

Name: SOLUTION

(Sign in ink, print in pencil)

Notes

1. There are four (4) problems in this exam. Please make sure that your copy has all of them.
2. Please show your work, indicating clearly what formula you used and what the symbols mean. Just writing the answer will not get you full credit. In stating vectors, give both magnitude and direction.
3. Write your answers on the sheets provided.
4. Do not forget to write the units.
5. Do not hesitate to ask for clarification at any time during the exam. You may buy a formula at the cost of one point.

Take Care! God Bless You!

$$k_e = 9 \times 10^9 \frac{N \cdot m^2}{C^2}$$

$$\epsilon_0 = 9 \times 10^{-12} F/m$$

$$\text{Mass of proton} \quad m_p = 1.6 \times 10^{-27} \text{ kg}$$

$$\text{Mass of electron} \quad m_e = 9 \times 10^{-31} \text{ kg}$$

$$\text{Elementary Charge} \quad e = 1.6 \times 10^{-19} C$$

$$\mu_0 = 4\pi \times 10^{-7} H/m$$

NO CALCULATORS!

Problem 1a

Write down the Faraday-Lenz Law in your own words and define the sum on the left of the equation precisely. (5)

A NON-COULOMB \vec{E} -field appears in every loop surrounding a region where flux of \vec{B} is changing with time.

$$\sum_{C \rightarrow} \vec{E}_{\text{nc}} \cdot d\vec{l} = - \frac{\Delta \Phi_B}{\Delta t}$$

The quantity on the left is the circulation of \vec{E}_{nc} around a closed loop. It is also the emf in the loop.

The minus sign on the right ensures that \vec{E}_{nc} opposes the change in flux of \vec{B} .

Problem 1b

Show that in an AC generator the emf is maximum when the flux of \vec{B} through the coil is zero and vice versa. (5)

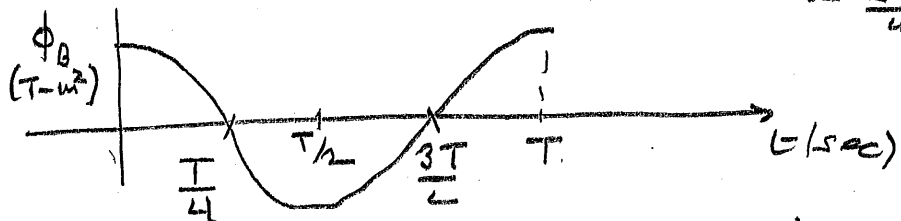
In an ac generator is rotated at a constant angular velocity ω in the presence of a \vec{B} field.



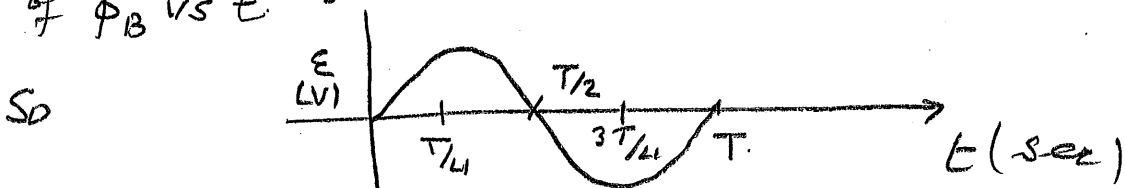
The rotation is about an axis perpendicular to \vec{B} so that the flux of \vec{B} through the coil is

$$\Phi_B = BA \cos(\hat{n}, \vec{B}) = BA \cos \omega t$$

$$\text{Period } T = \frac{2\pi}{\omega}$$

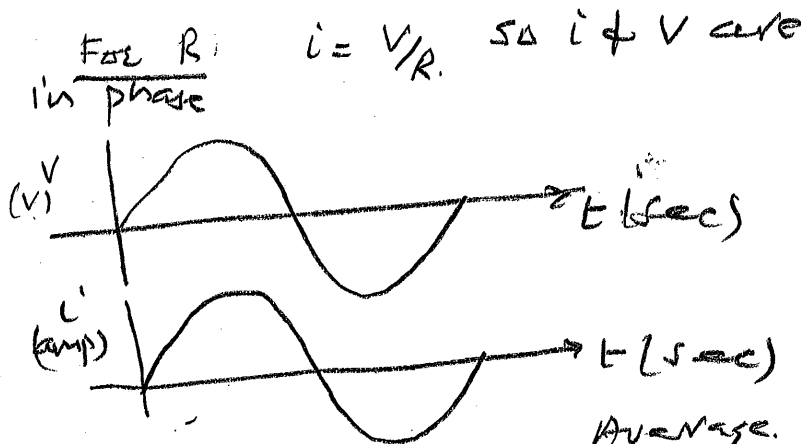
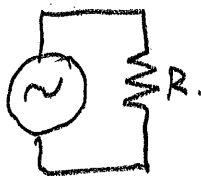


The emf $= - \frac{\Delta \Phi_B}{\Delta t}$ is the negative of the slope of Φ_B vs t .



Problem 1c

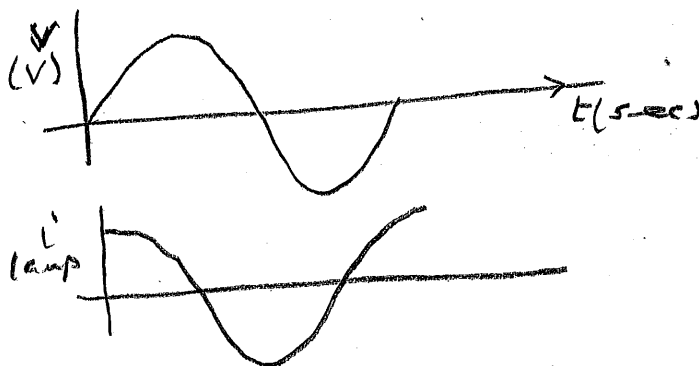
Explain why when you connect R to an AC generator it absorbs energy but if you connect C (Capacitor) or Inductor (L) there is no absorption on the average. (15)



Power $P_W = iV \sim \sin^2 \omega t$
 $\langle P_W \rangle$ is non-zero b/c $\langle \sin^2 \omega t \rangle = 1/2$. Average.



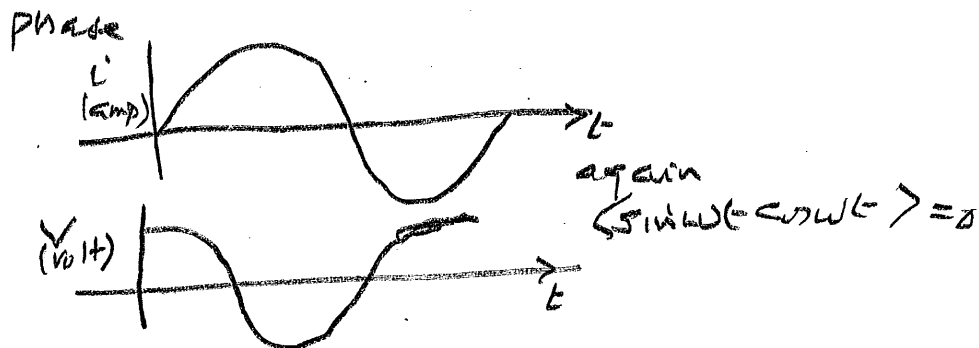
$q = CV$ so q & V are in phase
 $i = \frac{\Delta q}{\Delta t}$ so



$P_W = iV \sim \sin \omega t \cos \omega t$
 $\langle \sin \omega t \cos \omega t \rangle = 0$



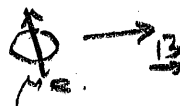
$\mathcal{E} - L \frac{\Delta i}{\Delta t} = 0$ so V and $\frac{\Delta i}{\Delta t}$ are in phase



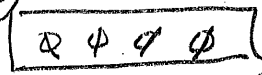
Problem 2a

What is a bar magnet? Discuss the various conceptual steps that take us from a single electron ($\mu_e = 9.2 \times 10^{-24} \text{ N-m/T}$) to a store bought bar magnet. (15)

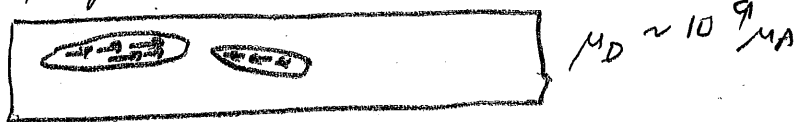
A bar magnet is any object which experiences a torque when placed in a \vec{B} field $\vec{\tau} = [\vec{\mu} \times \vec{B}]$, μ is magnetic moment.

Electron $\mu_e = 9.2 \times 10^{-24} \frac{\text{N-m}}{\text{T}}$ will feel $9.2 \times 10^{-24} \text{ N-m}$ in one Tesla 

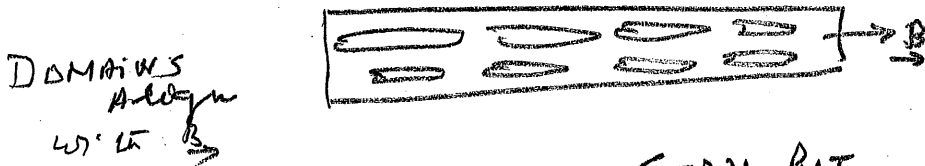
Atom Many Electrons, arrange that μ 's don't cancel $\Phi \Phi \Phi \Phi \Phi \mu_A = 5 \mu_e$
atom is a bar magnet

Put μ_A 's in a solid. At high thermal effects make $\langle \mu_A \rangle = 0$. 

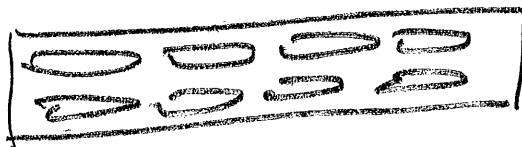
At low T in some materials a new force (Exchange) comes into play and aligns μ_A 's to form Domains



μ_D 's large. Need a solid in which μ_D 's have a preferred direction, say along x, Apply $\vec{B} \parallel \vec{x}$.



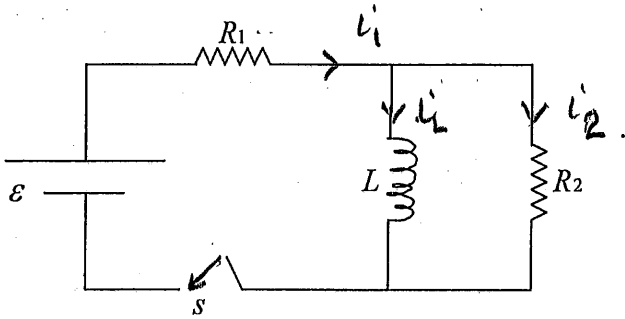
Remove \vec{B} DOMAINS STAY PUT



Store Bought bar magnet

Problem 2b

In the circuit shown, $R_1 = R_2 = 10\Omega$, $L = 1\text{mh}$, and $\varepsilon = 20\text{V}$. The switch is closed at $t = 0$. What is the current in the circuit (i) immediately after s is closed (ii) a long time later? (10)



Properties of an Inductor: In an L-R circuit
 (i) Current in L is zero when switch is just closed, (ii) Current in L is constant a long time later so $V_L = 0$.

$$\left[\varepsilon = -L \frac{\Delta i}{\Delta t} \right]$$

Apply Jn Rule

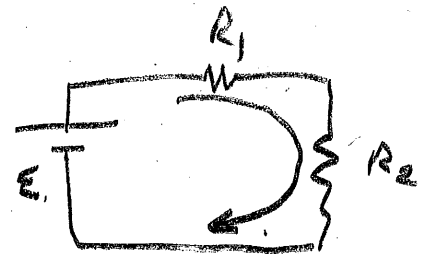
$$i_L + i_2 = i_1$$

Loop Rule

$$V_{R_2} - V_L = 0$$

$$t = 0^+, i_L = 0$$

$$i_1 = i_2 = \frac{\varepsilon}{R_1 + R_2} = 1 \text{ amp}$$

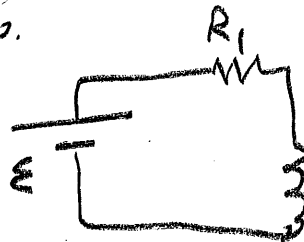


$$t = \infty, V_L = 0, V_{R_2} = 0, i_{R_2} = 0$$

$$i_1 = i_L = \frac{\varepsilon}{R_1}$$

$$= \frac{20}{10}$$

$$= 2 \text{ amps}$$



Problem 3a

Why did Maxwell propose the existence of a displacement current? How would you distinguish between a displacement current and a conduction current? (5,5)

Maxwell looked at the following fields

Eqs.
$$\sum_C \vec{B} \cdot \Delta \vec{l} = \mu_0 \sum I$$

and
$$\sum_C \vec{E}_{nc} \cdot \Delta \vec{l} = - \frac{\Delta \phi_B}{\Delta t}$$

and argued that nature must be symmetric between \vec{B} and \vec{E} fields so if time varying flux of \vec{B} creates \vec{E}_{nc} , time varying flux of \vec{E} must create \vec{B} . He knew that every current creates \vec{B} so he proposed that time varying flux of \vec{E} causes a displacement current

$$i_D = \epsilon_0 \frac{\Delta \phi_E}{\Delta t}$$

A conduction current, on the other hand, requires flux of charge inside conductor

$$i_C = \frac{\Delta q}{\Delta t}$$

Problem 3b

What is a travelling wave?

(5)

Any deviation from equilibrium which is a function of x and t such that x and t appear in the form

$$(x \mp vt)$$

will travel as a wave of velocity

$$\vec{v} = \pm v \hat{x}$$

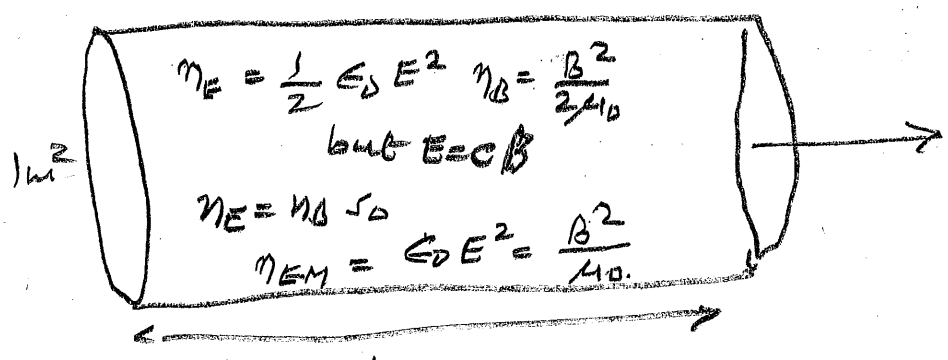
Problem 3c

Show that an E - M wave has the intensity $\langle I \rangle = \frac{1}{2} \epsilon_0 c E_m^2 = \frac{c B_m^2}{2 \mu_0}$ where E_m and B_m are the amplitudes of the \underline{E} and \underline{B} fields. (10)

E - M wave, E & B fields travel at speed of light c .

Intensity = Energy transport per m^2 per sec.

Take a tube of cross-section $1m^2$ and length $1m$



Both E & B vary as \sin fns.
 $E = E_m \sin ()$ $B = B_m \sin ()$

So time average

$$\langle \eta_{EM} \rangle = \frac{1}{2} \epsilon_0 E_m^2 = \frac{B_m^2}{2\mu_0} \quad \text{b/c } \langle \sin^2 \rangle = \frac{1}{2}$$

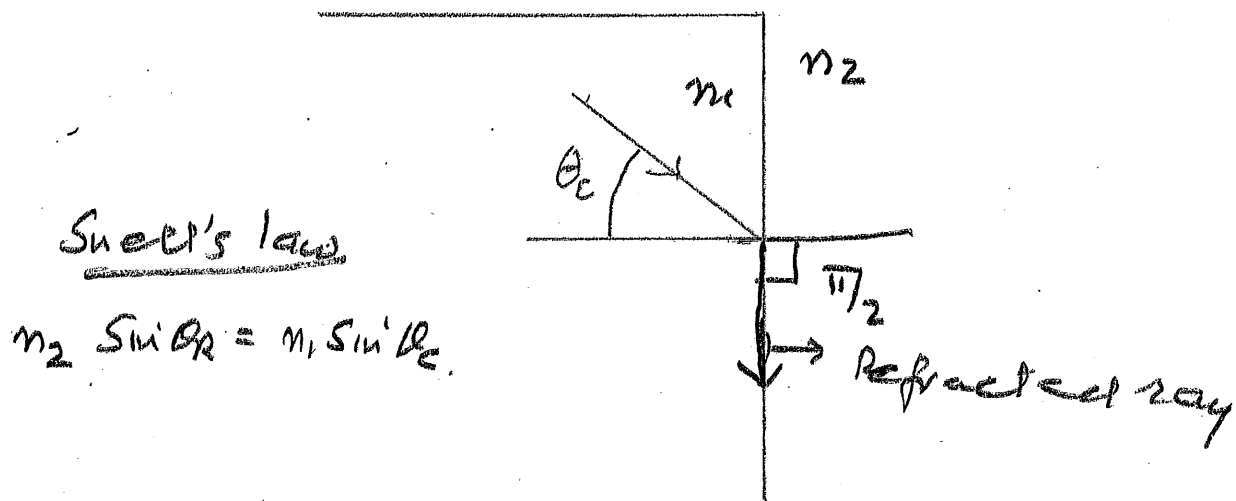
In one second all this energy in meters will flow through $1m^2$ hence

$$\begin{aligned} \langle I \rangle &= c \langle \eta_{EM} \rangle = \frac{1}{2} c \epsilon_0 E_m^2 \\ &= \frac{c B_m^2}{2\mu_0} \end{aligned}$$

Problem 4a

In the picture shown we are told that light is incident on the vertical face at the critical angle.

- (i) What does this tell you about the ratio $\frac{n_2}{n_1}$, (ii) locate the refracted ray. (5,5)



For critical angle:

$$\theta_R = \pi/2 \text{ which means } \theta_R > \theta_c$$

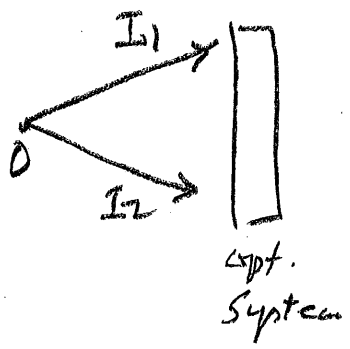
$$\text{hence } n_2 < n_1$$

$\theta_R = \pi/2$ so refracted ray travels along surface

Problem 4b

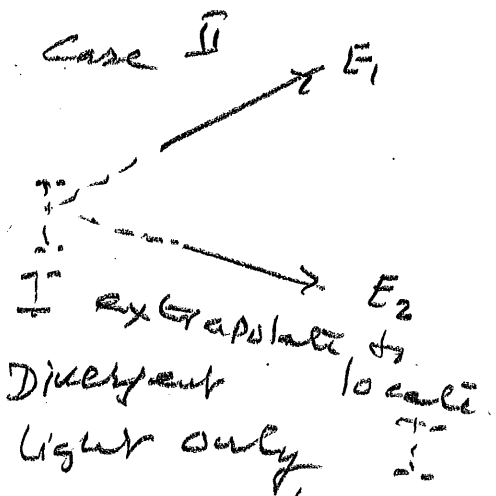
How would you distinguish between a real image and a virtual image? Support your answer with a diagram. (6)

Schem 1 for locating image
Case I



Emergent rays intersect.
 Light goes through pt. where image is located
 R.I. Image.

Case II
 Divergent light only appears to come from point where image is located
 Virtual Image



Problem 4c

Write down the dimensions of

(i) Capacitance

$$Q^2 M^{-1} L^{-2} T^2$$

(ii) \underline{B} -Field

$$M T^{-1} Q^{-1}$$

(iii) Magnetic Moment

$$Q T^{-1} L^2$$

(9)

$$C = \frac{Q}{V} \quad \frac{Q}{ML^2 T^{-2} Q^{-1}}$$

$$F_B = q [\underline{v} \times \underline{B}]$$

$$MLT^{-2} = Q L T^{-1} B$$

$$M = I A$$

$$Q T^{-1} L^2$$