

Name: Solution

(Sign in ink, print in pencil)

Notes

1. There are six (6) problems in this exam. Please make sure that your copy has all of them.
2. Please show your work, indicating clearly what formula you used and what the symbols mean. Just writing the answer will not get you full credit. In stating vectors, give both magnitude and direction.
3. Write your answers on the sheets provided.
4. Do not forget to write the units.
5. Do not hesitate to ask for clarification at any time during the exam. You may buy a formula at the cost of one point.

Take Care! God Bless You!

$$k_e = 9 \times 10^9 \frac{N \cdot m^2}{C^2}$$

$$\epsilon_0 = 9 \times 10^{-12} \frac{F}{m}$$

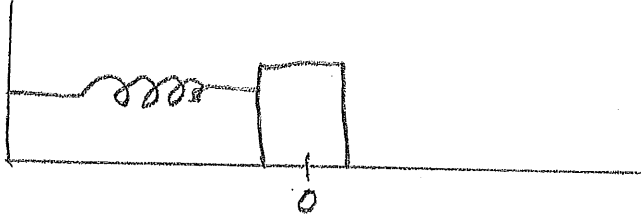
Mass of proton  $m_p = 1.6 \times 10^{-27} \text{ kg}$

Mass of electron  $m_e = 9 \times 10^{-31} \text{ kg}$

Elementary Charge  $e = 1.6 \times 10^{-19} \text{ C}$

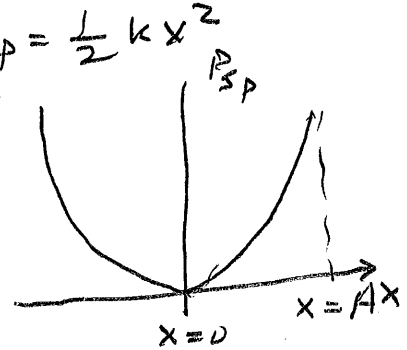
Problem 1a

A spring-mass system consists of a mass  $M$  attached to a spring of spring constant  $k$  and placed as shown on a horizontal frictionless table. The spring is unstretched at  $x = 0$ .

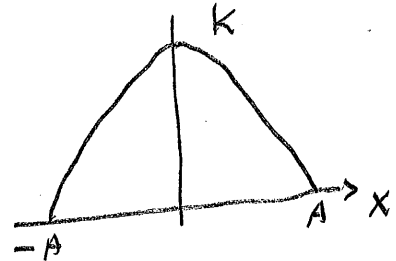


- a) If you pull the mass to  $x = A$ , what is the potential energy? (5)
- b) When you release the mass, why does it oscillate? (5)

a) This is a spring force so  $P_{sp} = \frac{1}{2} k x^2$   
 At  $x = A$ ,  $P_{sp} = \frac{1}{2} k A^2$ .



b) When the mass is released it will reduce its potential energy and gain kinetic energy  $k$ . At  $x = 0$   $P_{sp} \rightarrow 0$  and  $k$  is maximum. Since it has velocity at  $x = 0$ , it cannot stop so it keeps till  $x = -A$  when its potential energy is restored. Then it starts back.  $PE \rightarrow KE \rightarrow PE \dots$



Problem 1b

At what values of  $x$  will its kinetic energy equal its potential energy? Why?

(6)

By Conservation of Energy

$$\frac{1}{2} Mv^2 + \frac{1}{2} kx^2 = \frac{1}{2} kA^2 \quad \rightarrow \text{Total Energy stored to start the motion.}$$

$$\text{If } \frac{1}{2} kx^2 = \frac{1}{2} Mv^2$$

Each must be  $\frac{1}{2}$  of  $\frac{1}{2} kA^2$

$$\text{So } \frac{1}{2} kx^2 = \frac{1}{2} \cdot \frac{1}{2} kA^2$$

$$x^2 = \frac{A^2}{2}$$

$$x = \frac{A}{\sqrt{2}}$$

Problem 2a

What is a travelling wave?

(6)

A deviation from equilibrium which is a function of both  $x$  and  $t$  can be written in the form  $f(x \pm vt)$

will travel as a wave of velocity

$$\vec{v} = \pm v \hat{x}$$

Problem 2b

As written below the expression for the deviation from equilibrium  $D$  has several quantities missing

$$D = \dots \sin(x - vt)$$

here

$D$  is a physical quantity

$x$  is a length

$v$  a speed

and  $t$  a time

Justify the quantities you need to introduce and explain their physical significance. (10)

$D$  has dimensions/units  $\sin$  has so we need a multiplier  $A$  whose dimensions/units will be same as  $D$ , giving

$$D = A \sin(x - vt)$$

Since  $\sin$  is dimensionless, its argument  $(x - vt)$  cannot have dimensions, so we must divide by a length  $\lambda$  such that

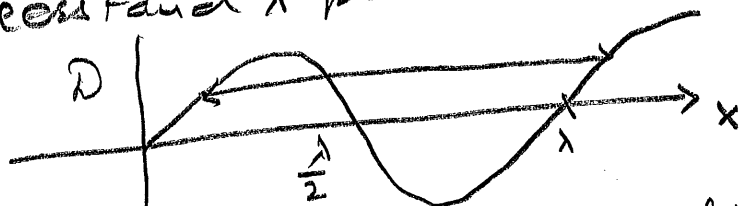
$$D = A \frac{\sin(x - vt)}{\lambda}$$

$\sin$  repeats every  $2\pi$  radians so to keep definition of  $\lambda$  simple include  $2\pi$

$$D = A \sin \frac{2\pi(x - vt)}{\lambda}$$

$A$  measures the largest value of  $D$  so we call it amplitude.

To understand  $\lambda$  plot  $D$  as a fn. of  $x$  at  $t=0$



$\lambda$  has the meaning of repeat distance so we call it wavelength.

Problem 3a

What is sound?

(6)

Any mechanical wave whose  
frequency is between 20 Hz  
and 20,000 Hz

Problem 3b

The speed of sound in air is 340 m/s. Can mechanical waves of wavelengths i) 100m, (ii) 17m, (iii) 0.017m and (iv) 0.010m be called "sound"? (10)

Since frequencies are between 20 Hz  
and 20,000 Hz

$$\text{Longest } \lambda = \frac{340}{20} = 17 \text{ m.}$$

$$\text{Shortest } \lambda = \frac{340}{20,000} = 17 \text{ mm.}$$

- So i) No  
ii) Yes  
iii) Yes  
iv) No.

## Problem 4a

At  $t = 0$ , draw a periodic sound wave as a displacement wave.

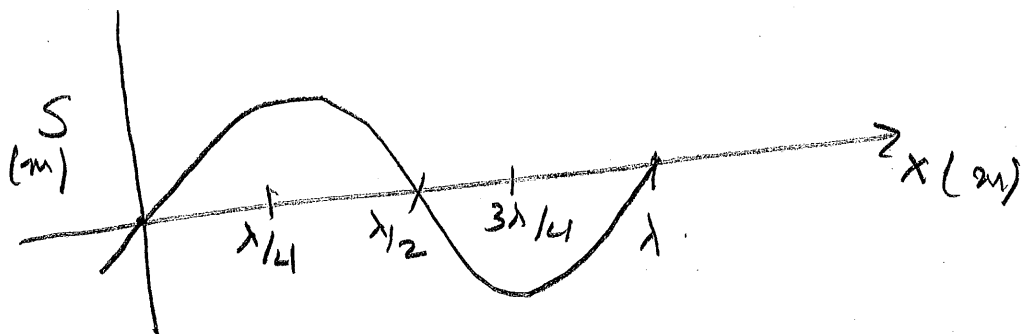
(5)

Displacement wave

$$S = S_m \sin(kx - \omega t)$$

$$t = 0$$

$$S = S_m \sin \frac{2\pi x}{\lambda}$$



$x = 0$   
 Max<sup>m</sup> Expansion  $\Delta P$  must -ive.

$$x = \frac{\lambda}{4}$$

$$\Delta V = 0 \quad \Delta P = 0.$$

$x = \lambda/2$   
 Max<sup>m</sup> Contraction  $\Delta P$  must +ive.

or recall

$$P = P_0 - \gamma S_m \rho_0 k c_0 (ax - \omega t)$$

for  $t = 0$

$$P = P_0 - \gamma S_m \rho_0 k c_0 k x$$

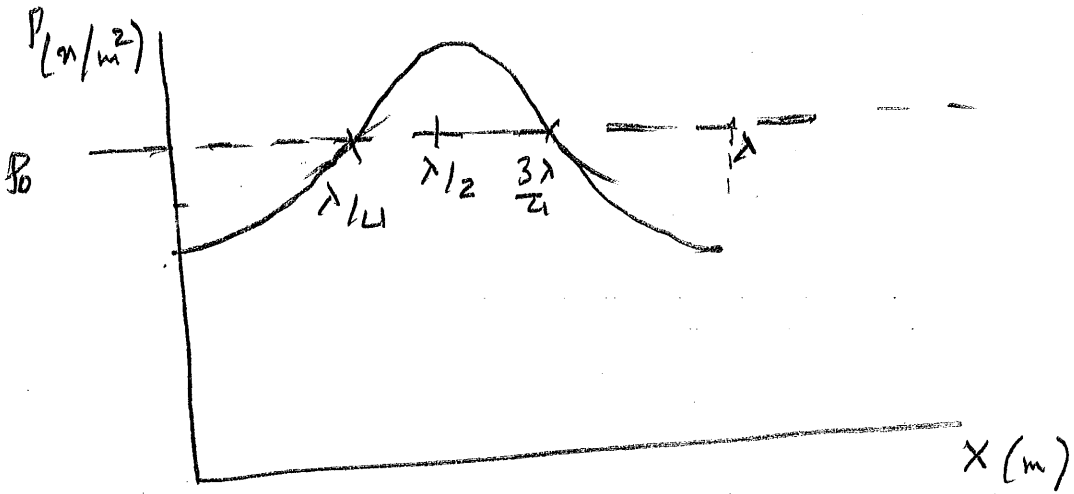
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Prob. 4b

Draw the same sound wave as a Pressure wave. Justify your answer.

(7)



Problem 4c

The speed of sound in a gas is written as

$$V_s = \sqrt{\frac{\gamma k_B T}{m}}$$

Why is there a  $\gamma$  on the right side of this equation?

(8)

Since displacement varies with position, volume must change. If volume changes, pressure must change. We need to know the relationship between these changes. Since the frequency of sound is high it is not possible to have thermal equilibrium, flow of heat is close to zero and the process is ADIABATIC. Consequently,  $P$  and  $V$  satisfy

$$P V^\gamma = \text{const}$$

$$\text{with } \gamma = \frac{C_p}{C_v}$$

and  $\gamma$  appears in the equations

Problem 5a

Show that the coulomb law

$$\underline{F_E} = \frac{k_e Q_1 Q_2}{r^2} \hat{r}$$

implies that like charges repel and unlike charges attract

(6)

If  $Q_1, Q_2$  have the same sign  $\underline{F_E}$  is along  $+\hat{r}$  and will increase the separation between them - Repulsion

$-|Q_2|$   
 $Q_2$

Two forces  
 pushing  
 charges apart

$-|Q_1|$

$Q_1$

If  $Q_1, Q_2$  have opposite signs  $\underline{F_E}$  is along  $-\hat{r}$  and will reduce the separation between them, attraction

$-|Q_2|$

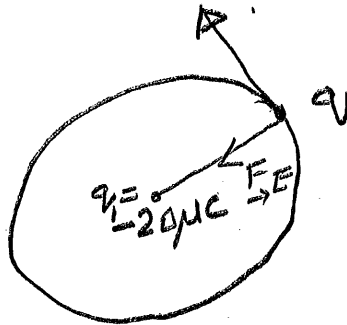
Two forces  
 pulling  
 charges  
 together

$Q_1$

Problem 5b

A charge of  $-20 \mu\text{C}$  is sitting at  $r = 0$  and a particle of mass  $0.1 \text{ kg}$  is going around it on a circle of radius  $0.5 \text{ m}$  at a speed of  $10 \text{ m/s}$ . What is the charge on the particle? Why? (Neglect gravity)

(10)



For a mass  $M$  to go around a circle of radius  $R$  with speed  $v$  we must provide a centripetal force

$$\vec{F}_c = -\frac{Mv^2}{R} \hat{r}$$

Here, the Coulomb force  $\vec{F}_E$  will do the job,  $q$  has to be positive to get

$$\vec{F}_E = -\frac{k_e |q_1 q|}{R^2} \hat{r}$$

So

$$\frac{k_e |q_1 q|}{R^2} = \frac{Mv^2}{R}$$

$$q = \frac{Mv^2 R}{k_e |q_1|} = \frac{0.1 \times 100 \times 0.5}{9 \times 10^9 \times 20 \times 10^{-6}} = 2.8 \times 10^{-5} \text{ C}$$

Problem 6a

A wave is written as

$$D = 2 \text{ N/m}^2 + 2.1 \text{ N/m}^2 \cos(6.28x + 12.56t)$$

Do you think that such a wave can exist? Why?

(5)

FROM THE UNITS we see that this is a pressure wave.

This wave cannot exist b/c when  $\cos(\ )$  becomes equal to  $-1$ , the pressure will become  $-ive$ , which is impossible

Problem 6b How would you discover the presence of an  $\underline{E}$ -field?

(5)

A stationary charge experiences a force in an  $\underline{E}$ -field. We take a test charge  $q$  and attach it to a spring balance. If we walk into an  $\underline{E}$ -field the balance will register a force. Measure  $\underline{F}_E$  and calculate  $\underline{E} = \frac{\underline{F}_E}{q}$

Problem 6c

What is Doppler Effect?

(6)

If either the detector or the source moves along the line joining them, the perceived/measured frequency is not equal to the emitted frequency. If D moves, it counts a different number. If S moves the wave is squeezed or stretched.