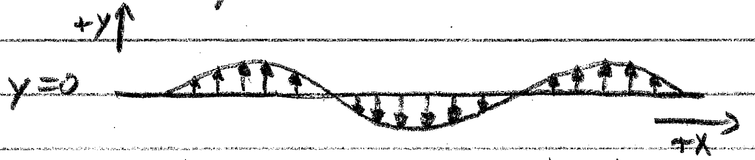
Equilibrium point $-Mg\hat{y} - ky\hat{y} = 0$

$$y = -\frac{Mg}{k}$$

Problem 2-3 If we hang the spring from the ceiling, its resonant frequency will <u>not</u> change. While the force pulling down on the mass affects where the mass will 'hang' in equilibrium, the force of gravity is effectively constant for any practical displacement of the spring.

Another, probably less useful but more rigorous explanation: ω for a spring is given by $\omega = \sqrt{k/m}$. 'g' is nowhere to be found in there; the spring's frequency doesn't care what g is.

Problem 2-4 i) Since the displacement is a function of x, but it points in the y-direction, the oscillation will look like:



$y = 0.01\sin(6.28x - 15.6t)\,\hat{y}$

This is a transverse wave - displacement is perpendicular to the line of travel.

ii) $k = 6.28\,\text{m}^{-1}$ from the equation. $\lambda = \frac{2\pi}{k} \approx 1.00\,\text{m}$

$\omega = 12.56\,\text{rad/s}$ from the equation

OR

$f = \frac{\omega}{2\pi} = 2.00\,\text{Hz}$ from the equation

$v = \omega/k = 2.00\,\text{m/s}$

Problem 2-5 i) Since P is proportionate to A^2 (henceforth: $P \sim A^2$), doubling A should increase P by a factor of $2^2 = 4$.
P will quadruple.

ii) $P \sim \omega^2$, so if we halve ω, P changes by a factor of $(\frac{1}{2})^2 = \frac{1}{4}$.
P will become one quarter of what it was.

iii) There's a hidden T here. $P \sim T/v$, or $\frac{T}{\sqrt{T/\mu}}$. So when we cancel, we get $P \sim \sqrt{T}$.
So tripling T increases P by a factor of $\sqrt{3}$.

Problem 2-6 i) ω and ω' have to be equal - they will 'match up'. This is simply a question of the numbers of waves - if each wave makes it from one string to the other, the number of waves per second must be the same on each string.*

ii) Since $v = \omega/k$, $v' = \omega'/k'$, and $\omega = \omega'$, we can show that the difference between k and k' is defined by

$$\frac{v}{k} = \omega = \frac{v'}{k'}$$
$$k'v = v'k$$
$$k' = \left(\frac{v'}{v}\right)k$$

Differences in k (and wavelength) come from changes in the speed waves travel at in different media.

iii) $\dfrac{A_r}{A_i} = \dfrac{v-v'}{v+v'}$ $\quad \dfrac{A_f}{A_i} = \dfrac{2v'}{v+v'}$ If $v' \ll v$, then...

$\dfrac{A_f}{A_i}$ becomes very small, while A_r/A_i becomes almost one. We obtain (as mentioned in the lecture notes),

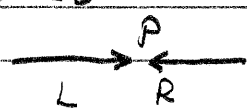
$$A_r \simeq A_i$$

$y_i = A_i \sin(kx + \omega t)$ $\quad y_r \simeq A_i \sin(kx - \omega t)$

at $x = 0$, $y_i = A_i \sin(\omega t + 0) \longrightarrow$ phase change

$y_r = -A_i \sin \omega t = A_i \sin(\omega t + \pi)$

* Hold your left and right hand index fingers like so



If you want them to move up and down so that there is NO break at P, they must both have the same frequency.

Problem 2-7 We have $y_i = A_i \sin(kx + \omega t)$ and $y_r = A_i \sin(kx - \omega t)$.
At $x=0$, $y_i = A_i \sin(\omega t)$, $y_r = A_i \sin(-\omega t)$.
When we superimpose, since $\sin(\text{blah}) = -\sin(-\text{blah})$, that means that

$$y_i(x=0) + y_r(x=0) = 0.$$

The waves cancel out, and at $x=0$ the displacement is permanently zero.

In general, waves superimpose to

$$y_i + y_r = A_i [\sin(kx+\omega t) + \sin(kx - \omega t)]$$

$k = 2\pi/\lambda$
$\lambda = 2\pi/k$
$\frac{n\lambda}{2} = n\pi/k$

We use the trig identity: $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$.

So $y_i + y_r = A_i \{[\sin(kx)\cos(\omega t) + \cos(kx)\sin(\omega t)] + [\sin(kx)\cos(\omega t) - \cos(kx)\sin(\omega t)]\}$

cancel

$$\Downarrow$$

$$y_i + y_r = A_i [2\sin(kx)\cos(\omega t)]$$
$$= 2A_i \sin\left(\frac{2\pi x}{\lambda}\right)\cos(\omega t).$$

If $x = 0, \lambda/2, \lambda, 3\lambda/2, \ldots$ then the sine will simply be zero, since $\sin(n\pi) = 0$ for integer n.
If $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$ then the sine will be $\sin(\pi/2)$, $\sin(3\pi/2)$, and so on. At these points, $y_i + y_r = \pm 2A_i\cos(\omega t)$, giving the double amplitude of an antinode.

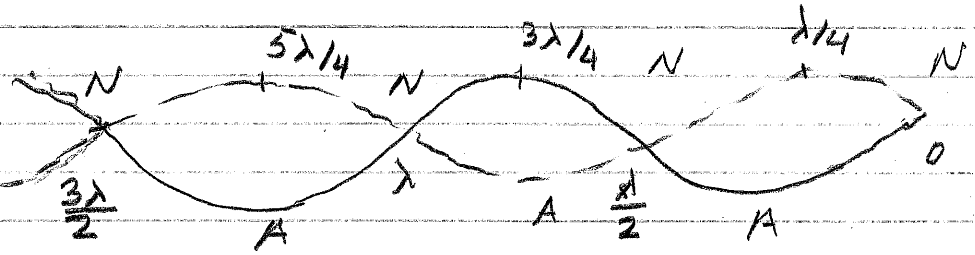
Problem 2-8 Wave velocity is $v = \sqrt{T/\mu} = \sqrt{\frac{1000\text{ N}}{0.001\text{ kg/m}}} = \sqrt{10^6 \frac{m^2}{s^2}}$

$$v = 10^3 \text{ m/s}$$

Now, at the third harmonic, $\lambda = \frac{2}{3}(1\text{m}) = 0.667\text{m}$; at the fifth, $\lambda = \frac{2}{5}(1\text{m}) = 0.4\text{m}$ - this can be illustrated by a scale diagram of the standing wave. $v = \lambda f$ thus gives us, for the third harmonic, $f_3 = \frac{1000\text{ m/s}}{0.667\text{m}} = 1500 \text{ 1/s}$
" " fifth " , $f_5 = \frac{1000\text{ m/s}}{0.400\text{m}} = 2500 \text{ 1/s}$

Likewise, $\omega = 2\pi f$, so $\omega_3 = 9425 \text{ 1/s}$
$\omega_5 = 15708 \text{ 1/s}$

Prob 2-7 Picture



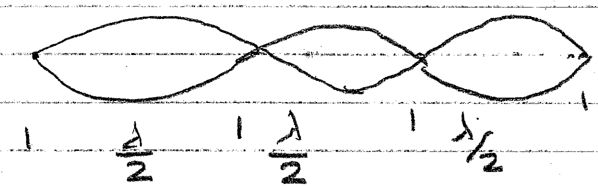
N → Nodes: No motion at any time

A → Antinodes – Motion with double the amplitude

Prob 2 8

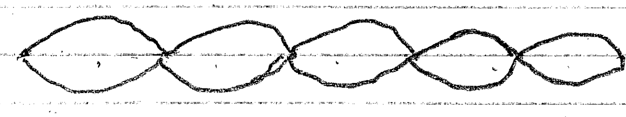
Third harmonic

3 half wavelengths



$$\frac{3\lambda_3}{2} = L$$

FIFTH HARMONIC



$$\frac{5\lambda_5}{2} = L$$

5 half wavelengths.

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Problem 2-9 Crucial point about the Experiment is that the FREQUENCY IS FIXED so to get different λ's you must change the speed because $\lambda = V/f$. so $\lambda_n = V_n/f$.

a) Looking at an equation of the form $f_n = \frac{n}{2L}\sqrt{\frac{F}{\mu}}$ (from lecture notes)

Suppose we keep f fixed, L fixed, and μ fixed, and want a low-n node. Since only n and F can change, and we want n to be small, F must be large.

The fact that we are varying F, but not f, to get different modes is very important to understanding the lab.

b) To make n 3 times larger, we need $\sqrt{F}$ to be one third the size, for balance. If

$$\sqrt{F_{new}} = \frac{1}{3}\sqrt{F_{old}}, \quad \text{then}$$

$$F_{new} = \left(\frac{1}{3}\right)^2 F_{old}$$

$$F_{new} = F_{old}/9$$

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Problem 2-10 $I = \frac{1}{2} s_m^2 \omega^2 \frac{\gamma P_0}{v_s}$

$$s_m^2 = \frac{2I}{\omega^2}\frac{v_s}{\gamma P_0} = \frac{2(10^{-12}\ W/m^2)(330\ \frac{m}{s})}{\omega^2(1.4)(10^5\ N/m^2)} \qquad \frac{W \cdot m/s}{N} = \frac{s^2}{kg \cdot m} \cdot \frac{kg \cdot m^2}{s^2} \cdot \frac{m}{s}$$

$$s_m^2 = \frac{1}{\omega^2}(4.71 \times 10^{-15}\ m^2)$$

$$s_m = (6.87 \times 10^{-8}\ m)\frac{1}{\omega} \quad \text{or}$$

$$\omega s_m = 6.87 \times 10^{-8}\ m$$

Without a given fixed value for ω, that's the best that can be done.

→ Assume $\omega = 10^3$ rad/s.

$$S_m = 6.87 \times 10^{-11}\ m$$

Which is roughly the size of the hydrogen atom. Now you know how powerful your ear is!

Problem 2-11 Pressure occurs where the volume of the gas is being changed. If all molecules move at the same pace, and aren't being pulled apart (low pressure zone) or packed together (high pressure zone), then there will be no change in pressure.

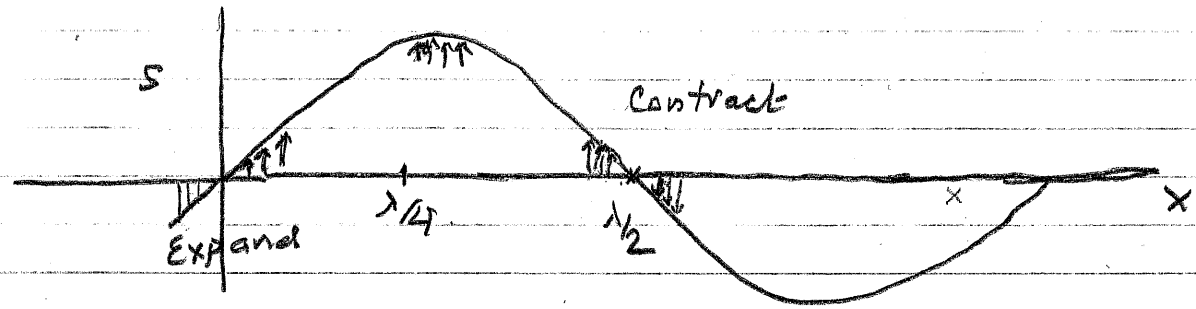
In a displacement wave, the regions where pulling apart and packing together occur at places where the displacement is positive on one side, negative at the other, and therefore zero at the actual point. So pressure maxima and minima occur at zero displacement.

(See Next page)

Problem 2-11 (Picture).

Draw displacement wave at $t=0$.

$$S = S_m \sin \frac{2\pi x}{\lambda}$$



Near $x=0$ displacements look like.

← ← ← → → →

Zero at $x=0$, +ive and increasing with x
−ive and increasing with $-x$
Gas is Expanding maximally, pressure will be lowered the most.

displ → 0.
ΔP → Max. −ive

at $\lambda/4$, all displacements nearly equal to S_m

↑ ↑ ↑ ↑

No change in Vol. — $\Delta P \to 0$

displ. maximum
$\Delta P = 0.$

→ → → × ← ← ←

at $x = \lambda/2$ Displ. positive reduce as x-increase
Displ. −ive increases as x-increase
Gas is contracting maximally, Pressure drops
displ → 0
ΔP → Max +ive

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Problem 2-12

Classically, we define 'sound' as lying in the range between 20 Hz and 20000 Hz. This is, theoretically, based on the range of human hearing (although many people can't hear the high end of this range).

The physics of vibrations of air below 20 Hz (infrasound) and above 20 kHz (ultrasound) is the same, by and large; all that changes is that you won't be able to hear it anymore.

Given $v = \lambda f$, we can easily calculate:

$$\lambda = v/f = (330\,m/s)/f$$

For 20 Hz, $\lambda = 16.5\,m$

For 20000 Hz, $\lambda = .0165\,m = 16.5\,mm$

So, figure on a wavelength range of 16.5m to 16.5mm.

Prob 2-53

In a sound wave in the gas displacement varies with position so volume changes. Of course if volume changes pressure must change. To determine this we must know the way pressure and volume are related. Since the frequencies are high it is not possible for heat to flow to maintain thermodynamic equilibrium. Hence sound becomes an ADIABATIC PROCESS. ($DQ \equiv 0$). Consequence is that the pressure-volume equation is

$$PV^{\gamma} = \text{Const.}$$

where $\gamma = \dfrac{C_p}{C_V}$, $\gamma > 1$ always.

C_p = sp. ht. at const. vol.

C_V = sp. ht at const pressure

Problem 2-14 "mp" here should be m_p, the mass of the proton $m_p = 1.67 \times 10^{-27}$ kg.

For helium, $\gamma = \frac{5}{3}$, $m = 4m_p = 6.68 \times 10^{-27}$ kg,
" air, $\gamma = 7/5$, average $m \simeq 30 m_p = 5.01 \times 10^{-26}$ kg

Take the equation and abstract out $k_B T$ (which is not given). v_s is <u>proportional to</u> $\sqrt{\gamma/m}$.

For helium, $\sqrt{\gamma/m} = \sqrt{\frac{5/3}{4m_p}} = \sqrt{\frac{5}{12m_p}}$

For air, $\sqrt{\gamma/m} = \sqrt{\frac{7/5}{30m_p}} = \sqrt{\frac{7}{150m_p}}$

So we can say that the speed of sound in helium is larger than in air, as $\sqrt{5/12}$ is larger than $\sqrt{7/150}$.

Problem 2-15 $v_s = \sqrt{\frac{\gamma k_B T}{m}}$ $\quad v_{rms} = \sqrt{\frac{3k_B T}{m}}$

$$\frac{v_{rms}}{v_s} = \frac{\sqrt{3k_BT/m}}{\sqrt{\gamma k_BT/m}}$$

Cancel: $\frac{v_{rms}}{v_s} = \frac{\sqrt{3}}{\sqrt{\gamma}}$

$$\gamma v_{rms}^2 = 3v_s^2$$

or

$$v_s^2 = \frac{\gamma}{3} v_{rms}^2$$

or

$$v_s = \sqrt{\gamma/3}\, v_{rms}$$