

Name: Solu

(Sign in ink, print in pencil)

Notes

- 1) There are four (4) problems in this exam. Please make sure that your copy has all of them.
- 2) Please show your work indicating clearly what formula you used and what the symbols mean. Just writing the answer will not get you full credit. In stating vectors give both magnitude and direction.
- 3) Write your answers on the sheet provided.
- 4) Do not forget to write the units
- 5) Do not hesitate to ask for clarification at any time during the exam. You may buy a formula at the cost of one point.

Take Care! God Bless You!

$$k = 9 \times 10^9 \frac{N \cdot m^2}{C^2}$$

$$\epsilon_0 = 9 \times 10^{-12} \frac{F}{m}$$

$$\text{Mass of proton} \quad m_p = 1.6 \times 10^{-27} \text{ kg}$$

$$\text{Mass of electron} \quad m_e = 9 \times 10^{-31} \text{ kg}$$

$$\text{Elementary Charge} \quad e = 1.6 \times 10^{-19} \text{ C}$$

**NO CALCULATORS!**

Prob 1a Write down Gauss's law for  $\underline{E}$ -fields.

(6)

Total flux of  $\underline{E}$  through  
a closed surface  
is determined solely  
by the charges  
enclosed by the surface

$$\sum \underline{E} \cdot \underline{\Delta A} = \frac{1}{\epsilon_0} \sum Q_i$$

Prob 1b Given a conducting sphere, if you wish to put a charge  $Q$  on it, where will  $Q$  reside under stationary conditions? Why? (5)

Under stationary conditions mobile charges inside conductor are not allowed to move hence

$E_{\text{inside}} = 0$ . By Gauss' law therefore

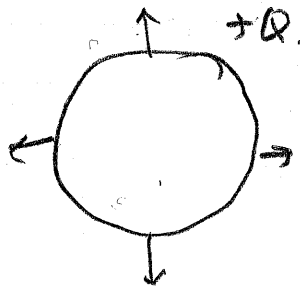
→ there can be no excess charge

inside ( $N_+ = N_-$ ). All the charge

$Q$  must be on the surface.

Prob 1c What is the direction of the  $\underline{E}$ -field on the surface of the sphere? Why?

(5)



Since charges cannot move

$\underline{E}$  must be perpendicular to surface

$$\underline{E} \parallel \hat{r}$$

If  $\underline{E}$  had any component parallel to surface charges would move along the surface.

Prob 1d Place the sphere with its center at  $r = 0$ . Show that the magnitude of  $\underline{E}$  on the surface is  $\frac{\sigma}{\epsilon_0}$  where  $\sigma = \frac{Q}{4\pi r^2}$  is the charge density.

(10)

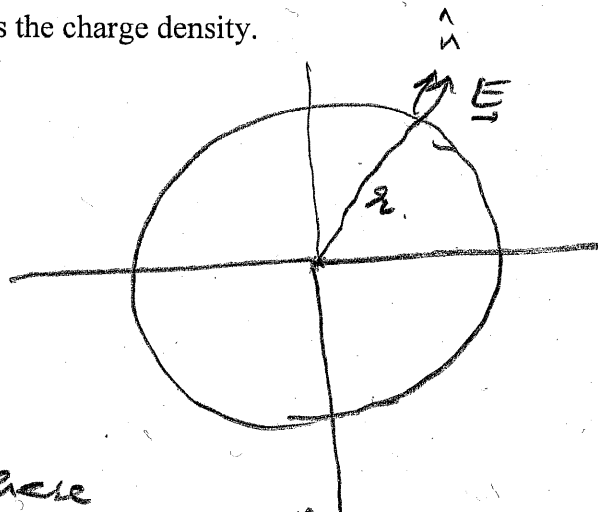
$\underline{E}$  is radial  
so gaussian surface can be a sphere

$$\underline{E} \parallel \hat{r}$$

$$\sum \underline{E} \cdot \underline{\Delta A} \text{ for sphere} = E(r) \cdot 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E(r) = \frac{Q}{4\pi r^2 \epsilon_0} = \frac{\sigma}{\epsilon_0}$$

$E$  inside is zero so  $E$  jumps by  $\frac{\sigma}{\epsilon_0}$  on crossing spherical sheet



Prob 2a

Show that the Coulomb force

$$\underline{F}_E = \frac{Q_1 Q_2}{4\pi \epsilon_0 r^2} \hat{r}$$

is a conservative force.

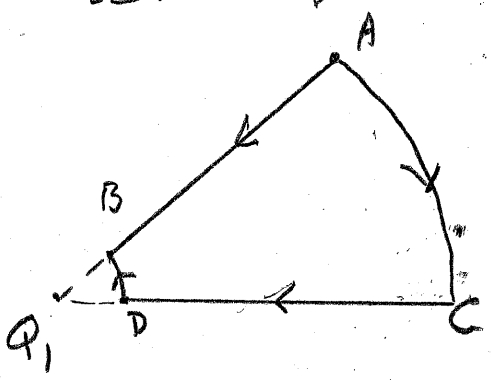
CRUCIAL POINT:

FORCE ACTS ONLY ALONG LINE JOINING  $Q_1$  TO  $Q_2$ !

(20)

In A conservative force work done is independent of path & is controlled only by end-points.

Let us fix  $Q_1$  at  $r=0$  & move  $Q_2$



First path, start at A  
 go along radius AB  
 $F_E \parallel \hat{r}$ , calculate  $\Delta W_{AB}$

Second Path first go  
 from A  $\rightarrow$  C along  
 circumference,  
 $F_E \perp \Delta s$  work done  
 $\rightarrow$  IS ZERO.  $\Delta W_{AC} = 0$

Next, go along radius CD = AB,  
 $\Delta W_{CD} = \Delta W_{AB}$ , same force, same displacement

D  $\rightarrow$  B again  $\Delta W_{DB} = 0$ .  
 (circumference)

Hence  $\Delta W_{ACDB} = \Delta W_{AB}$   
 work done is independent of path!

Prob 2b Since  $\underline{F}_E$  is conservative, one can define a potential energy  $P_E$  and the change in  $P_E$  is given by

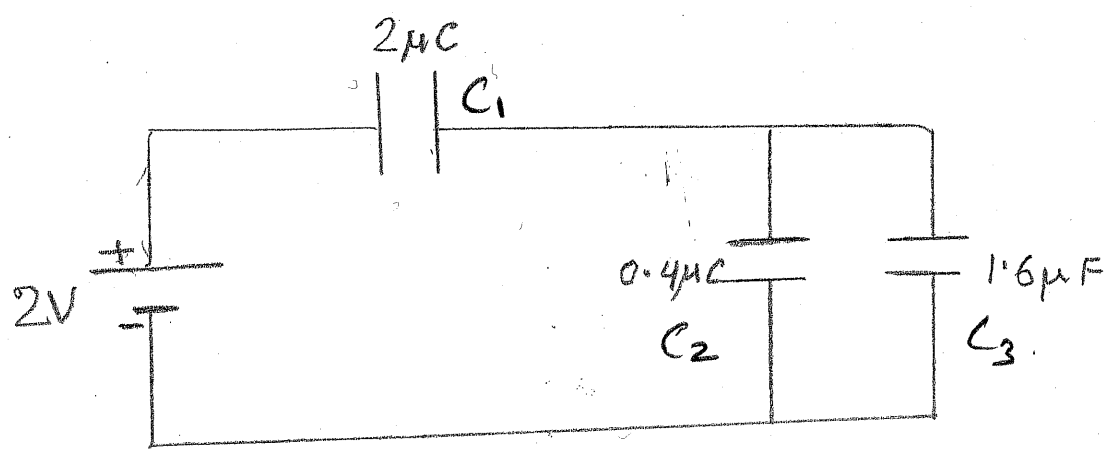
$$\Delta P_E = -\underline{F}_E \cdot \underline{\Delta S}$$

Why is there a negative sign on the right side of this equation? (5)

Potential energy is work stored in system when it is assembled in presence of a conservative force (Prob 2a). To perform the assembly, the applied force must be equal but opposite to the conservative force  $\underline{F}_E$

( In the calculations of 2a  $Q_2$  had to be moved with  $-\underline{F}_E$  otherwise it would be pushed away by  $Q_1$  )  
otherwise,  $Q_2$  would accelerate and change its kinetic energy.

Prob 3a In the circuit shown, which capacitor has (i) the most charge, (ii) the least charge? Why? (10)

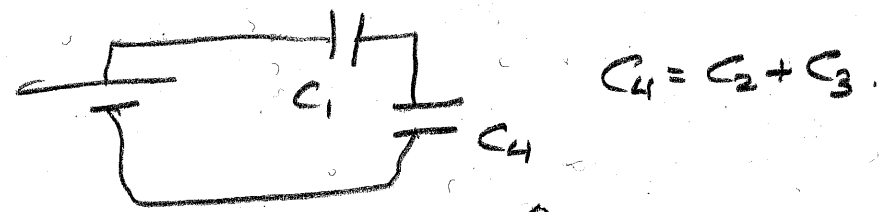


Capacitance  $C = Q/V$

Hence In series: where  $Q$  is common and  $V$ 's add  
 $\frac{1}{C_s} = \sum \frac{1}{C_i}$

In parallel  $V$  is common and  $Q$ 's add  
 so  $C_p = \sum C_i$

In circuit  $C_2, C_3$  in parallel give



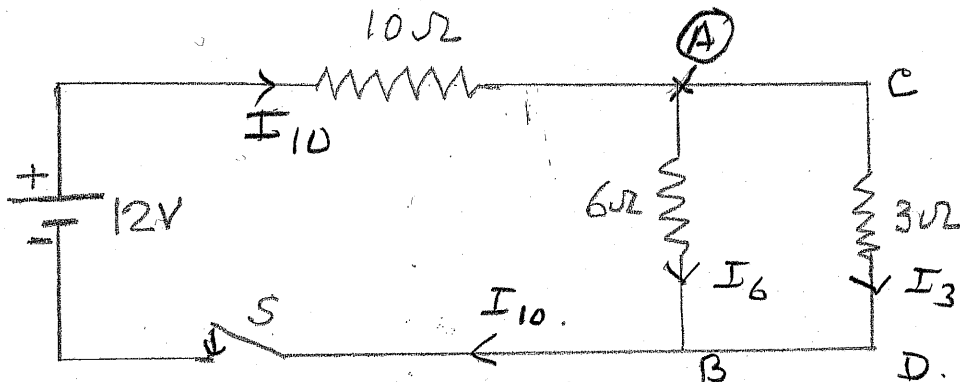
$C_1, C_4$  in series so  $Q_1 = Q_4$   
 But  $Q_4 = Q_2 + Q_3$  [ $Q$ 's add]

So  $Q_1$  has most charge

Next,  $V_2 = V_3$  so  $Q_2 = C_2 V_2$   
 $Q_3 = C_3 V_3$

So  $Q_2$  is least since  $C_2 = C_3/4$ .

Prob 3b In the circuit shown, when the switch is closed, which resistor has the (i) largest current, (ii) smallest current? Why? (10)



Use Kirchhoff's Rules

Loop Rule:  $\sum_{Loop} \Delta V = 0$

Jn Rule:  $\sum I_{out} = \sum I_{in}$

Apply Jn. Rule at  $\textcircled{A}$

$$I_6 + I_3 = I_{10}$$

So  $I_{10}$  is largest

Apply Loop Rule to ACBD.

$$V_{CD} - V_{BA} = 0$$

$$V_{CD} = V_{BA} = V \text{ (say)}$$

$$I_6 = \frac{V}{6} \text{ amp}$$

$I_6$  is smallest

$$I_3 = \frac{V}{3} \text{ amp}$$

Prob 3c

In an RC circuit, why does the time constant depend on both R and C?

(5)

Process involves transferring charge from Battery to C where  $Q$

accumulates.

Charge must go through R to reach C so larger the R, smaller the current, longer it takes

charge accumulates on C  $Q = CV$

larger C, more Q, longer it takes.



Prob 4a

What is the difference between an  $\underline{E}$ -field and a  $\underline{B}$ -field?

(5,5)

SI: In an  $\underline{E}$  field charge experiences force in  $\underline{E}$  field

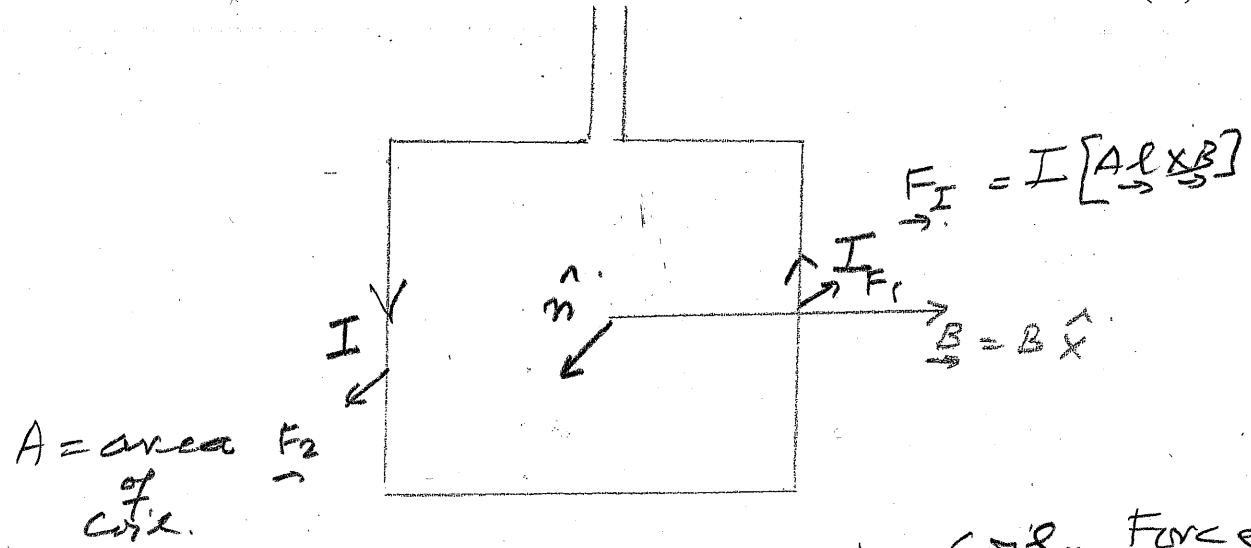
$$\underline{F}_E = q \underline{E}$$

In  $\underline{B}$  a Moving charge experiences a Force which is always perpendicular to its velocity

$$\underline{F}_B = q [\underline{v} \times \underline{B}]$$

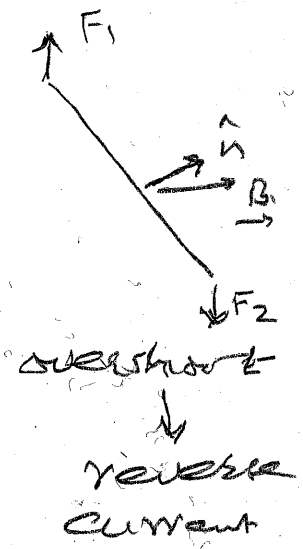
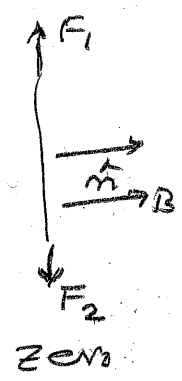
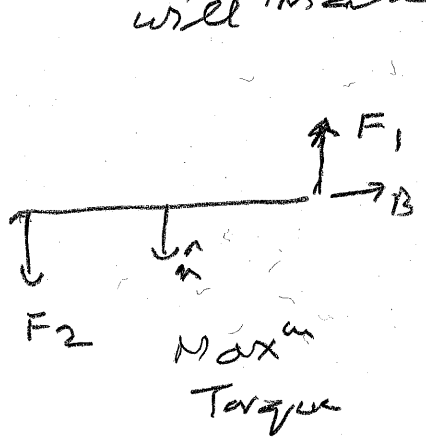
Prob 4b Shown is a coil of width  $b$  and length  $l$  suspended vertically in a  $\underline{B}$ -field. How would you make it work like a motor? The coil is free to rotate about its vertical axis.

(15)

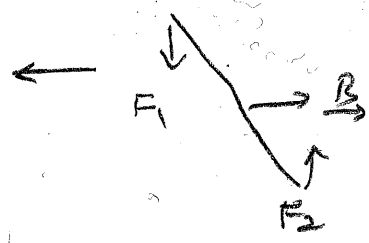


Establish a current in coil. Forces

$\vec{F}_I$  applied, coil experiences a torque  $\vec{\tau} = I A \hat{n} \times \underline{B}$  which will make it turn



Keeps rotating



So Reverse current every time  $\hat{n}$  parallel or antiparallel to  $\underline{B}$