EXERCISES - 5

FORMULAE

GAUSS'S LAW

$$\sum_{c} \vec{E} \cdot \vec{\Delta A} = \frac{1}{\varepsilon_0} \Sigma Q_i$$

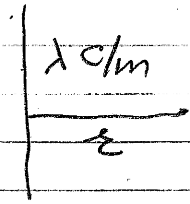
FIELDS

Point Charge $\pm Q$ at $r = 0$

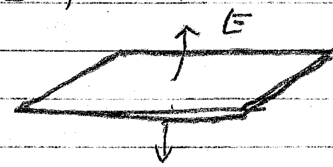
$$\vec{E}(r) = \pm \frac{Q}{4\pi \varepsilon_0 r^2} \hat{r}$$

Line of charge λ c/m

FIELD AT r $\quad \vec{E}(r) = \dfrac{\lambda}{2\pi \varepsilon_0 r} \hat{r}$



Sheet of charge σ C/m² in xz-plane



$y > 0 \quad \vec{E} = \dfrac{\sigma}{2\epsilon_0}$

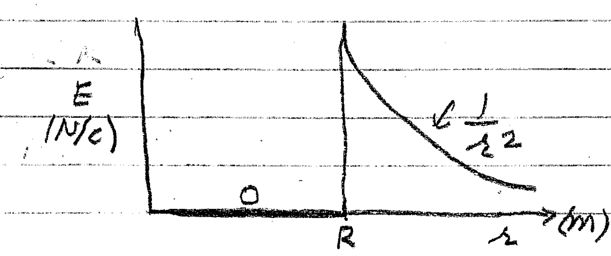
$y < 0 \quad \vec{E} = -\dfrac{\sigma}{2\epsilon_0} \hat{y}$

→ E-field jumps by $\dfrac{\sigma}{\varepsilon_0}$ on crossing charge sheet.

Spherical shell of charge Q and radius R centered at $r = 0$

$r < R, \quad \vec{E} = 0$

$r > R, \quad \vec{E} = \dfrac{Q}{4\pi \varepsilon_0 r^2}$



Uniformly charged solid sphere of charge

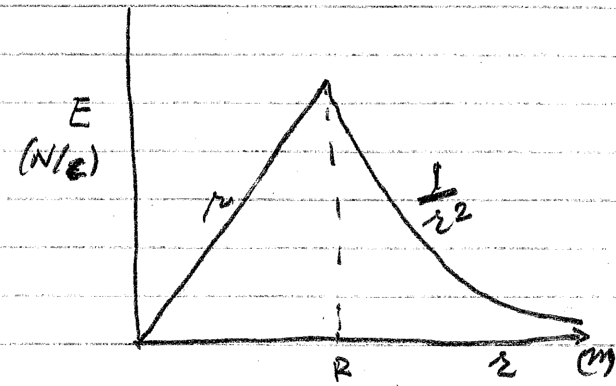
Q and radius r centered at $r = 0$.

1(5)

Charge density $\rho = \dfrac{Q}{\frac{4\pi}{3} R^3}$.

$r < R \quad \vec{E}(r) = \dfrac{\rho r}{3\epsilon_0} \hat{r}$

$r > R \quad \vec{E}(r) = \dfrac{Q}{4\pi\epsilon_0 r^2} \hat{r}$

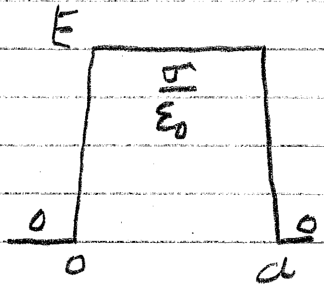


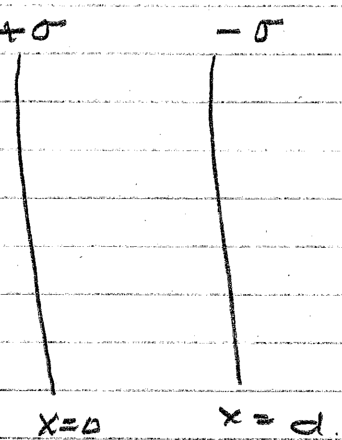
Two sheets parallel to yz-plane, $+\sigma$ at $x = 0$, $-\sigma$ at $x = d$

$x < 0 \quad \vec{E} = 0$

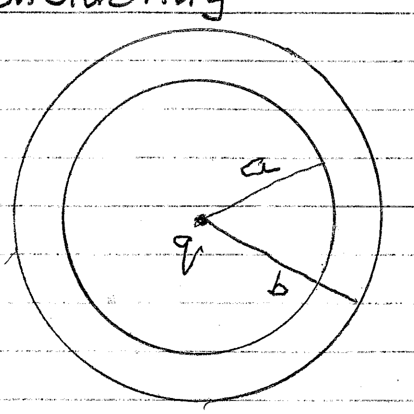
$0 < x < d \quad \vec{E} = \dfrac{\sigma}{\epsilon_0} \hat{x}$

$x > d \quad \vec{E} = 0$





E5-1 An uncharged thin conducting spherical shell has a point charge $q = 10\mu C$ at its Center. The radii are $a = 0.95m$ $b = 1.00m$.



Calculate (i) $\underline{E}$-field at $r<a$
(ii) Charge on surface at a
(iii) Charge on surface at b
(iv) $\underline{E}$-field at $a<r<b$
(v) $\underline{E}$-field $r>b$

(i) By Gauss' Law

$$\sum \underline{E} \cdot \Delta \underline{A} = \frac{1}{\varepsilon_0} \Sigma Q_i$$

$$r<a \qquad \underline{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{10^{-5}}{4\pi \times 9 \times 10^{-12} r^2}\hat{r}$$

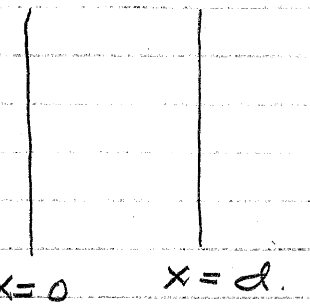
(ii) Inside Conductor charge is at rest, so $\underline{E}$ must be Zero. Requires induced charge on Surface at a to be $-10\mu C$.

(iii) Total charge on Conductor is zero so q on surface at b must $+10\mu C$

(iv) $\underline{E} = 0. \quad a<r<b$

(v) $$\underline{E} = \frac{10^{-5}}{4\pi \times 9 \times 10^{-12}}\hat{r}$$

E5-2 Two large metal plates of area $10 m^2$ face each other. They are about 5cm apart and carry equal but opposite charges. If the $\underset{\rightarrow}{E}$ field between them is $60 N/C\ \hat{x}$ what are the charges on the plates?



$$\underset{\rightarrow}{E} = \frac{\sigma}{\varepsilon_0}\hat{x}$$

so $\sigma = \varepsilon_0 E = (9\times10^{-12}\times 60)\ C/m^2 = 5.4\times10^{-10} C/m^2$

Left plate $Q = 5.4\times10^{-9} C$

Right plate $Q = -5.4\times10^{-9} C$

E-5-3 An electron with energy 1.6×10^{-17} Joules is fired directly toward a metal plate that has a surface charge density $-2\times10^{-6} C/m^2$. From what distance must the electron be fired if it is to just fail to strike the plate? (

The Electron has initial velocity given by

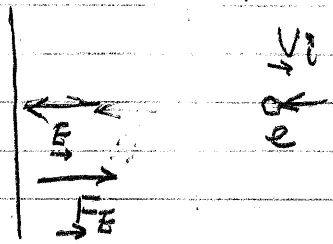
$$\frac{1}{2} m_e v_i^2 = 1.6\times10^{-17} J$$

$$\underset{\rightarrow}{V_i} = -\left(\frac{2\times1.6\times10^{-17}}{9\times10^{-31}}\right)^{1/2} m/s\ \hat{x} = -6\times10^{6} m/s$$

Therefore we assume that its motion is along -ive x

The E-field is $\vec{E} = -\frac{2\times10^{-6}}{2\times9\times10^{-12}}\hat{x} = -1.11\times10^{+5} N/C \left(\frac{\sigma}{2\epsilon_0}\right)$

Force on electron is



$$\vec{F}_E = +\frac{1.6\times10^{-19}\times2\times10^{-6}}{2\times9\times10^{-12}}\hat{x}$$

$$= 1.77\times10^{-14} N\,\hat{x}$$

acceleration

$$\vec{a} = +\frac{1.6\times10^{-19}\times2\times10^{-6}}{2\times9\times10^{-12}\times9\times10^{-31}}\hat{x} = 2\times10^{16} m/s^2\,\hat{x}$$

Motion at constant acceleration

$$v^2 = v_i^2 + 2a(x-x_i)$$

Final vel. is zero so

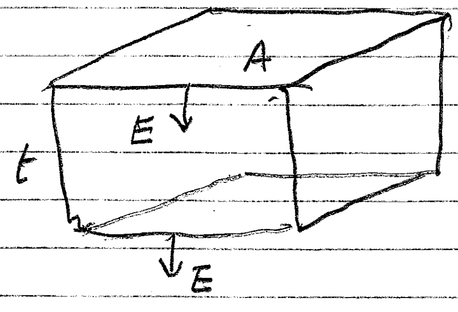
$$(x-x_i) = -\frac{v_i^2}{2a} = -\frac{3.55\times10^{13}}{2\times10^{16}} m$$

$$= -1.8\times10^{-3} m$$

E5-4 Near Earth's surface one measures $\vec{E} = -150 N/C\,\hat{z}$ at $h = 250m$ and $-170 N/C\,\hat{z}$ at $h = 400m$. Estimate the volume charge density of the atmosphere.

Gauss's Law

$$\sum \vec{E}\cdot\Delta\vec{A} = \frac{\sum Q_i}{\epsilon_0}$$



Here $Q = \rho A t$.

and $\sum \underset{\rightarrow}{E} \cdot \underset{\rightarrow}{\Delta A} = -20 A \ \frac{N\text{-}m^2}{C}$.

so $$\frac{\rho \times 150 \times A}{\varepsilon_0} = -20 A \ \frac{N\text{-}m^2}{C}.$$

$$\rho = \frac{20}{150} \times 9 \times 10^{-12} \ c/m^3$$

$$= 1 \cdot 2 \times 10^{-12} \ c/m^3$$

E5-5 In the Van de Graaf demo the air breaks down (sparks!) when the surface field is 3×10^6 N/c. If the sphere has a radius of $0 \cdot 15$m what is the charge on its surface?

$$\underset{\rightarrow}{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \hat{r} \quad r > R \qquad \underset{\rightarrow}{E} = 0, \ r < R.$$

$$Q = 4\pi \varepsilon_0 r^2 E$$

$$= 4\pi \times 9 \times 10^{-12} \times (0 \cdot 15)^2 \times 3 \times 10^6 \ C$$

$$= 7 \cdot 6 \times 10^{-6} \ C$$

Note: Charge density is $\sigma = \frac{Q}{4\pi r^2}$ so field jumps by $\frac{\sigma}{\varepsilon_0}$!