EXERCIZES – 9.

<u>Recall</u> A stationary mass experiences a force in a Gravitational field

$\rightarrow \quad \underset{\rightarrow}{F_G} = m\, \underset{\rightarrow}{G_F}$

A stationary mass creates a $\underset{\rightarrow}{G_F} = -\dfrac{GM}{r^2}\hat{r}$

A stationary charge experiences a force

$\rightarrow$ in an $\underset{\rightarrow}{E}$-field $\quad \underset{\rightarrow}{F_E} = q\,\underset{\rightarrow}{E}$

A stationary charge creates a Coulomb $\underset{\rightarrow}{E}$ field. $\quad \underset{\rightarrow}{E} = \dfrac{Q}{4\pi\epsilon_0 r^2}\hat{r}$

New Field: If a moving charge experiences a force which is perpendicular to its velocity at <u>all times</u> it must be moving in a $\underset{\rightarrow}{B}$ field.

$$\underset{\rightarrow}{F_B} = q\left[\underset{\rightarrow}{V} \times \underset{\rightarrow}{B}\right]$$

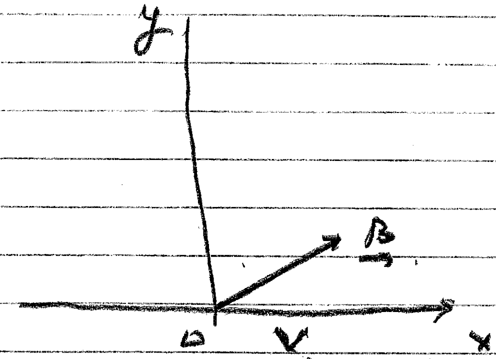
$\left[\underset{\rightarrow}{V}\times\underset{\rightarrow}{B}\right]$ is cross-product of Vector $\underset{\rightarrow}{V}$ and Vector $\underset{\rightarrow}{B}$. Its magnitude is $VB\sin(\underset{\rightarrow}{V},\underset{\rightarrow}{B})$

Its direction is given by right-hand

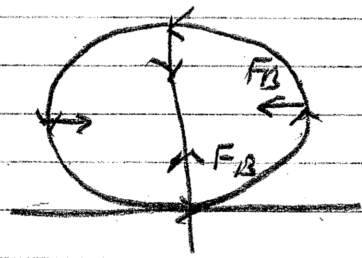
$\left[\underset{\rightarrow}{V}\times\underset{\rightarrow}{B}\right]$ is perpendicular to both $\underset{\rightarrow}{V}$ and $\underset{\rightarrow}{B}$.

$q\underset{\rightarrow}{V} \parallel$ Thumb, $\underset{\rightarrow}{B} \parallel$ Fingers, $q\left[\underset{\rightarrow}{V}\times\underset{\rightarrow}{B}\right] \perp$ Palm-of right hand.

If a charge $+q$ is moving at $\vec{V} = V\hat{x}$ and you switch on a $\vec{B} = -B\hat{z}$, q will move on a circle of radius



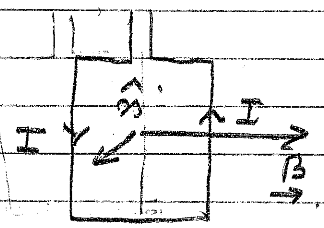
$$R = \frac{MV}{qB}$$ [because



$\vec{F_B}$ provides centripetal force] with an angular velocity $\vec{\omega} = +\frac{qB}{M}\hat{z}$

Current is a moving charge so a current I in a conductor of length $\vec{\Delta l}$ experiences $\vec{F_I} = I\left[\vec{\Delta l} \times \vec{B}\right]$

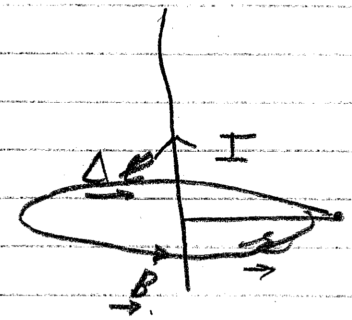
So if you support a coil of "area" $A\hat{n}$ so that it can turn freely around y-axis, place it in a field $\vec{B} = B\hat{x}$ and establish a current I in it, the coil will experience a Torque



$$\vec{\tau} = IA\left[\hat{n} \times \vec{B}\right]$$

If you reverse the current every half-cycle when $\hat{n} \parallel \vec{B}$ you can construct a motor.

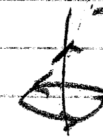
A current I generates $\vec{B}$ field at a point $\vec{r}$ and the $\vec{B}$ field is given by



$$\Delta\vec{B} = \frac{\mu_0 I}{4\pi} \frac{[\Delta\vec{l} \times \vec{r}]}{r^3}, \quad \mu_0 = 4\pi \times 10^{-7} \frac{T\text{-}m}{A}$$

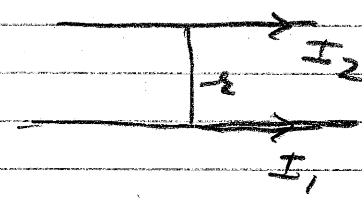
The $\vec{B}$ field circulates around I.

Rt. hand rule Thumb along I
Fingers curl along $\vec{B}$



Current - Current force : One current creates a $\vec{B}$ field, a parallel current feels a force/meter

$$\vec{F}_{I_1 I_2} = -\frac{\mu_0 I_1 I_2}{2\pi r}\hat{r}$$



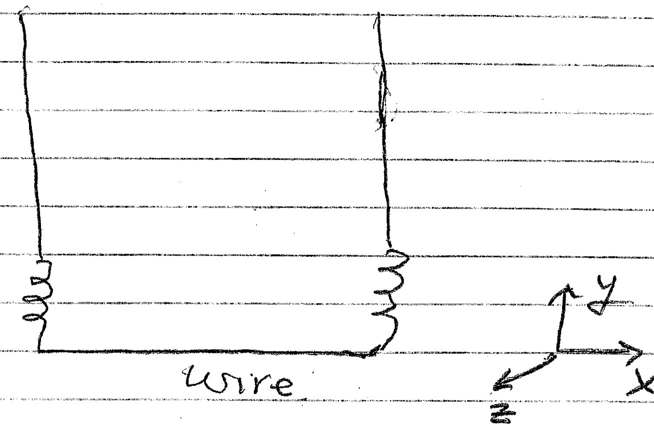
∴ Parallel currents attract one another
Anti parallel currents repel.

Ampere's Law

$$\boxed{\sum_{C} \vec{B} \cdot \Delta\vec{l} = \mu_0 \sum I}$$

Circulation of $\vec{B}$ around a closed loop is determined solely by the currents threading the surface on which the loop is drawn.

E9-1 As shown, a wire of length 50cm and mass 0.015kg is suspended by a pair of "spring" leads in a field of $-0.4T\,\hat{z}$. What current would be needed to make sure that the springs are unstretched.



For Em we need

$$\vec{F}_I - Mg\,\hat{y} = 0$$

so $\quad \vec{F}_I = +Mg\,\hat{y}$

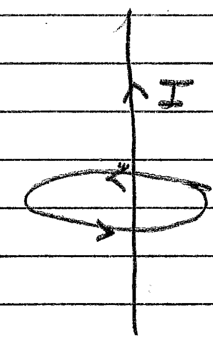
$\vec{F}_I = I\left[\vec{\Delta l}\times\vec{B}\right]$ so we need current to flow from left to right to make $\vec{F}_I \parallel +\hat{y}$ $\left[\vec{\Delta l} = \Delta l\,\hat{x}\right]$

Also, $\quad I\,\Delta l\,B = Mg$

$$I = \frac{0.015\times 9.8}{0.5\times 0.4} = 7.35 \text{ Amps.}$$

E9-2 Magnetoencelography (MEG) is a noninvasive method for studying electrical activity in the brain. It depends upon the fact that an electrical current generates a B-field. Current technology can detect fields as small as 10^{-15} T. If you think of a neuron as a straight wire what electrical current will be needed to produce such a field at 5 cm.

For a straight wire



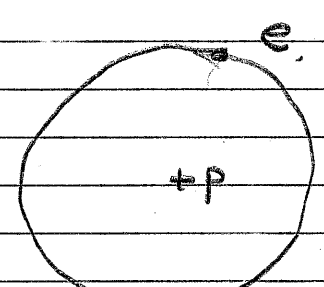
$$\vec{B}(r) = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

$$\mu_0 = 4\pi\times10^{-7}\ \frac{\text{T-m}}{\text{A}}$$

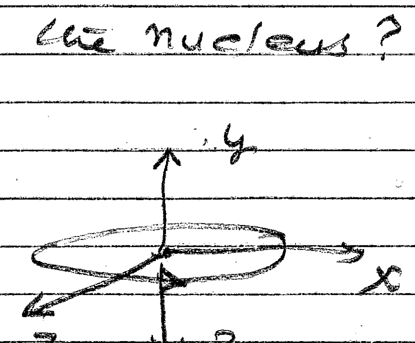
$$\text{so}\quad I = \frac{B\cdot 2\pi r}{\mu_0} = \frac{10^{-15}\times 2\pi\times5\times10^{-2}}{4\pi\times10^{-7}}$$

$$= 2.5\times10^{-10}\ \text{Amp.}$$

<u>E9-3</u> In the Bohr model an electron goes around the proton in a circular orbit of radius 5×10^{-11} m at a speed of about 2×10^{6} m/s. What is the $\vec{B}$ field at the nucleus?



At the center of a ring of radius a, lying in xz-plane



$$\underline{B} = \frac{\mu_0 I}{2a}\,\hat{y}$$

here $I = \frac{e}{T}$ T = period of electron

$e = -1.6\times10^{-19}\,C$

$T = \frac{2\pi a}{V}$ $I = \frac{eV}{2\pi a} = \frac{1.6\times10^{-19}\times2\times10^{6}}{2\times\pi\times5\times10^{-11}}$

$\simeq 1\,mA$

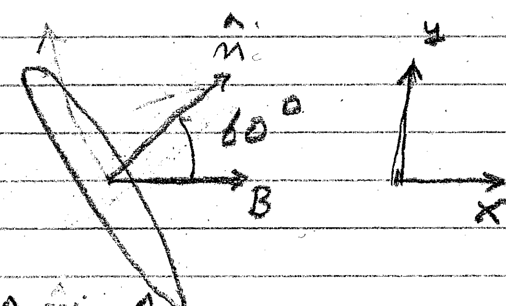
so $\underline{B} = \frac{\mu_0 V e}{2\pi a}\cdot\frac{1}{2a}\,\hat{y}$

$$= \frac{\mu_0\, eV}{4\pi\, a^2}\,\hat{y} = -\frac{10^{-7}\times1.6\times10^{-19}\times2\times10^{6}}{25\times10^{-22}}\,\hat{y}$$

$$= -12.8\,T\,\hat{y}$$

E 9-4 A 10A current flows in a circular loop of radius 5cm. The axis of the loop in xy-plane is 60^0 to a $\underline{B}$-field of $0.1\,T\,\hat{x}$.

What is the Torque on the loop?

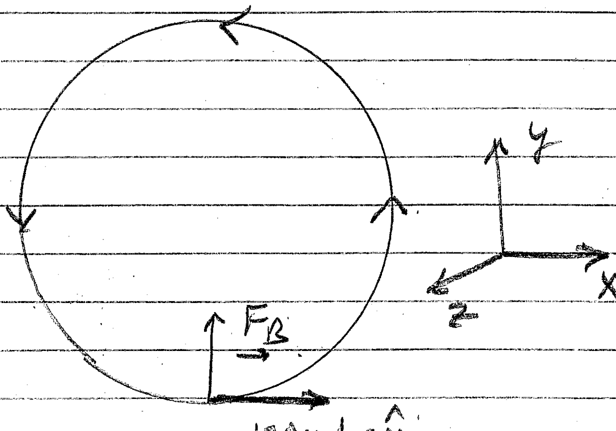


$\underline{\tau} = I A\,\hat{n}\times\underline{B}$

$= -(10\times\pi\times25\times10^{-4}\times\sin 60\times0.1)\,N\text{-}m\,\hat{z}$

$= -6.8\times10^{-3}\,N\,\hat{z}$

E-9-5 A charged particle with an initial velocity of 100 m/s $\hat{x}$ enters a $\underset{\rightarrow}{B}$ field of $-0.5\,T\,\hat{z}$



The picture shows that it travels on a circle of radius 0.5 m counterclockwise.

(i) Is the charge +ive or −ive? Why?

(ii) What is q/M its charge-to-mass ratio?

(i) The charge has to be positive. At $x=0$, $\underset{\rightarrow}{V} \parallel \hat{x}$, $\underset{\rightarrow}{B} \parallel -\hat{z}$ so $\underset{\rightarrow}{F_B} \parallel \hat{y}$. Charge velocity turns up!

(ii) $\underset{\rightarrow}{F_B}$ provides $\underset{\rightarrow}{F_c}$

$$\underset{\rightarrow}{F_B} = -qvB\,\hat{r} \qquad \underset{\rightarrow}{F_c} = -\frac{Mv^2}{r}\hat{r}$$

$$qvB = \frac{Mv^2}{r}$$

$$\frac{q}{M} = \frac{v}{rB} = \frac{100}{0.5 \times 0.5} = 400\ C/kg$$