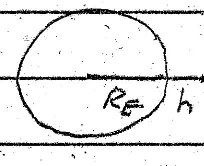
SOLUTIONS - WEEK 1.

1-1 No this is not a case of harmonic vibration. The only force acting on the Ball is its weight apart from the force during the elastic collision, (when its velocity reverses) with Earth.

1-2
$$P_g = 0.98\,J$$
$$P_{sp} = 0.95\,J$$

1-3
$$P_G = -\frac{GM_E M}{R_E + h}$$

$r = (R_E + h),\ h \ll R_E$



$$= -\frac{GM_E M}{R_E\left(1 + \frac{h}{R_E}\right)}$$

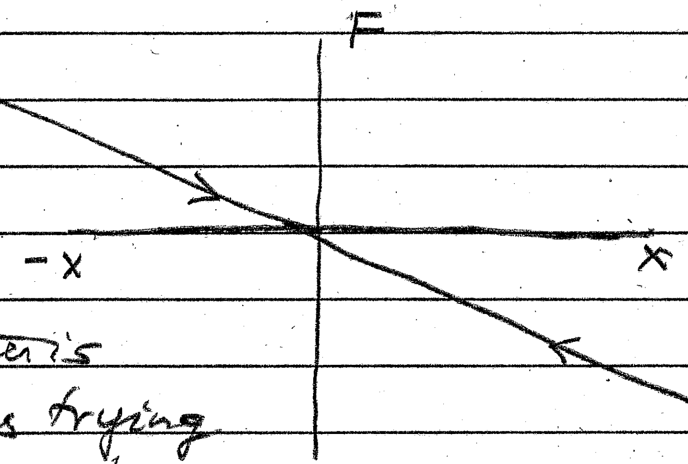
$$= -\frac{GM_E M}{R_E}\left(1 + \frac{h}{R_E}\right)^{-1}$$

$$= -\frac{GM_E M}{R_E}\left(1 - \frac{h}{R_E}\right)$$

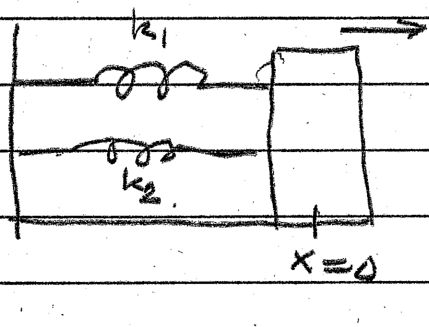
$$= -\frac{GM_E M}{R_E} + Mgh$$

Since $\frac{GM_E}{R_E^2} = g$

so Mgh is O.K. as long as we are interested only in changes of potential energy. Note: the large negative term keeps us firmly "tied" to the Earth!



1-4 The negative sign ensures that this is a "restoring force" That is, this force is always trying to bring the mass back to its equilibrium position at $x=0$. The oscillation goes on because at $x=0$, the velocity is finite and so the mass keeps moving and overshoots.



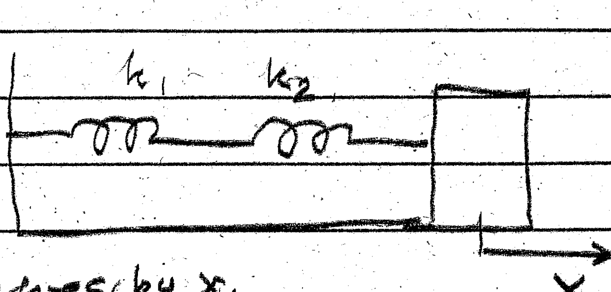
1-5 In this case when mass is displaced by x the total force

$$\underset{\sim}{F} = -k_1 x \hat{x} - k_2 x \hat{x}$$
$$= -(k_1 + k_2) x \hat{x}$$

so

$$k = k_1 + k_2 = 150 N/m$$

1-6 Now if displacement is x,



spring 1 stretches by x_1
and spring 2 stretches by x_2 so that

$$x_1 + x_2 = x$$

springs are in series so

$$F_1 = F_2 = F$$

so $\quad x_1 = \frac{F_1}{k_1} \quad , \quad x_2 = \frac{F_2}{k_2}$

$$x = (x_1 + x_2) = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$\vec{F} = -\frac{k_1 k_2}{k_1 + k_2}\, x\, \hat{x}$$

$$k_{eff} = 33.3\ N/m$$

1-7 75 beats in 60 secs

$$T = \frac{60}{75}\ sec = 0.8\ secs$$

frequency

$$f = \frac{1}{T} = 1.25\ Hz$$

1-8 $\omega = \sqrt{\frac{k}{m}}$ to double ω increase k by factor of 4.

$T_0 = 2\pi\sqrt{\frac{m}{k}}$, to double T_0 reduce k by factor of 4.

1-9 $\omega = \sqrt{\frac{k}{m}}$ To double ω reduce m by factor of 4.

$T = 2\pi\sqrt{\frac{m}{k}}$, To double T, increase m by factor of 4.

1-10 $x = A \sin(\omega t + \theta_0) = 0.01 \sin\left(12.56t + \frac{\pi}{6}\right)$ m.

amplitude $A = 0.01m$

frequency $f = \frac{\omega}{2\pi} = \frac{12.56}{6.28} = 2Hz$

phase $\theta_0 = \frac{\pi}{6}$

Maximum Vel. $V_{max} = A\omega = 0.01 \times 12.56 = 0.13m.$

$a_{max} = A\omega^2 = 0.01 \times (12.56)^2 = 1.58\ m/s^2$

1-11 Mass of Cu atom $= \frac{0.064}{6 \times 10^{23}}$ kg.

$\omega = \sqrt{\frac{k}{M}}$

$k = \omega^2 M = (2\pi)^2 \times 10^{26} \times \frac{0.064}{6 \times 10^{23}}$

$= 420 N/m$

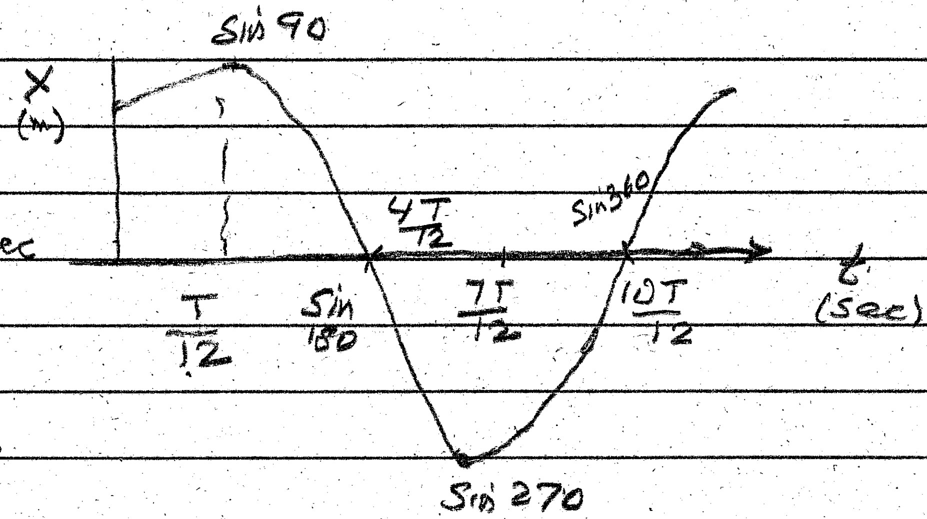
1-12 $\theta = \pi/3$

$T = 1\ sec.$

$x = A \sin(\omega t + \pi/3)$

$\omega = 2\pi$ radians

$\sin 60 = 0.866$



first max

$\frac{T}{12} = \frac{1}{12}$ sec

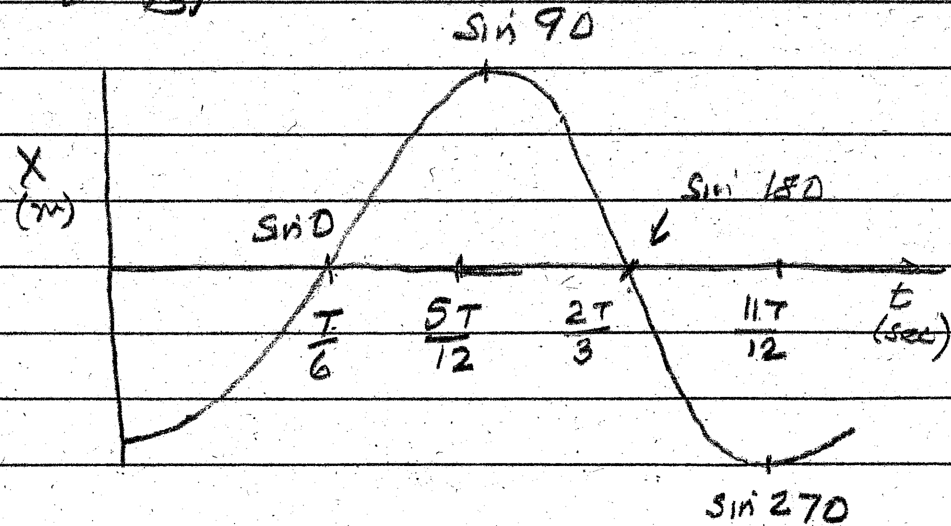
first zero

$T/3 = \frac{1}{3}$ sec

$\theta = -\pi/3$

$x = A \sin(\omega t - \pi/3)$

$\sin 60 = 0.866$



First zero

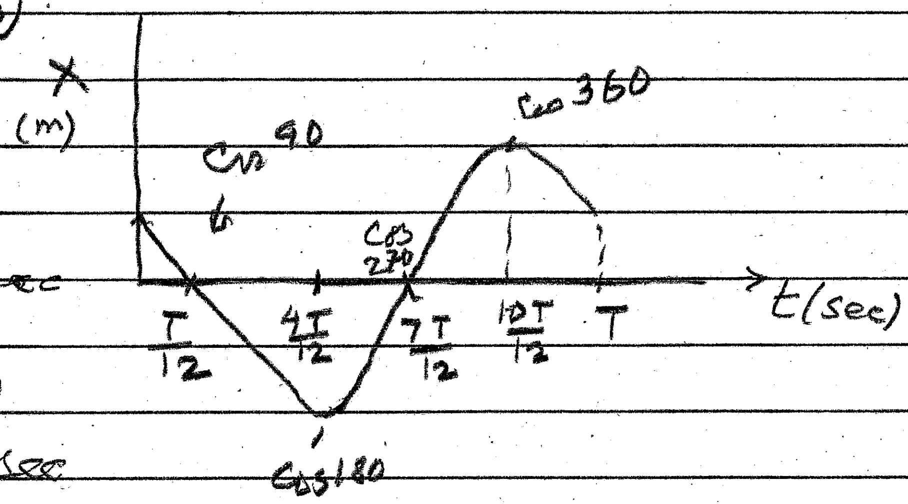
$T/6 = \frac{1}{6}$ sec

First max

$\frac{5T}{12} = \frac{5}{12}$ sec

1-13 $\theta = \frac{4\pi}{3}$ T = 1 sec, $\omega = 2\pi$ radians

$x = A\cos(\omega t + \pi/3)$



first zero

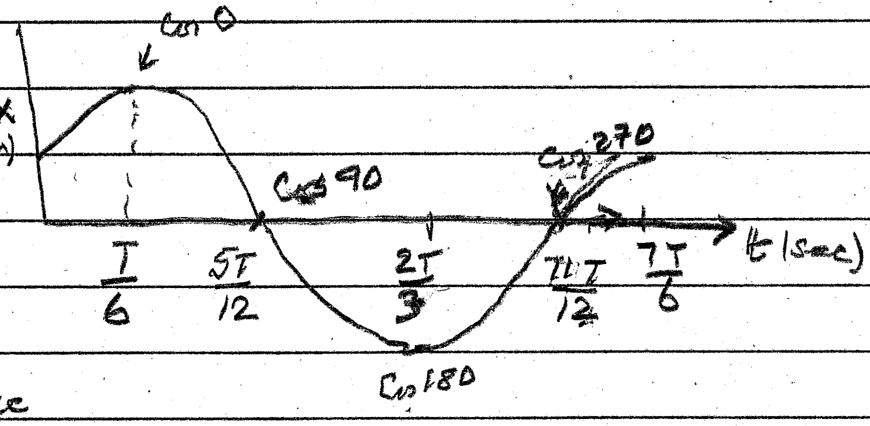
$\frac{T}{12} = \frac{1}{12}$ sec

first max (-ive)

$\frac{4T}{12} = \frac{1}{3}$ sec

$\theta = -\pi/3$

$x = A\cos(\omega t - \pi/3)$



first Max $\frac{T}{6} = \frac{1}{6}$ sec

first zero $\frac{5T}{12} = \frac{5}{12}$ sec

1-14 To solve this problem one needs to understand the working of a grandfather clock which consists of a pendulum and a mechanism so that for each period of the pendulum the second hand moves by a pre-determined amount — the click.

Supposing the click is 1 sec. that is in each period the hand will advance by 1 sec.

If the period $T = 2\pi\sqrt{\frac{l}{g}}$ is also one second the clock will indicate the correct time.

If $T > 1$ sec. then clock seconds 1 sec when the actual time elapsed is larger so it will lose some time.

If it loses 1 min in every hour it loses 60 secs in 3600 secs.

That is 1 part in 60.

To make it run true you will have to reduce its period by 1 part in 60.

That is T must go to $T\left(1-\frac{1}{60}\right)$.

So you must reduce the length to l' where

$$\sqrt{\frac{l'}{l}} = 1-\frac{1}{60}$$

$$\frac{l'}{l} = \left(1-\frac{1}{60}\right)^2 \doteq 1-\frac{1}{30}$$