

Name: SOLUTION

(Sign in ink, print in pencil)

Notes

- 1) There are four (4) problems in this exam. Please make sure that your copy has all of them.
- 2) Please show your work indicating clearly what formula you used and what the symbols mean. Just writing the answer will not get you full credit. In stating vectors give both magnitude and direction.
- 3) Write your answers on the sheet provided.
- 4) Do not forget to write the units
- 5) Do not hesitate to ask for clarification at any time during the exam. You may buy a formula at the cost of one point.

Take Care! God Bless You!

$$\epsilon_0 = 9 \times 10^{-12} \frac{F}{m}$$

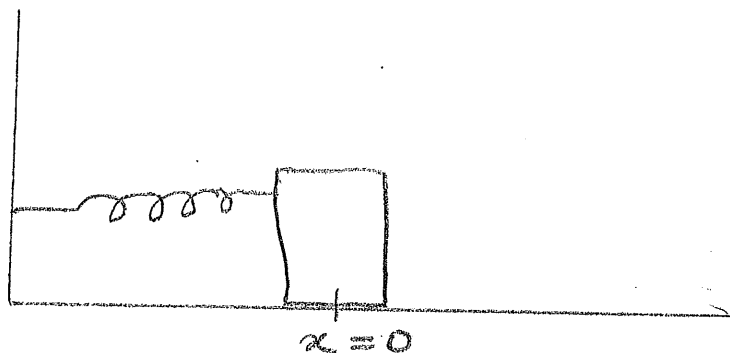
$$\text{Mass of proton} \quad m_p = 1.6 \times 10^{-27} \text{ kg}$$

$$\text{Mass of electron} \quad m_e = 9 \times 10^{-31} \text{ kg}$$

$$\text{Elementary Charge} \quad e = 1.6 \times 10^{-19} \text{ C}$$

Problem 1

A spring mass oscillator consists of a mass  $M$  attached to a spring of spring constant  $k$  and placed on a frictionless horizontal table. The spring is unstretched at  $x=0$ .

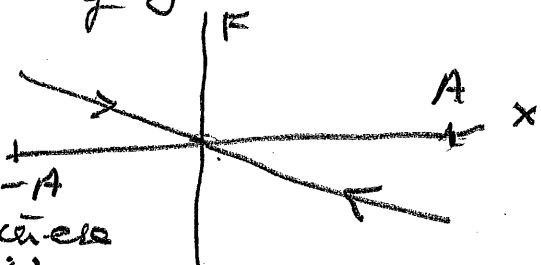


- a) If you pull the mass to  $x=A$  and release it, why does it oscillate? (7)
- b) What is its potential energy at  $x=A$ ? (5)
- c) What is its kinetic energy at  $x=0$ ? (5)
- d) At what value of  $x$  is its kinetic energy equal to its potential energy? (8)

a) Because when it is released it is acted upon by the spring force

$$\vec{F} = -kx\hat{x}$$

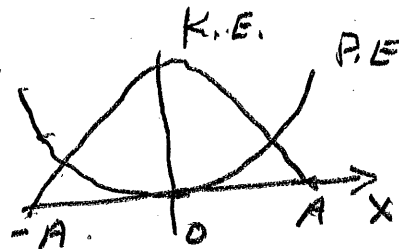
This force will bring it back to  $x=0$ , but when it gets there it has a velocity so it keeps going until  $x=-A$  and then  $\vec{F}$  again brings it back....



b) spring  $P_{sp} = \frac{1}{2} kx^2$  so at  $A$   $P_{sp} = \frac{1}{2} kA^2$

c) at  $x=0$  all the energy is kinetic

$$\frac{1}{2} Mv_{max}^2 = \frac{1}{2} kA^2$$



d) Conservation of Energy

$$\frac{1}{2} kA^2 = \frac{1}{2} Mv^2 + \frac{1}{2} kx^2$$

If  $K.E. = P.E.$  each must be  $\frac{1}{2}$  of total.

$$\frac{1}{2} kx^2 = \frac{1}{2} \cdot \frac{1}{2} kA^2$$

$$x^2 = \frac{A^2}{2} \quad x = \frac{A}{\sqrt{2}}$$

Problem 2a

As written below the expression for the deviation from equilibrium D has several quantities missing

$$D = \dots \sin(x - vt)$$

here

- D is a physical quantity
- x is a length
- v a speed
- and t a time

Justify the quantities you need to introduce and explain their physical significance. (15)

i) D has dimensions/units. Sin has none, so we need a multiplier A which has same dimensions/units as D giving

$$D = A \sin(x - vt)$$

ii) Since Sin has no dimensions, its ARGUMENT cannot have dimensions. We need to divide by a length  $\lambda$  and write

$$D = A \sin \frac{(x - vt)}{\lambda}$$

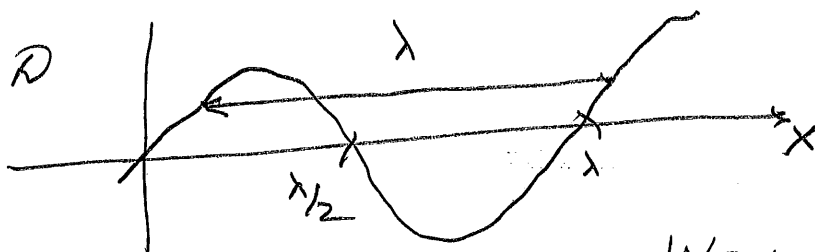
(iii) Sin repeats every  $2\pi$  so we write

$$D = A \sin \frac{2\pi(x - vt)}{\lambda}$$

because it makes definition of  $\lambda$  simple

Physical significance A measures largest values of D so it is called Amplitude.

To get  $\lambda$ , plot D at  $t=0$



$\lambda$  is repeat distance - Wavelength

Problem 2b

A wave is written as

$$P = 10 \text{ N/m}^2 + (10.1 \text{ N/m}^2) \sin(kx - \omega t)$$

- (i) Looking at the unit, can you tell what variation does it represent? (5)

- (ii) Can such a wave exist? Justify your answers. (5)

(i) Yes, it is a pressure wave

(ii). This wave cannot exist because when  $\sin(\ )$  becomes equal to  $-1$  total  $P$  will become negative. Pressure cannot be negative.

Problem 3a  
What is sound?

(5)

Any mechanical wave whose  
frequency lies between  
20 Hz and 20,000 Hz

Problem 3b

The speed of sound in a gas is written as

$$v_s = \sqrt{\frac{\gamma k_B T}{m}}$$

Why is there a  $\gamma (= \frac{C_p}{C_v})$  in this equation?

(10)

Sound is a displacement wave. If displacement (of gas layer) varies with position, there is a volume change ( $\Delta V$ ). If volume changes there must be a pressure change ( $\Delta P$ ). We need to know the relationship between these changes. Since the frequency is high, there is no flow of heat to ensure thermal equilibrium. Instead the process becomes ADIABATIC. For an adiabatic process the P-V equation is

$$P V^\gamma = \text{const}$$

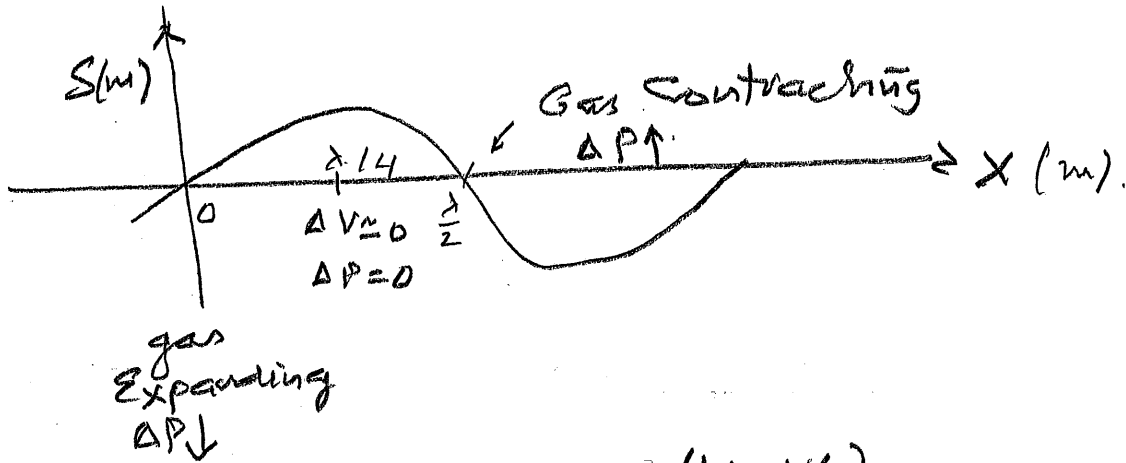
with  $\gamma = \frac{C_p}{C_v}$

Problem 3c

At  $t = 0$ , draw a periodic sound wave as (i) a displacement wave and (ii) its corresponding pressure wave. (3,7)

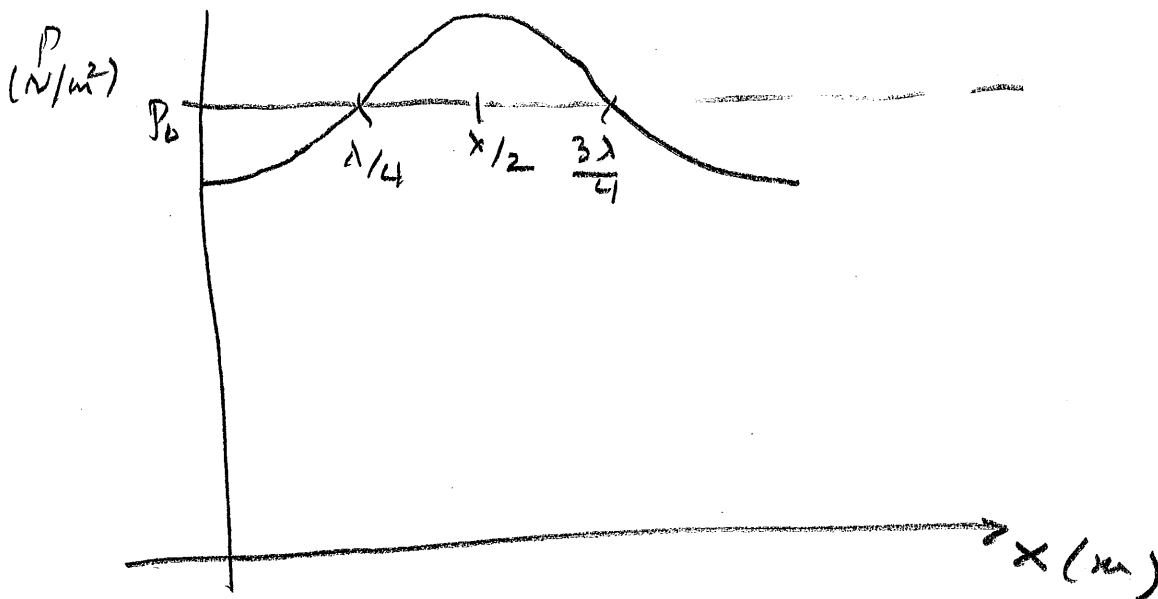
$$S = S_m \sin(kx - \omega t)$$

$$t=0, S = S_m \sin kx$$



$$P = P_0 - \gamma S_m k P_0 \cos(kx - \omega t)$$

$t=0$



Problem 4a

The Coulomb force is written as

$$\underline{F}_E = \frac{k_e Q_1 Q_2}{r^2} \hat{r}$$

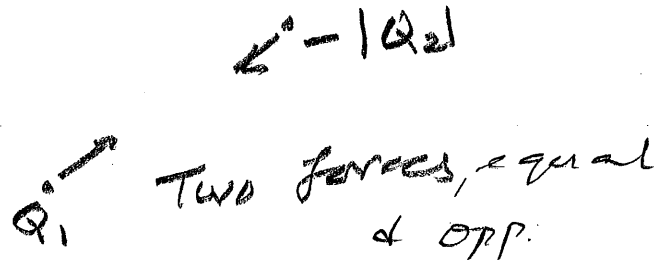
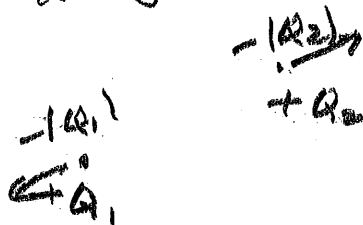
Show that this equation is consistent with Newton's Third Law of Motion. (5)

Newton's 3rd law says if two objects exert forces on one another the forces form an action-reaction pair  $\underline{F}_{12} = -\underline{F}_{21}$

Here,  $Q_1, Q_2$  same sign

$Q_1, Q_2$  different signs

Two forces equal & opp.



Problem 4b

How would you discover the presence of an  $\underline{E}$ -field? (5)

A stationary charge experiences a force in an  $\underline{E}$ -field. So we take a test charge  $q$  and attach it to a spring balance. If it experiences a force it must be in an  $\underline{E}$ -field.

Measure  $\underline{F}_E$ ,

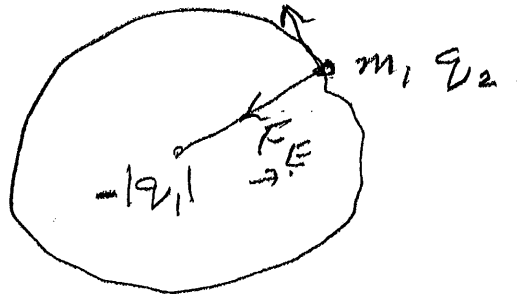
$$\underline{E} = \frac{\underline{F}_E}{q}$$



Problem 4c

A Charge of  $-20\mu\text{C}$  is sitting at  $r=0$  and a particle of mass  $0.1\text{kg}$  is going around it in a circle of radius  $0.25\text{m}$  at a speed of  $10\text{m/s}$ . What is the charge on the particle?

(15)



In order to go on a circle of radius  $R$  with speed  $v$  the mass must be provided with a centripetal force

$$\vec{F}_c = -\frac{mv^2}{R} \hat{r}$$

This must come from the Coulomb force

$$\vec{F}_E = \frac{k_e Q_1 Q_2}{R^2} \hat{r}$$

Since  $Q_1$  is  $-|q_1|$ ,  $Q_2$  must be positive

↓ give

$$\vec{F}_E = -\frac{k_e |q_1| q_2}{R^2} \hat{r}$$

Hence

$$\frac{k_e |q_1| q_2}{R^2} = \frac{mv^2}{R}$$

$$q_2 = \frac{mv^2 R}{k_e |q_1|} = \frac{0.1 \times (10)^2 \times 0.25}{9 \times 10^9 \times 20 \times 10^{-6}}$$
$$= 1.4 \times 10^{-5} \text{C}$$