EXERCIZES – 6

FORMULAE

Coulomb Force $\underset{\to}{F_E} = \frac{Q_1 Q_2}{4\pi \varepsilon_0 r^2}\hat{r}$ acts ONLY along the line drawn from Q_1 to Q_2 and this property is used to prove that this force is CONSERVATIVE: WORK DONE IS INDEPENDENT OF THE PATH. Consequently, one can define:

Electrical Potential Energy: Its change is

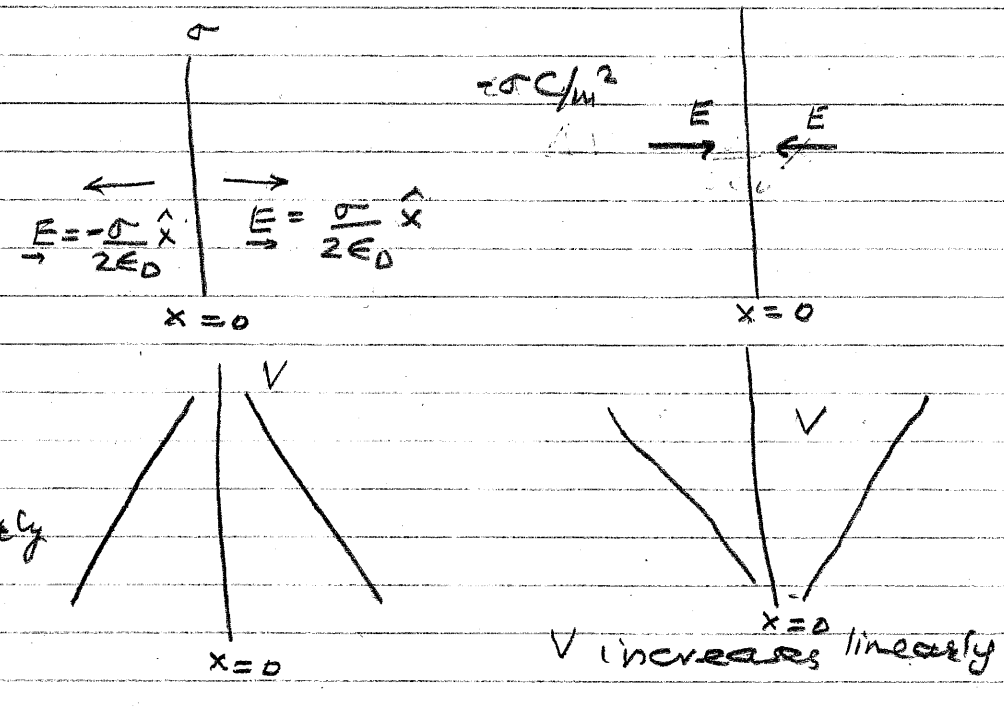
$$\Delta P_E = -\vec{F}_E \cdot \Delta\vec{S} \qquad \text{(Joules)}$$

AND USE IT TO define Electric Potential Difference.

$$\underset{\text{Volts}}{\Delta V} = \frac{\Delta P_E}{q} = -\vec{E}\cdot\Delta\vec{S} \quad \text{(where } q \text{ is test charge)}$$

Special Cases

Charge sheet
σ C/m²

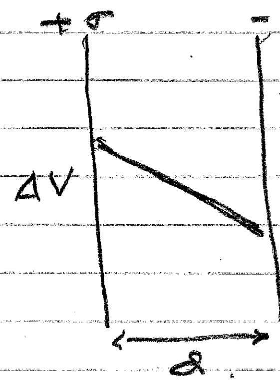


$$\Delta V = -\frac{\sigma}{2\epsilon_0}x$$

V drops linearly

V increases linearly

Two Sheets



$$\Delta V = -\frac{\sigma}{\varepsilon_0} d$$

Note: Along y and z, $\vec{E} = 0$ so $\Delta V = 0$,

EQUIPOTENTIALS ARE PLANES PARALLEL TO SHEETS.

Point Charge at $r = 0$

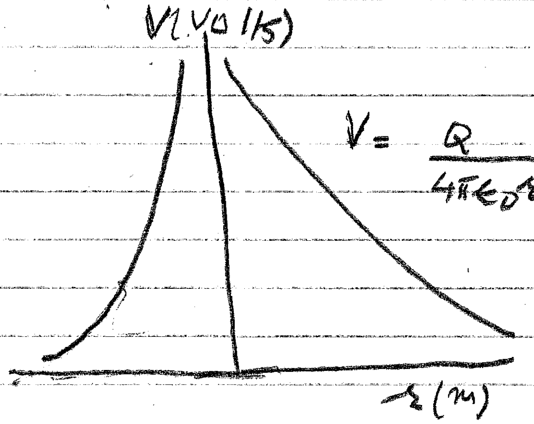
$+Q$

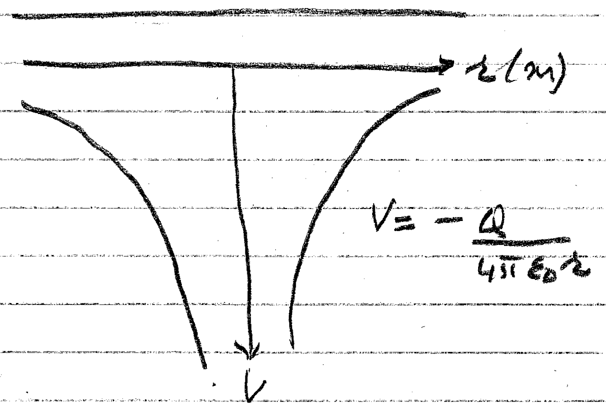
$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

$-Q$

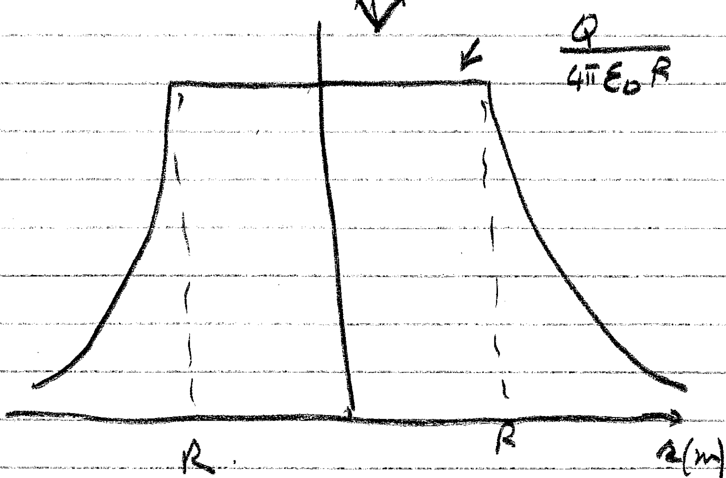
$$\vec{E} = -\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

$V = 0$ at $r \to \infty$

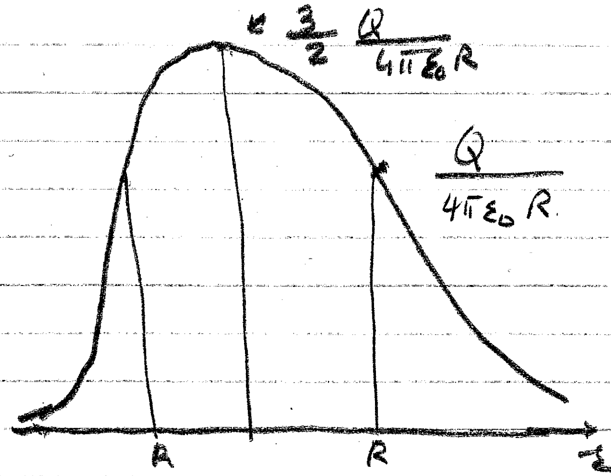




Spherical shell Q +ive



Solid sphere Q +ive



CONSERVATION OF ENERGY

$$K_f + P_{fg} + P_{fsp} + P_{fE} = K_i + P_{ig} + P_{isp} + P_{iE}$$

K: Kinetic Energy

P_g: Gravitational Potential Energy. Earth-Mass system

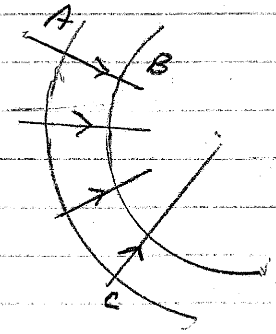
P_{sp}: Potential Energy stored in spring

P_E: Potential Energy of charge in $\vec{E}$-field

Potential Energy Point charges

$$P_E(q_1, q_2) = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

E6-1 In the picture shown the $\vec{E}$-field lines represent an $\vec{E}$-field which does 4×10^{-19} J of work on an Electron in going from A to B.



The circles are equipotentials. Calculate 1. $\Delta V_{AB} = V_B - V_A$

2. $\Delta V_{AC} = V_C - V_A$ 3. $\Delta V_{BA} = V_A - V_B$

$$\Delta V_{AB} = -\frac{4\times10^{-19}}{1.6\times10^{-19}} = -2 \text{ Volts.}$$

$\vec{E}$ field is doing +ive work so electron is increasing it's potential Energy in going from A to B hence the potential ΔV must drop.

2. A and C are on an equipotential so

$V_A - V_C = 0.$

3. $\Delta V_{BC} = \Delta V_{BA} + \Delta V_{AC}$

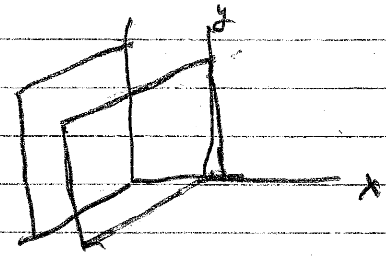
$= +2$ Volts.

E6-2 An Infinite sheet of charge parallel to YZ-plane has a charge density $\sigma = 0.1\,\mu C/m^2$. What are the equipotential surfaces?

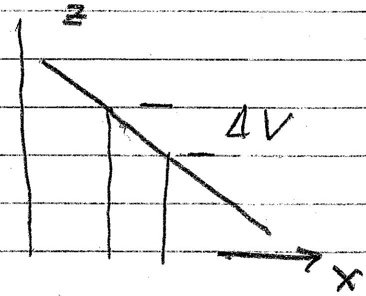
$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{x}$

(ii) What is the separation between two surfaces whose $\Delta V = -50V$?

(i) Since $\underset{\rightarrow}{E} = \frac{\sigma}{2\epsilon_0}\hat{x}$, NO work will be done if a charge moves along y or z so equipotentials are all parallel to YZ-planes.



$$\Delta V = -\frac{\sigma}{2\epsilon_0}\Delta X$$



So $\quad \Delta X = -\frac{\epsilon_0 \Delta V}{\sigma}$

$$= \frac{9\times10^{-12}\times 50}{10^{-7}} \text{ m}$$

$$= 4.5\times10^{-3} \text{ m}$$

E6-3 In the quark model of a proton there are supposed to be two "up" quarks with charges $+\frac{2}{3}\times1.6\times10^{-19}$C and one "down" quark with charge $-\frac{1}{3}\times10^{-19}$c. The quarks are separated by 1.3×10^{-15} m

C $-\frac{1}{3}e$

$+2/3\,e$ A $\qquad$ 1.3×10^{-15} m $\qquad$ $+2/3\,e$ B

Calculate the total potential Energy of this configuration

When first up ($+\frac{2}{3}e$) is placed at A it costs no energy.

Once there is an $+\frac{2}{3}e$ at A putting $+\frac{2}{3}e$ at B costs

$$P_E(A,B) = \frac{\left(\frac{2}{3} \times 1.6 \times 10^{-19}\right)^2}{4\pi \varepsilon_0 \times 1.3 \times 10^{-15}} \text{ J}$$

$$= 7.8 \times 10^{-14} \text{ J}$$

Next we put $-\frac{4}{3}e$ at C.

Now
$$P_E(A,B,C) = \frac{2.56 \times 10^{-38}}{4\pi \varepsilon_0 \times 1.3 \times 10^{-15}} \left[\frac{4}{9} - \frac{2}{9} - \frac{2}{9}\right]$$

$$= 0.$$

E6-4 We are told that a spherical charge Q has a potential energy of

$$P_E = \frac{3}{5}\,\frac{Q^2}{4\pi\varepsilon_0 R}$$

where R is the radius of the sphere.

Estimate the potential energy of an electron if you assume that it is a sphere of radius 10^{-15} m.

$$P_E = \frac{3}{5} \times \frac{(1.6\times 10^{-19})^2}{4\times\pi\times 9\,10^{-12}\times 10^{-15}}$$

$$= 13.5\times 10^{-14}\ \text{J}$$

Next, suppose we buy Einstein's famous formula

$$m = \frac{E}{c^2} \qquad c = 3\times 10^{8}\ \text{m/s}$$

and estimate the mass of the electron

$$m = \frac{13.5\times 10^{-14}}{9\times 10^{+16}} = 1.5\times 10^{-30}\ \text{kg}$$

The true value is 9×10^{-31} kg. Also, it is now well established that the Electron has **no** size.

E6-5 When U^{235} captures a neutron it fissions into two nuclei each of $+46e$ and they are at rest just after fission, being about 1.3×10^{-14} m apart. (i) Estimate their potential energy (ii) About how many fissions per second will be needed to produce 1 MW of power in a reactor?

$$P_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = \frac{1}{4\pi \times 9 \times 10^{-12}} \frac{46 \times 46 \times (1.6 \times 10^{-19})^2}{1.3 \times 10^{-14}}$$

$$= 3.75 \times 10^{-11} \text{ J}$$

In terms of MeV
Since $1 \text{ MeV} = 1.6 \times 10^{-13}$ J

we get $P_E = \dfrac{3.75 \times 10^{-11}}{1.6 \times 10^{-13}} = 234.$ MeV.

Note: This estimate is essentially the energy released per fission

$$\# \text{ of fissions/sec} = \frac{10^6}{3.75 \times 10^{-11}} = 2.67 \times 10^{16} \text{ per sec.}$$

Since $1 \text{ MW} = 10^6$ J/s

E6-6 In a typical living cell the electrical potential inside is 0.07 V lower than the potential outside. If the cell membrane is 10^{-7} m thick what is the magnitude and direction of the $\vec{E}$ field within the membrane?

Magnitude of $E = \dfrac{\Delta V}{\Delta X} = \dfrac{0.07}{10^{-7}} = 7 \times 10^{5}$ N/m.

directed inward