SOLUTIONS - WK 14.

14-1 Both sound waves as well as light waves diffract and spread out when they go through an opening. For a narrow opening (slit) the spread angle is given by

$$\sin\theta_1 = \frac{\lambda}{w}$$

where λ = wavelength
w = width of opening.

Sound has wavelengths between 17 mm and 17 m so θ_1 is large for large openings such as doors or windows, hence sound spreads while going around a corner. The wavelengths of light are 10^{-7} m. Therefore θ_1 is close to zero for openings of 1 m. Effectively no spreading. For $w \gg \lambda$ light travels in straight lines (Geometrical optics)

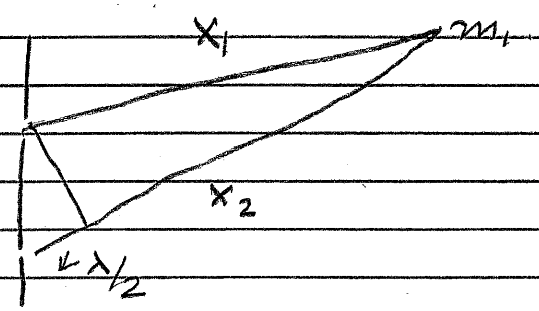
14-2 The separation between any two dark neighbors is $y_n - y_{n-1} = \frac{D\lambda}{d}$

Here $D = 1$ m
$\lambda = 589 \times 10^{-9}$ m
$d = 0.2$ mm $= 2 \times 10^{-4}$ m

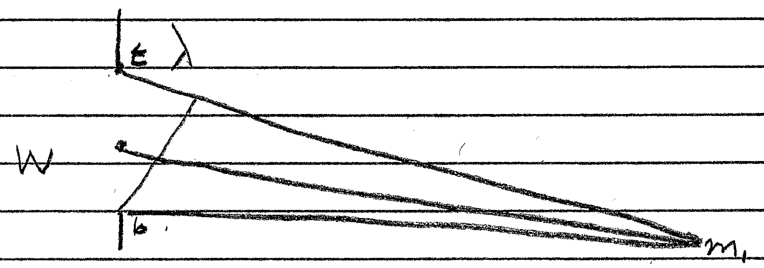
Hence $y_n - y_{n-1} = 2.95 \times 10^{-3}$ m [~ 3 mm]

14-3 Both involve superposition of waves. However, in interference one is dealing with a finite (2, 3, 4 ...) numbers of coherent sources while in diffraction essentially an infinite number of sources are effective.

14-4 In 2-Slit Interference one is dealing with 2 waves which start in phase. The first minimum occurs when the phase difference between them is π which happens when the path difference is $\frac{\lambda}{2}$

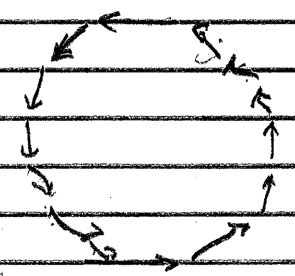


In Single slit diffraction, the first minimum occurs when all the waves starting from the slit add together to produce a zero.



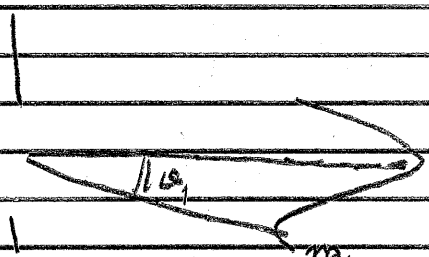
This will happen when the path difference between the wave from the top point t and that from the bottom point b is equal to λ, because in that case for every wave coming from the bottom half of the slit there is a wave coming from the top half which is $\lambda/2$ behind

So that they cancel one another. One can visualise this by drawing a ~~d~~ vector diagram representing the E vectors of the waves



The Resultant vector is **ZERO**!

14-5 $\sin\theta_1 = \frac{\lambda}{W}$

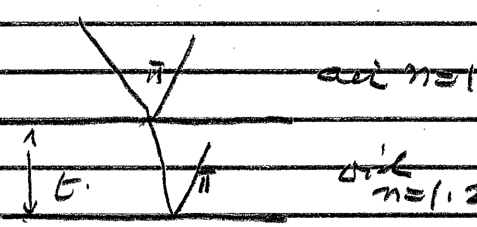


$\theta_1 \ll 1$ so $\sin\theta_1 = \theta_1$

$\theta_1(400nm) = 4\times10^{-3}$ radian

$\theta_1(700nm) = 7\times10^{-3}$ radian

14-6 There is a phase change of π for both reflections, therefore the condition for maxima is



$$2 n_{oil} t = m\lambda \quad m = 1, 2, 3.$$

$$1080\ nm = m\lambda$$

$m = 2, \quad \lambda = \underline{\underline{590}}\ nm.$ (Orange Color)

14-7 Again there is a phase change of π at each surface. To get destructive interference

$n = 1$ (air)

$n = 1.25$ (coating)

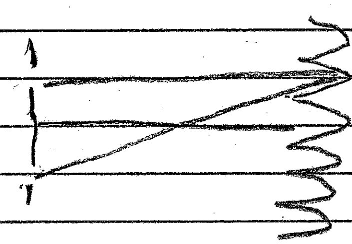
$n = 1.5$ (glass).

$$2 \times 1.25 \times t = \frac{\lambda}{2}$$

$$t = \frac{\lambda}{4 \times 1.25} = 120\ nm.$$

14-8 The Intensity on the screen is

$$I = \frac{1}{2} \epsilon_0 c\ 4E_m^2 \cos^2\left(\frac{\phi_1 - \phi_2}{2}\right).$$



when

$$(\phi_1 - \phi_2) = 2m\pi$$

Taking average.

$$\langle I \rangle = \frac{1}{2} \epsilon_0 c\ 4E_m^2 \left\langle \cos^2\frac{(\phi_1 - \phi_2)}{2} \right\rangle$$

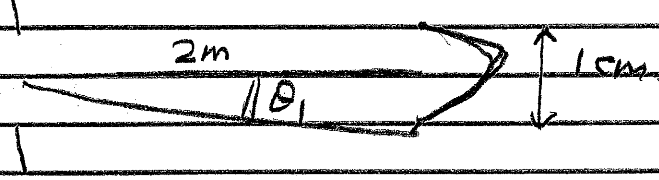
We know that $\langle \cos^2 (\ ) \rangle = \frac{1}{2}$.

so

$$\langle I \rangle = \frac{1}{2} \epsilon_0 c\ 2E_m^2$$

which is twice the intensity due to one source of amplitude E_m.

14-9 Again, angles are small so



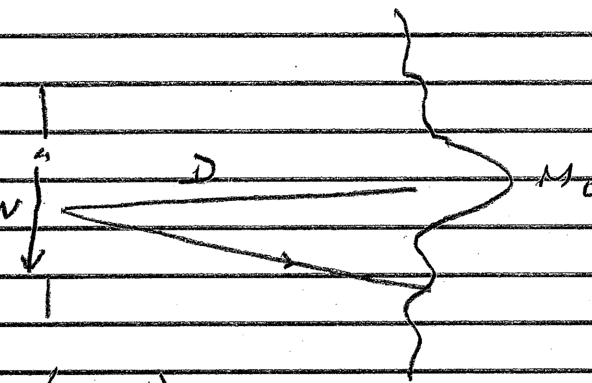
$$\sin\theta_1 = \theta_1 = \frac{\lambda}{W}$$

$$\frac{0.5}{200} = \frac{\lambda}{W}$$

$$W = \frac{200\lambda}{0.5} = 400 \times 436 \times 10^{-9}\ \text{m.}$$

$$= 1.74 \times 10^{-4}\ \text{m.}$$

14-10 The single slit diffraction pattern is produced by the superposition of a very large number N ($\gg 1$) of waves all of which start from the slit in phase. In order to calculate the intensity produced by them on a screen we can think of N, E vector each of magnitude E_m arriving at the screen.

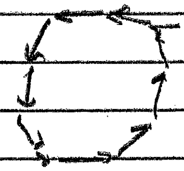


For the main maximum they are all in phase [Maximum path difference is $\frac{W^2}{4D}$ which is much smaller than λ] so we get the total vector

$$\rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \rightarrow \quad NE_m$$

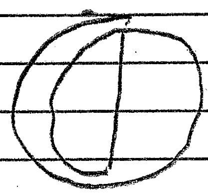
and Intensity is proportional to $N^2E_m^2$

For first Minimum [see prob 14-4] the vectors form a closed loop



Total $E = 0$

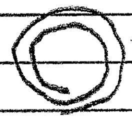
Next maximum the "string" of length NE_m winds further to produce a diameter D_1



$$\frac{3\pi}{2} D_1 = NE_m$$

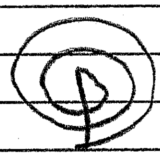
So amplitude $D_1 = \frac{2}{3\pi} NE_m$ Intensity prop. to $\frac{4}{9\pi^2} N^2 E_m^2$

2nd Minimum : Circle closes twice



$E = 0$

Next, maximum, wind 2.5 times



$$\frac{5\pi}{2} D_2 = NE_m$$

$D_2 = \frac{2}{5\pi} NE_m$ Intensity proportional to $\frac{4}{25\pi^2} N^2 E_m^2$

and so on.