EXERCIZES 2

FORMULAE

Travelling Wave $D = f(x \mp vt)$

has velocity $\underline{v} = \pm v\hat{x}$

Periodic or Harmonic Wave

$$D = A\sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right) = A\sin(kx - \omega t)$$

$A \rightarrow$ Amplitude
$\lambda \rightarrow$ Wavelength () Wave speed $v = \lambda f$
$T \rightarrow$ Period
$\frac{1}{T} \rightarrow$ frequency (f), $\omega = 2\pi f$

$\underline{A} \parallel \hat{x} \rightarrow$ Longitudinal, $\underline{A} \perp \hat{x} \rightarrow$ Transverse
Speed of wave on Stretched string

$$v = \sqrt{\frac{T}{\mu}}$$

$T =$ Tension
$\mu =$ Mass per meter

Power transport by harmonic wave on stretched string.

$$P_w = \frac{1}{2} A^2\omega^2 \frac{T}{v} = \frac{1}{2} A^2\omega^2 \mu v$$

Speed of sound in a gas

$$v_s = \sqrt{\frac{\gamma P_0}{\rho_0}} = \sqrt{\frac{\gamma k_B T}{m}}, \quad \gamma = \frac{c_p}{c_v}$$

Sound level in dB $\beta = 10 \log \frac{I}{I_0}$ $\quad I_0 = 10^{-12}$ Watt/m^2

$$I = \frac{P_w}{4\pi R^2}$$

Periodic Sound wave.

DISPLACEMENT	**PRESSURE**
$S = S_m \sin(kx - \omega t)$.	$P = P_0 - \gamma P_0 S_m k \cos(kx - \omega t)$

Intensity of periodic Sound wave.

$$I = \frac{1}{2} S_m^2 \omega^2 \frac{\gamma P_0}{V_s}$$

Standing waves

Both ends fixed $\quad \frac{n \lambda_n}{2} = L \quad n = 1, 2, 3, \ldots$

One end ~~fixed~~ one open $\quad \frac{(2n+1)\lambda_n}{4} = L \quad n = 1, 2, 3$

Reflexion at fixed End produces

$$y = 2 A_i \sin kx \cos \omega t.$$

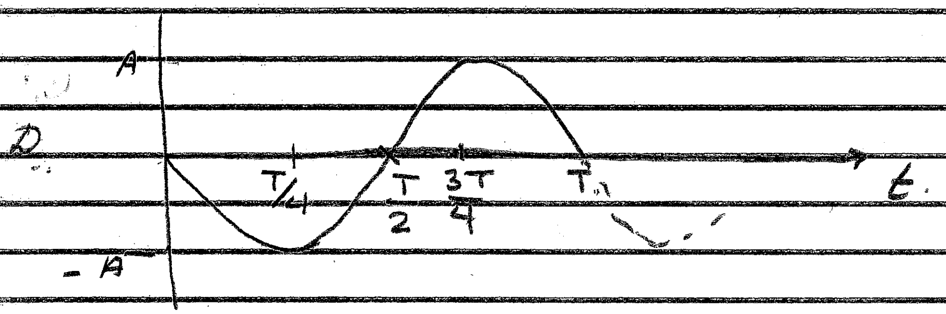
Reflexion at open end produces

$$y = 2 A_i \cos kx \sin \omega t.$$

E2-1 A sinusoidal wave travels along a string. The time for a particular point to move from maximum displacement to zero is 0.5 sec. What are the (a) period (b) frequency. If the wave speed is 5 m/s, what is the wavelength.

The wave is $D = A\sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right)$.

For fixed x, say x = 0 it will look like



In going from maximum to zero it will take $\frac{T}{4}$ secs.

Hence period $T = (4 \times 0.5) = 2$ sec

frequency $f = \frac{1}{2}$ Hz.

Wave speed $V = \lambda f$

Hence $\lambda = \frac{V}{f} = 5 \times 2 = 10$ m

E2-2 A transverse periodic wave is travelling on a String, At $x = 10 cm$ the displacement of the particles is given by

$$y = (5.0\ cm) \sin[1.0 - (4.0\ rad/s)t)$$

(i) What is the wavelength of the wave?

(ii) what is the frequency of the wave

(iii) If the linear density is 4 g/cm what is the tension in the string?

The equation of the wave is

$$y = A \sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right).$$

at $x = 0.1 m$

$$\frac{2\pi x}{\lambda} = 1 \quad so \quad \lambda = 2\pi \times 0.1 = 0.628 m.$$

$$2\pi f = 4\ rad/s \quad so \quad f = \frac{4}{2\pi}\ Hz.$$

speed

$$V = \lambda f = 0.628 \times \frac{4}{2\pi} = 40\ cm/s.$$

$$v = \sqrt{\frac{T}{\mu}} \qquad T = \mu v^2 = \frac{4 \times 10^{-3}}{1 \times 10^{-2}} \times (0.4)^2 = 0.064\ N$$

E2-3 A string 2.7m long has a mass of 270g. The tension in the string is 40N. What must be the frequency of waves of amplitude 0.8 cm in order that the average power transmitted is 80W?

Power $P_W = \frac{1}{2} A^2 \omega^2 \mu V$

$$= \frac{1}{2} A^2 \omega^2 \mu \sqrt{\frac{F}{\mu}}$$

Here $\mu = \frac{0.27}{2.7} = 0.1 \text{ kg/m}$

$$v = \sqrt{\frac{40}{0.1}} = 20 \text{ m/s}$$

$$A = 0.8 \text{ cm} = 8 \times 10^{-3} \text{ m}$$

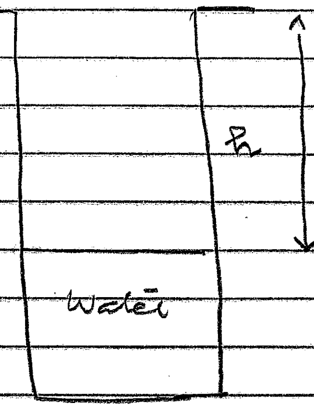
$$\omega = \sqrt{\frac{2 P_W}{A^2 \mu V}} = \sqrt{\frac{2 \times 80}{8 \times 10^{-3} \times 0.1 \times 20}} = 10^2 \text{ rad/s}$$

$$f = \frac{\omega}{2\pi} = 15.9 \text{ Hz}$$

E2-4 A Stone is dropped into a well. The sound of the splash is heard 3s later. What is the depth of the well?

Assume speed of sound $V_s = 340 \text{ m/s}$

Total time



$$t = t_{drop} + t_{rise}$$

During drop it is free fall

$$y = y_0 - 4.9t^2$$

$$0 = h - 4.9\,t_{drop}^2$$

$$t_{drop} = \sqrt{\frac{h}{4.9}} \text{ sec}$$

$$t_{rise} = \frac{h}{340} \text{ sec}$$

so $$\frac{h}{340} + \sqrt{\frac{h}{4.9}} = 3.$$

$$2.94\times10^{-3}\,h + 0.45\sqrt{h} - 3 = 0$$

This is a quadratic Eqn. in $\sqrt{h}$.

$$\sqrt{h} = \frac{-0.45 \pm \sqrt{(0.45)^2 + 4\times3\times2.94\times10^{-3}}}{2\times2.94\times10^{-3}} \quad m^{1/2}$$

$$= 6.46\ m^{1/2}$$

$$\therefore h = 41.76\,m$$

E2-5 A "point" source of sound emits 25W. What is the (i) INTENSITY (ii) sound level at 2.5m away.

(i) $$I = \frac{P}{4\pi r^2} = \frac{25}{4\pi\times(2.5)^2} = 0.32\,W/m^2$$

(ii) Sound Level

$$\beta = 10 \log \frac{I}{10^{-12}}$$

$$= 10 \left[\log_{10} 0.32 + 12\right]$$

$$= 115\,dB$$

When we recall that 120 dB is the threshold of pain, this answer tells us that buying 100W amplifiers for use at home is rather unwise because we will get 130 dB at 2.5 m

E7-6 The G string on a violin is 30 cm long. When played without fingering it vibrates at a frequency of 196 Hz. The next higher notes are A (220 Hz), B (247 Hz), C (262 Hz) and D (294 Hz). How far from the end of the string must a finger be placed to play Each of these notes?

In the fundamental mode the wavelength obeys $\frac{\lambda_1}{2} = l$

hence frequency goes $f_1 = \frac{V}{2L}$

Given $\frac{v}{0.60} = 196 Hz$

or $\frac{v}{196} = 0.6 m.$

To get A we need $l = \frac{0.6 \times 196}{220} = 0.535m$

So place finger at 0.065m from End

for B $l = \frac{0.6 \times 196}{247} = 0.476m.$

put finger 0.124m from End

for C $l = \frac{0.6 \times 196}{262} = 0.449m.$

put finger 0.151m from End

for D $l = \frac{0.6 \times 196}{294} = 0.4$

put finger 0.2m from End