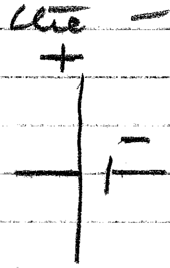
EXERCIZES-7

FORMULAE

Devices

Battery generates a Coulomb $\underline{E}$-field using chemical Energy. Typically, consists of two metal plates dipping in a chemical mixture in which +ive and -ive ions are separated. The +ive ions get attached to one plate and the -ives to the other



and a potential difference appears. For historic reasons this potential difference is called Electro motive force or EMF.

Output of Battery

$$\mathcal{E} = \text{EMF in Volts.}$$

EMF is the amount of work needed to move a unit charge.

Capacitor : is like a "Bucket" for an $\underline{E}$-field. It will not generate an

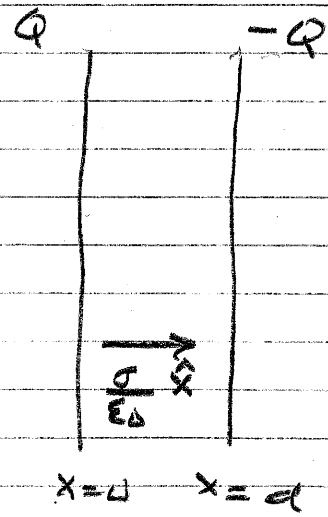
$\underline{E}$ - field but it can trap one:

Essentially one establishes a potential difference V between two plates and charges $\pm Q$ appear on the plates thereby trapping an $\underline{E}$-field between them. The "size" of a capacitor is called "Capacitance"

$$C = \frac{Q}{V}$$

The larger the C the larger the Q it can hold for a given V.

Simplest Capacitor - Two plates of Area A separated by a small distance with air or vacuum between the plates.



The sheet charges are

$$\sigma = \frac{Q}{A}$$

and $\underline{E}$-field is

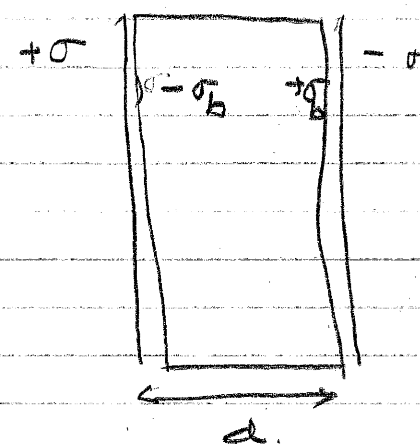
$$\underline{E}_0 = \frac{\sigma}{\varepsilon_0}\hat{x}$$

giving $|V| = \frac{\sigma}{\varepsilon_0} d.$

so $$C_0 = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0} d}$$

$$C_0 = \frac{\varepsilon_0 A}{d}.$$

If you put a dielectric between the plates, the dipoles inside it line up so bound charge sheets $\pm \sigma_b$ appear making the $\underline{E}$-field inside



$$\underrightarrow{E}_k = \frac{\sigma - \sigma_b}{\varepsilon_0} \hat{x}$$

and $|V_k| = \left(\frac{\sigma - \sigma_b}{\varepsilon_0}\right) d$

$$C_k = \frac{\varepsilon_0 \sigma A}{(\sigma - \sigma_b) d.} = k \frac{\varepsilon_0 A}{d.}$$

$k = \frac{\sigma}{\sigma - \sigma_b}$ is called Dielectric Const.

In order to place a charge $\pm Q$ on C_0 we must perform $\frac{Q^2}{2C_0}$ Joules of work which becomes the potential Energy

$$U_E = \frac{Q^2}{2C_0}$$

as it is stored in the $\underline{E}$-field between the plates. Consequently, $1 m^3$ of an $\underline{E}$-field stores

$$\eta_E = \frac{1}{2} \epsilon_0 E_0^2 \quad J/m^3 \quad \text{of Energy}$$

for a Capacitor with a dielectric

$$U_E(k) = \frac{Q^2}{2C_k}$$

and $\eta_E(k) = \frac{1}{2} k \epsilon_0 E_k^2 \quad J/m^3$

where $E_k = \frac{E_0}{k}$

Combinations of Capacitors

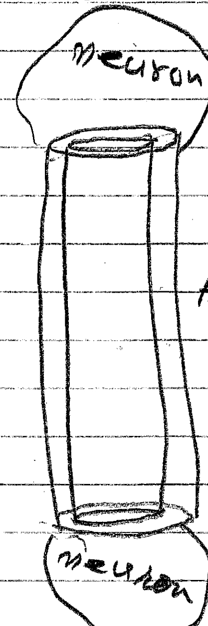
Series Connection Q's are Common
V's add

$$\frac{1}{C_s} = \Sigma \frac{1}{C_i}$$

Parallel Connection V's are Common, Q's add

$$C_p = \Sigma C_i$$

7-1 An axon is a long tail like part of a nerve cell. Its membrane is a thin cylindrical shell of radius 10^{-5} m, length $L = 0.1$ m and thickness $d = 10^{-8}$ m. It has a positive charge on one side and negative on the other and acts like a parallel plate capacitor of area $2\pi r L$ and plate separation d. The dielectric constant $k = 3$.



(i) Calculate the capacitance
(ii) If the potential difference across the membrane is 70 mV what is (a) the charge on the "plates" and (b) the $\vec{E}$-field through the membrane.

(i) $$C_k = k\frac{\varepsilon_0 A}{d} = \frac{3 \times 9 \times 10^{-12} \times 2 \times \pi \times 10^{-5} \times 0.1}{10^{-8}}$$
$$= 1.7 \times 10^{-8}\ F$$

(ii) a) $$\frac{Q}{V_k} = C_k$$
$$Q = V_k C_k = 7 \times 10^{-3} \times 1.7 \times 10^{-8} = 1.19 \times 10^{-10}\ C$$

b) $$E = \frac{V_k}{d} = \frac{70}{10^{-8}} = 7 \times 10^{9}\ N/C.$$

E7-2 Estimate the Electrical Energy stored in the atmosphere if the E-field of the Earth is 200 V/m on the average and extends up to 1000m above the Earth [radius of Earth 6400 km].

Volume of E-field $= 4\pi R_E^2 \times h$

$R_E = 6400$ km
$h = 1$ km.
Dielectric Const = 1.

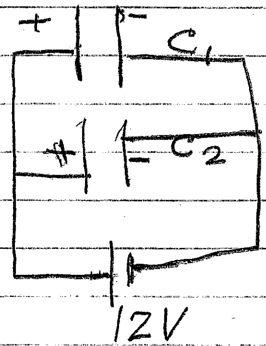
Total $U_E = \frac{1}{2} \epsilon_0 E^2 \times$ Volume

$= \frac{1}{2} \times 9 \times 10^{-12} \times (200)^2 \times 4\pi \times (6400 \times 10^3)^2 \times 1$

$= 9.27 \times 10^7$ Joules

E7-3 Two Capacitors $C_1 = 4\mu F$ and $C_2 = 12\mu F$ are first connected as shown

i What are the charges on the plates



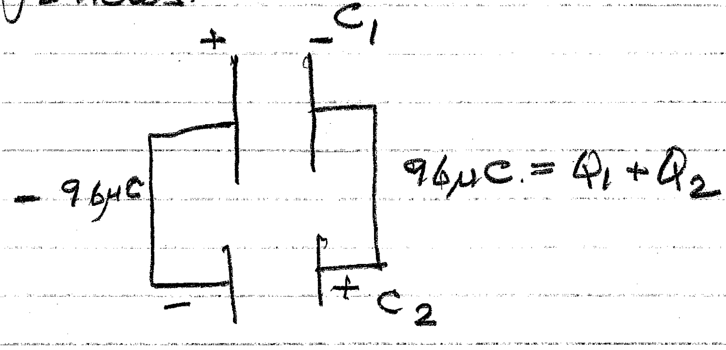
$Q_1 = C_1 V = 48 \mu C$

$Q_2 = C_2 V = 144 \mu C$

(ii) What is the energy stored in them

$U_E = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2 = \frac{1}{2} \times 16 \times 10^{-6} \times 144 = 1.15 \times 10^{-3}$ J

Next carefully disconnect them from the battery and connect them to one another as follows.



Now what is the potential across them. Again, they are in parallel so potentials must be same $V_1 = V_2 = V$

Also charge is conserved so total charge must be $+96\mu C$ on right plates and $-96\mu C$ on left plate. That is

$$(12\mu F \times V) + (4\mu F \times V) = 96\mu C. \qquad (Q = CV)$$

So $V = 6$ Volts

$Q_1 = 24\mu C$

$Q_2 = 48\mu C.$

$$\text{Total Energy} = \frac{1}{2} C_1 V^2 + \frac{1}{2} C_2 V^2$$

$$= \frac{1}{2} \times 16 \times 10^{-6} \times 36 = 2.88 \times 10^{-4} J$$

What happened to the Energy? Why did it reduce by a factor of 4?

E7-4 Show that the plates of a parallel plate capacitor attract one another with a force given by

$$F = \frac{q^2}{2\epsilon_0 A}$$

Recall that pressure is energy per unit volume, Hence,

$$\eta_E = \frac{1}{2}\epsilon_0 E^2$$

represents the pressure pulling on the plates

Force due to this will be attractive

$$F = \eta_E \times A = \frac{1}{2}\epsilon_0 E^2 A$$

But $E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}$

so

$$F = \frac{1}{2}\epsilon_0 \cdot \frac{q^2}{\epsilon_0^2 A^2} A = \frac{q^2}{2\epsilon_0 A}$$

E-7.5 An air filled capacitor $\left[C_0 = \frac{\epsilon_0 A}{d}\right]$ is connected to a battery until the plates are charged to $\pm Q$ $\left[u_E(0) = \frac{Q^2}{2C_0}\right]$. It is then disconnected and a plate of dielectric constant k is introduced $\left[u_E(k) = \frac{Q^2}{2C_k}\right]$.

Note that $C_k = k\, C_0$.

Hence $\dfrac{U_E(0)}{U_E(k)} = \dfrac{1}{k}$

and $k > 1$ so Energy is lost. Why?

When the dielectric is in the $\underline{E}$-field between the plates the dipoles in it must rotate. That costs Energy.