EXERCISES - 4

FORMULAE

Coulomb Force

$$\underset{\rightarrow}{F_E} = \frac{k_e Q_1 Q_2}{r^2}\hat{r}$$

$$k_e = 9\times10^9 \frac{N\text{-}m^2}{C^2}$$

$\underset{\rightarrow}{E}$ - field

$$\underset{\rightarrow}{E} = \frac{\underset{\rightarrow}{F_E}}{q} \qquad q = \text{test charge.}$$

Coulomb $\underset{\rightarrow}{E}$ Pt. charge Q at $r=0$

$$\underset{\rightarrow}{E} = \frac{k_e Q}{r^2}\hat{r}$$

FLUX OF $\underset{\rightarrow}{E}$

$$\Delta\Phi_E = \underset{\rightarrow}{E}\cdot\underset{\rightarrow}{\Delta A}$$

$$= E\,\Delta A\,\mathrm{Cos}(\underset{\rightarrow}{E},\hat{n})$$

GAUSS'S LAW

TOTAL FLUX OF $\underset{\rightarrow}{E}$ THROUGH A CLOSED SURFACE IS DETERINED SOLELY BY THE CHARGES ENCLOSED BY THE SURFACE

$$\sum_c \underset{\rightarrow}{E}\cdot\underset{\rightarrow}{\Delta A} = \frac{1}{\varepsilon_0}\sum Q_i$$

$$\varepsilon_0 = \frac{1}{k_e} = 9\times10^{-12}\ F/m$$

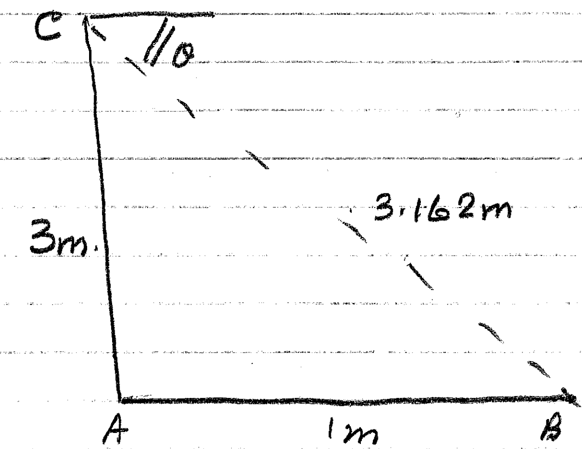
E4-1

$Q_A = 1\mu C$

$Q_B = 2\mu C$

$Q_C = -3\mu C.$

What is the total $\underset{\rightarrow}{F_E}$ on (i) Q_A (ii) Q_C.



(i) There are two forces on Q_A.

Due to B. $$\underset{\rightarrow}{F_E}(B,A) = -k_e \frac{1\times2\times10^{-12}}{1} N\hat{x}$$

$$= -\frac{9\times10^{9}\times 2\times10^{-12}}{1} N\hat{x}$$

$$= -0.018N\,\hat{x}$$

Due to C $$\underset{\rightarrow}{F_E}(C,A) = +\frac{9\times10^{9}\times3\times1\times10^{-12}}{9} N\,\hat{y}$$

$$= +0.003N\ \hat{y}$$

Total $\underset{\rightarrow}{F_E}$ on Q_A

$$\underset{\rightarrow}{F_E} = -0.018N\,\hat{x} + 0.003N\,\hat{y}$$

$18.2\times10^{-3}N$ at angle of 170.5 from X-axis.

(ii) Q_c

$$\underset{\rightarrow}{F_E}(C,A) = -0.003N\ \hat{y}$$

$$\underset{\rightarrow}{F}(C,B) = \frac{9\times10^{9}\times6\times10^{-12}}{10}\left[\cos\theta\,\hat{x} - \sin\theta\,\hat{y}\right] N$$

$$= 4.5\times10^{-3}\left[\frac{1}{3.162}\hat{x} - \frac{3}{3.162}\hat{y}\right]N$$

$$= 1.423\times10^{-3}N\,\hat{x} - 4.269\times10^{-3}N\,\hat{y}$$

$$\underset{\rightarrow}{F_E}(C) = 1.423 \times 10^{-3} N \hat{x} - 4.272 N \hat{y}$$

E 4-2 What is the magnitude and direction of an $\underset{\rightarrow}{E}$-field such that (i) an electron and (ii) a proton would feel a force equal to its weight?

Electron Weight $\underset{\rightarrow}{W_g} = Mg \hat{z}$

$$= -9 \times 10^{-31} \times 9.8 \, N \hat{z}$$

$$\underset{\rightarrow}{F_E} = q \underset{\rightarrow}{E}$$

$$= -1.6 \times 10^{-19} \underset{\rightarrow}{E}$$

so $\underset{\rightarrow}{E} = + \dfrac{9 \times 10^{-31} \times 9.8}{1.6 \times 10^{-19}} \hat{z} = 5.5 \times 10^{-11} \dfrac{N}{C} \hat{z}$

Proton Weight $\underset{\rightarrow}{W_g} = -1.6 \times 10^{-27} \times 9.8 \, N \hat{z}$

$$\underset{\rightarrow}{F_E} = q \underset{\rightarrow}{E} = +1.6 \times 10^{-19} \underset{\rightarrow}{E}$$

$$\underset{\rightarrow}{E} = \frac{-1.6 \times 10^{-19} \times 10^{-27}}{1.6 \times 10^{-19}} N \hat{z} = 10^{-8} \frac{N}{C} \hat{z}$$

E-4-3 The Intensity of the Earth's $\underset{\rightarrow}{E}$-field near its surface is $\sim 130 \, N/C \, \hat{z}$. What is the Earth's charge, assuming that Earth is a sphere and cause this $\underset{\rightarrow}{E}$-field?

On the surface of a sphere

$$\underset{\rightarrow}{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

So $$\frac{Q}{4\pi\varepsilon_0 R_E^2}\hat{r} = -130\,N/C\,\hat{r} \qquad R_E = 6400\,km.$$

$$Q = -4\pi\varepsilon_0 R_E^2 \times 130\ C.$$

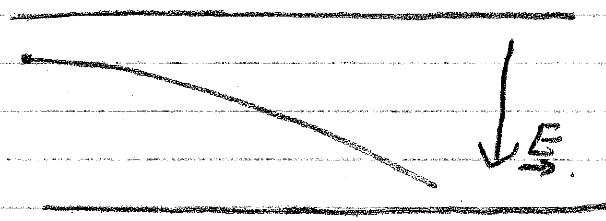
$$= -4\times\pi\times 9\times 10^{-12}\times(6400\times 10^3)^2\times 130\ C$$

$$= -4\times\pi\times 9\times 10^{-12}\times(6.4)^2\times 10^{12}\times 130\ C.$$

$$\simeq -6.02\times 10^5\,C$$

<u>E4-4</u> A Constant $\underset{\rightarrow}{E}$ - field.

$\underset{\rightarrow}{E} = -10^3\,N/C\,\hat{y}$ exists between two low wide parallel plates. If a 10^{-4} kg mass of charge of $10\mu C$ is released near the top plate at a velocity of $100\,m/s\,\hat{x}$ what is its motion?



$$\underset{\rightarrow}{F_E} = q\underset{\rightarrow}{E} = -10^{-5}\times 10^{3}\,N\,\hat{y} = -10^{-2}\,N\,\hat{y}$$

acceleration
$$\underset{\rightarrow}{a} = -\frac{10^{-2}}{10^{-4}} = -100\,m/s^2\,\hat{y}$$

$\therefore$ Motion is like projectile motion when a projectile is launched horizontally.

$\underset{\rightarrow}{v} = 100\,m/s\ \hat{x} - (100\,t)\,m/s\ \hat{y}$

$\underset{\rightarrow}{y} = (y_0 - 50t^2)\,m\ \hat{y}$

E4-5

An Electron moves in a circular orbit about a stationary proton. (i) What provides the centripetal force? (ii) The Electron has a kinetic Energy of 2.18×10^{-18} J, what is (i) its potential Energy, its speed and the radius of the orbit?

(i) The required centripetal force is

$$\underset{\rightarrow}{F_G} = -\frac{m_e v_e^2}{r}\hat{r}$$

m_e = mass of Electron
v_e = speed of Electron

It is provided by the Coulomb Force:

$$\underset{\rightarrow}{F_E} = -\frac{9\times10^9 \times (1.6\times10^{-19})^2}{r^2}\hat{r}$$

(ii) For a circular orbit we showed in Phys121 that the potential energy (-ive) is twice the kinetic Energy so

Potential Energy $P_e = -4.36 \times 10^{-18}$ J speed $\quad K_e = \frac{1}{2} m_e V_e^2$

$$V_e = \sqrt{\frac{2K_e}{m_e}} = \sqrt{\frac{2 \times 2.18 \times 10^{-18}}{9 \times 10^{-31}}}$$

$$= 2.2 \times 10^{6} \, m/s.$$

radius

$$\frac{m_e V_e^2}{r} = \frac{k_e (1.6 \times 10^{-19})^2}{r^2}$$

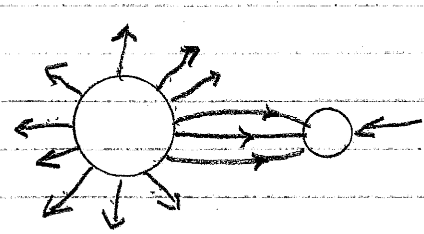
$$r = \frac{k_e (1.6 \times 10^{-19})^2}{m_e V_e^2}$$

$$= \frac{9 \times 10^9 \times 2.56 \times 10^{-38}}{4.36 \times 10^{-18}}$$

$$= 5.3 \times 10^{-11} \, m$$

This is close to the so-called Bohr radius, the putative "size" of the hydrogen atom. Caution, This is not the appropriate model for the hydrogen atom.

E 4-6 Two conducting spheres are held close together so that the E-field lines are as shown. What is



the sign (+ive or -ive) on them and what is the ratio of the charge on the small sphere relative to that on the large sphere?

Large sphere $\rightarrow$ +ive

Small sphere $\rightarrow$ −ive

# of lines is proportional to Q so

$$\frac{Q_{Large}}{Q_{Small}} = 3.$$