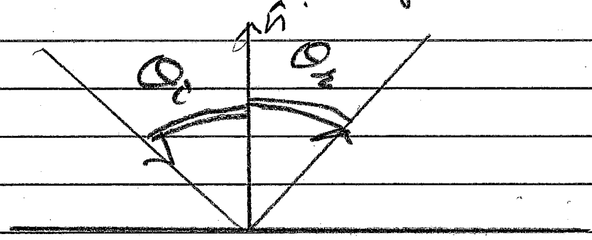
FORMULAE FOR WK12

GEOMETRICAL OPTICS

ALL OPENINGS/OBSTACLES LARGE COMPARED TO WAVELENGTH OF LIGHT: TRANSVERSE EM WAVE, speed in Vac. $c = 3 \times 10^8 m/s$, Wavelengths in Vacuum $400nm < \lambda < 700nm$.

Reflection All angles measured from normal to surface.



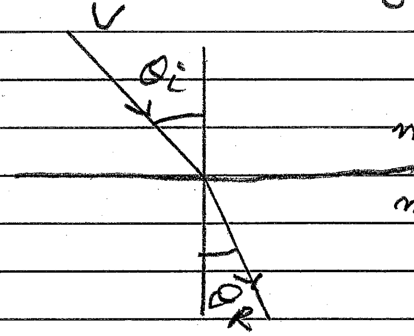
FERMAT'S PRINCIPLE GIVES:

$$\theta_r = \theta_i$$

Refraction Refractive Index

$$n = \frac{\text{speed in Vac}}{\text{speed in medium}} = \frac{c}{v} \quad \text{always} \geq 1.$$

Snell's law



$$n_2 \sin\theta_R = n_1 \sin\theta_i$$

If $n_2 < n_1$ $\theta_R > \theta_i$ (Not shown)

Critical angle $\theta_R = \frac{\pi}{2}$ $\sin\theta_c = \frac{n_2}{n_1}$

MIRRORS Reflection only: Distances on Light side are "positive" on Dark side "negative"

Mirror Equation:

p = Object distance

q = image distance

r = radius of sphere from which mirror its cut:

PLANE $r \to \infty$

$q = -p \qquad m = 1 \quad [\text{magnification } m = -\frac{q}{p} = 1]$

All images virtual (q is -ive).

Concave r is +ive
paraxial rays.

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{r} = \frac{1}{f} \qquad f \to \text{focal length.}$$

$$m = -\frac{q}{p}.$$

All images real and inverted except when $p < f$ image is virtual (q -ive), and magnified $m > 1$), and upright.

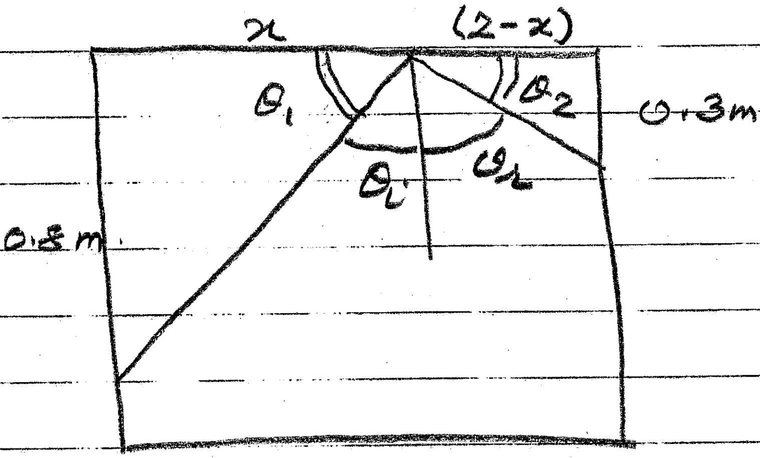
Convex r is -ive, f is -ive

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{r} = \frac{1}{f}$$

All images, virtual, upright and reduced.

WK-12 SOLUTIONS

12-1



$\theta_r = \theta_i$

hence

$\theta_2 = \theta_1$

$\tan\theta_2 = \tan\theta_1$

$$\frac{0.3}{2-x} = \frac{0.8}{x}$$

$$\frac{x}{2-x} = \frac{8}{3}$$

$3x = 16 - 8x$

$x = \frac{16}{11} = 1.45m$

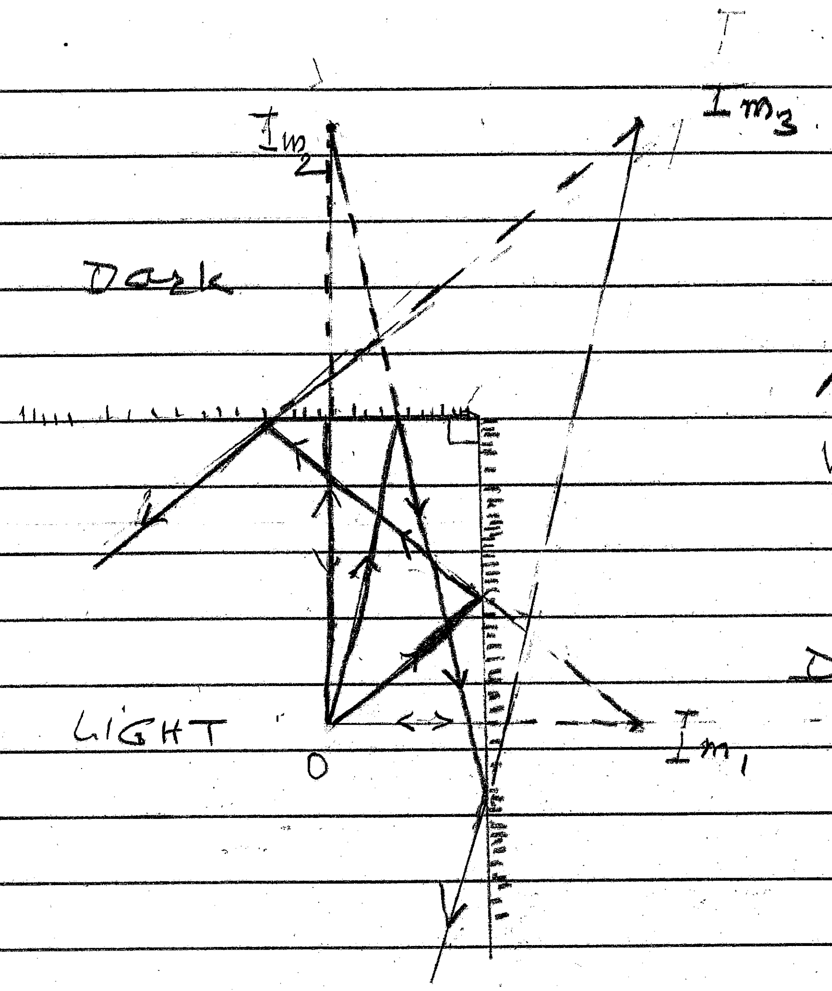
FORMULAE

$\theta_r = \theta_i$ $\qquad$ $n_i \sin\theta_i = \text{Constant}$

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{r} = \frac{1}{f}$$

$$m = -\frac{q}{p}$$

12-2



ALL IMAGES VIRTUAL

FORMS THREE IMAGES BECAUSE AFTER TWO REFLECTIONS LIGHT BECOMES ANTI-PARALLEL TO ITS ORIGINAL DIRECTION

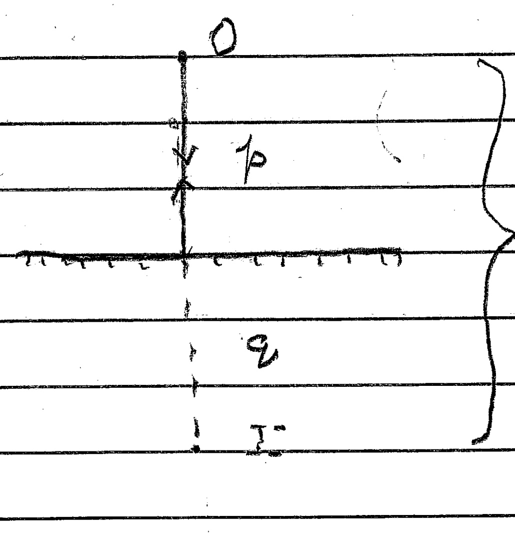
12-3 At all times $q = -p$.
so as $p = (1 - 4.9t^2)$ m

The OI distance varies as

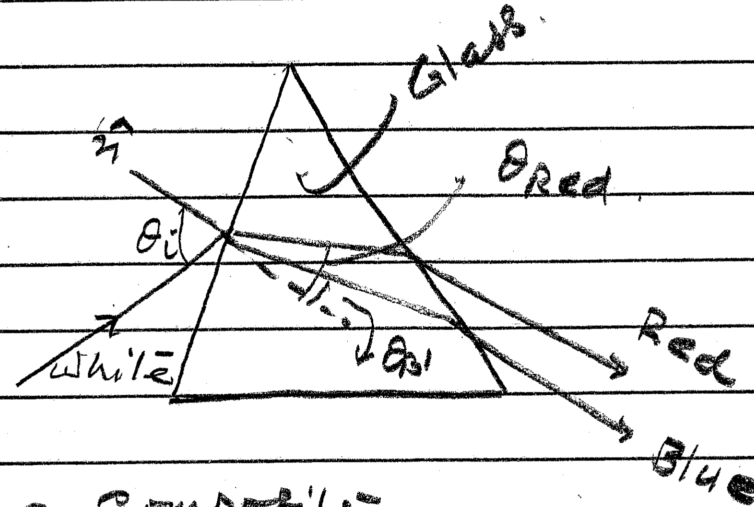
$d = (2 - 9.8t^2)$ m

where

$0 < t < 0.45$ sec.



12-4 5 Inferences drawn from Newton's Expts.



1. White light is a composite of many colors - Violet, Indigo, Blue, Green, Yellow, Orange, Red...

2. In air and vacuum speed of light is the same for all colors.

3. In glass speed of light varies with color so refractive index varies with color since

$$n = \frac{c}{V}.$$

4. For the refracted rays

$$\theta_{Red} > \theta_{Blue}$$

5. Both must satisfy Snell's law

$$n_{Red} \sin\theta_{Red} = n_{Bl} \sin\theta_{Bl} = n_{air} \sin\theta_i$$

so $\quad n_{Red} < n_{Bl}$

6. Since $n = \frac{c}{v}$

$v_{red} > v_{Bl}$ in Glass.

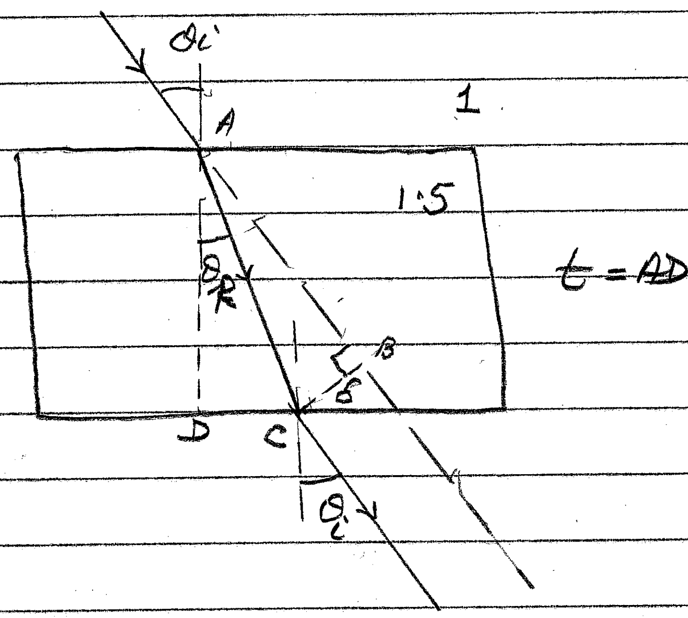
12-6

In ΔABC

$\angle BAC = (\theta_i - \theta_R)$

So

$\sin(\theta_i - \theta_R) = \frac{\delta}{AC} \rightarrow ①$



In ΔADC

$\angle CAD = \theta_R$ so

$\frac{AD}{AC} = \cos\theta_R \rightarrow ②$

Hence $\delta = AC \sin(\theta_i - \theta_R)$

$= \frac{t}{\cos\theta_R}\left[\sin\theta_i \cos\theta_R - \cos\theta_i \sin\theta_R\right]$

$= t \sin\theta_i \left[1 - \frac{\cos\theta_i \sin\theta_R}{\sin\theta_i \cos\theta_R}\right]$

$1.5 \sin\theta_R = \sin\theta_i$

$\frac{\sin\theta_R}{\sin\theta_i} = \frac{1}{1.5}$

$\delta = 10 \sin 30\left[1 - \frac{\cos 30}{1.5 \cos\theta_R}\right]$

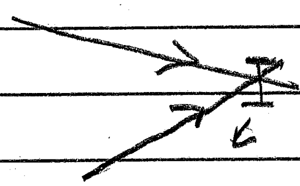
$\sin\theta_R = \frac{1}{2\times 1.5} = \frac{1}{3}$

$= 10 \times \frac{1}{2}\left[1 - \frac{0.866}{0.94 \times 1.5}\right] = 5[1 - 0.61]$

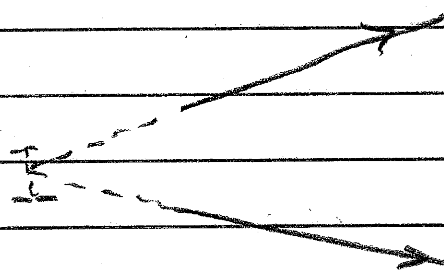
$\cos\theta_R = 0.94$

$= 1.92 cm$

12-7 Real Image is formed when light actually goes through the point where the Image is located so it can be projected on a screen.



Virtual Image is located by extrapolati-ng the rays. No light actually goes through that point, it only appears to come from that point

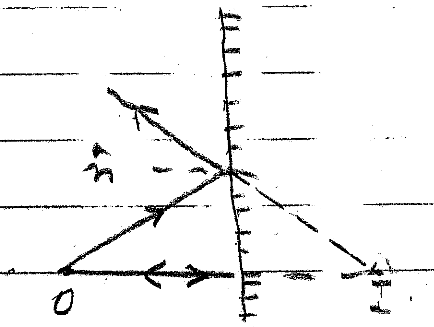


It cannot be projected on a screen!

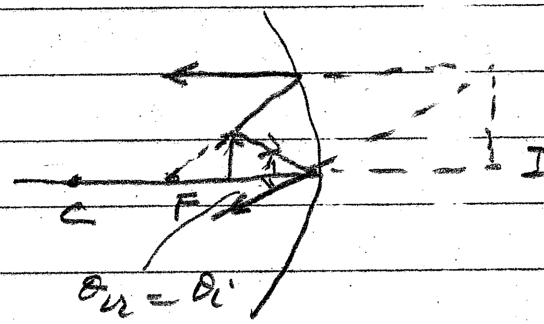
12-8 For real images q is positive, the Image is inverted m is -ive.
For virtual images q is -ive the image is upright, m is +ive.

12-9 $$\frac{1}{p} + \frac{1}{q} = \frac{2}{r}$$

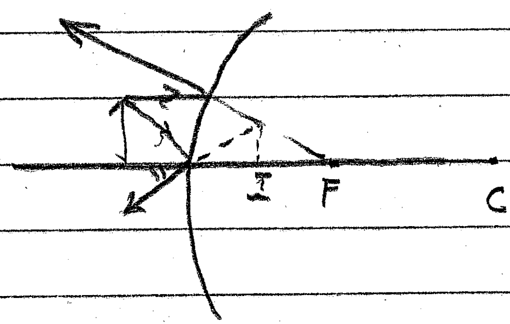
<u>PLANE MIRROR</u> $r \to \infty$, $q = -p$.
all Images are virtual, upright, same size as object.



<u>CONCAVE MIRROR</u> r is +ive.
As long as $p > f$ $\left[= \frac{r}{2}\right]$ all images are real and inverted
If $p < f$, virtual, upright, enhanced image



Convex Mirror r is −ive,
All Images, are virtual, upright, reduced and lie within focal point



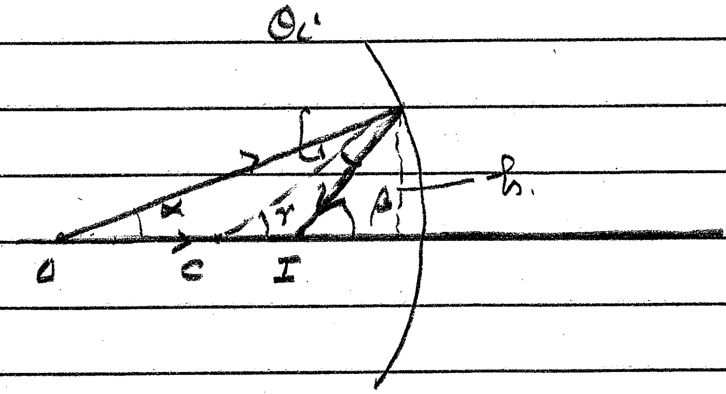
12-10

we write

$\beta = \gamma + \theta_r$

$\gamma = \alpha + \theta_i$

of course $\theta_r = \theta_i$



So $\beta = 2\gamma - \alpha$

$\beta + \alpha = 2\gamma$ — (1)

If rays are paraxial all angles are small and Eq (1) becomes

$\tan\beta + \tan\alpha = 2\tan\gamma$ or $\frac{h}{p} + \frac{h}{q} = \frac{2h}{r}$.

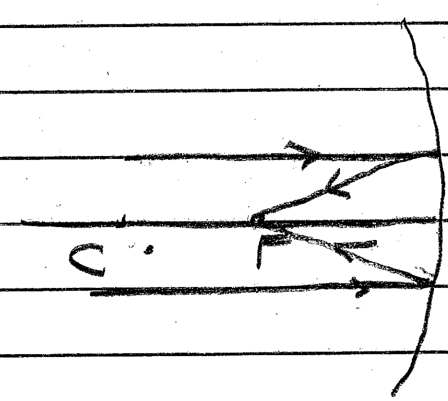
Otherwise, we can no longer get the simple eqn.

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{r}$$

12-11 Concave mirror

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{r}$$

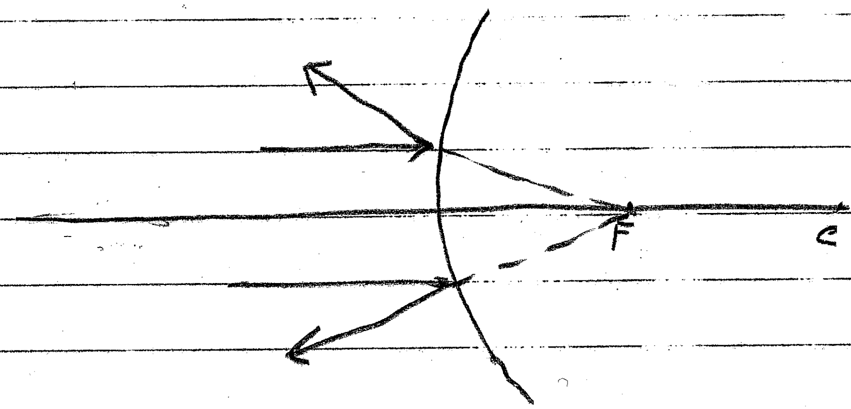
Smallest value of q is when $\frac{1}{p} = 0$ or $p \to \infty$, object very far. Then $q = \frac{r}{2} = f$, so Rl. image can never be closer than F.



12-12 i, No, for a convex mirror r is -ive so q has to be -ive.

ii) $\frac{1}{p} + \frac{1}{q} = \frac{2}{r}$

r -ive, q -ive, farthest will be when $\frac{1}{p} \to 0$ or $p \to \infty$

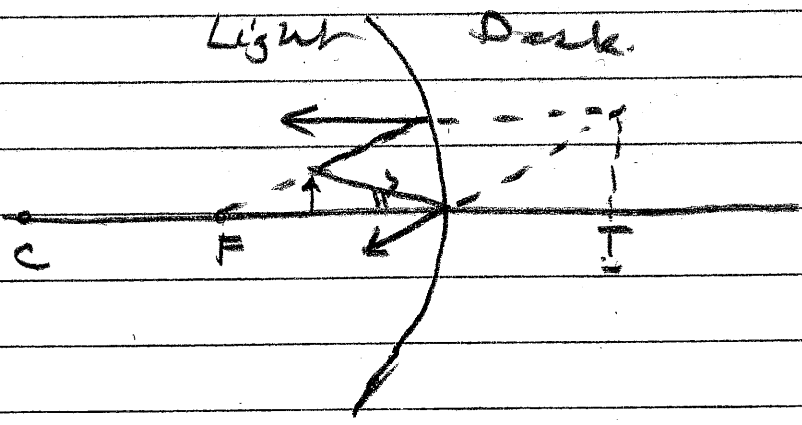


12-13 $\frac{1}{p} + \frac{1}{q} = \frac{2}{r} = \frac{1}{f}$

$m = \frac{-q}{p}$

To produce an upright image, q must be -ive so $p < f$.
$m > 1$

Virtual enlarged Image



12-14 $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$$m = -\frac{q}{p}$$

here $p = 20$ cm.

$m = -\frac{1}{2}$

so $q = 10$ cm.

$$\frac{1}{f} = \frac{1}{20} + \frac{1}{10} = \frac{3}{20}$$

$$f = \frac{20}{3} = 6.67 \text{ cm.}$$