EXERCIZES - 11

Maxwell's Equations and E-M waves

$$\sum_c \underset{\rightarrow}{E} \cdot \underset{\rightarrow}{\Delta A} = \frac{1}{\varepsilon_0} \sum Q_i \qquad (1) \quad \text{Gauss}$$

because stationary Q generates Coulomb

$$\underset{\rightarrow}{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \hat{r}$$

$$\sum_c \underset{\rightarrow}{B} \cdot \underset{\rightarrow}{\Delta A} = 0 \qquad (2) \quad \text{Gauss.}$$

because generators of $\underset{\rightarrow}{B}$ are dipoles of no size or currents for which $\underset{\rightarrow}{B}$ circulates around current.

$$\sum_c \underset{\rightarrow}{B} \cdot \underset{\rightarrow}{\Delta l} = \mu_0 \sum I_c + \mu_0 \varepsilon_0 \frac{\Delta \phi_E}{\Delta t} \qquad (3)$$

$I_c \rightarrow$ Conduction Current

$\varepsilon_0 \frac{\Delta \phi_E}{\Delta t} = i_d$, displacement current

$$\sum \underset{\rightarrow}{E_{NC}} \cdot \underset{\rightarrow}{\Delta l} = -\frac{\Delta \phi_B}{\Delta t} = \mathcal{E}$$

because $\underset{\rightarrow}{E_{NC}}$ appears in every loop surrounding

$\frac{\Delta \phi_B}{\Delta t}$ MINUS SIGN ON RIGHT IS <u>ESSENTIAL</u>

In Vacuum $Q = 0$, $I_c = 0$

Maxwell showed that both $\underset{\rightarrow}{E}$ and $\underset{\rightarrow}{B}$ are travelling waves

with speed $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8$ m/s.

so $\underset{\rightarrow}{E} = \underset{\rightarrow}{E_m} \sin \frac{2\pi}{\lambda}(z \mp ct)$

$$\underset{\rightarrow}{B} = \underset{\rightarrow}{B_m} \sin (z \mp ct).$$

Wave is totally transverse $\underset{\rightarrow}{E_m} \perp \hat{z}$, $\underset{\rightarrow}{B_m} \perp \hat{z}$ and

$\underset{\rightarrow}{E_m} \perp \underset{\rightarrow}{B_m}$. Also $E_m = c B_m$.

The electro-magnetic wave transports energy

Intensity at distance r from point source which emits P_w Watts

$$I(r) = \frac{P_w}{4\pi r^2}.$$

Average Intensity of E-M wave.

$$\langle I \rangle = \frac{1}{2} \epsilon_0 c E_m^2 = \frac{c B_m^2}{2\mu_0} = \frac{E_m B_m}{2\mu_0}$$

Radiation $\Longleftrightarrow$ EM wave

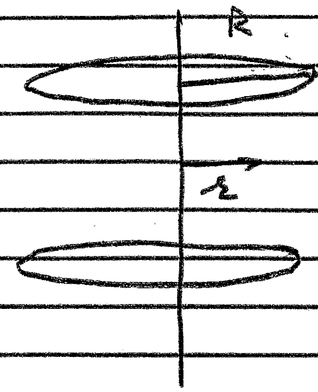
Light: Transverse EM-wave, speed $c = 3 \times 10^8$ m/s

Wavelengths $400\,nm < \lambda < 700\,nm$ in vacuum

In media speed $v < c$.

$n = \frac{c}{v}$ refractive index.

E11-1 Shown is a capacitor whose plates are circular of radius R. Calculate the $\underset{\rightarrow}{B}$ field due to the displacement current I_D as a function of r.



Since I_D is uniform one can define a current density $J_D = \frac{I_D}{\pi R^2}$.

The symmetry is cylindrical so $\underset{\rightarrow}{B}$ can be a function of r only.

Maxwell Eqn $\sum_c \underset{\rightarrow}{B} \cdot \underset{\rightarrow}{\Delta l} = \mu_0 i_D$

where i_D is the current inside the loop of radius r.

For $r < R$ $\quad B(r)\, 2\pi r = \mu_0 J_D \pi r^2$

$$\underset{\rightarrow}{B}(r) = \frac{\mu_0 J_D r}{2} \hat{\phi}$$

for $r > R$ $\quad B(r)\, 2\pi r = \mu_0 J_D \pi R^2$

$$\underset{\rightarrow}{B}(r) = \frac{\mu_0 I_D}{2\pi r} \hat{\phi}$$

The problem is exactly the same as a current density $\vec{J}$ flowing through a conducting wire of radius R.

E11-2 The magnetic field in an EM-wave is given by

$$\vec{B}_y = 2\times10^{-7}\sin(0.5\times10^{3}z - 1.5\times10^{11}t)\,\hat{y}\ T.$$

a) Calculate the amplitude, wavelength and frequency of this wave.

b) Write an expression for the Electric field.

The wave is $B_y = B_m \sin(kz - \omega t)\,\hat{y}\ T.$

$\vec{B}_m = 2\times10^{-7}\,\hat{y}\ T.$

$k = \frac{2\pi}{\lambda}$ $\quad \lambda = \frac{2\pi}{0.5\times10^{3}} = 4\pi\times10^{-3}\ m.$

$\omega = 2\pi f$

$$f = \frac{\omega}{2\pi} = \frac{1.5\times10^{11}}{2\pi}\ Hz$$

velocity

$$\lambda f = 4\pi\times10^{-3}\times\frac{1.5}{2\pi}\times10^{11} = 3\times10^{8}\ m/s.$$

Corresponding E wave

$$E_m = cB_m = 3\times10^{8}\times2\times10^{-7}\ V/m$$

$$\vec{E} = 60\sin(0.5\times10^{3}z - 1.5\times10^{11}t)\,\hat{x}\ V/m$$

E11-3 You walk 200m directly toward a street lamp and find that the intensity increases by a factor of 2. How far were you when you started walking. Treat the lamp as a point source.

$$I = \frac{P_W}{4\pi r^2}$$

At the start point let $I = \frac{P_W}{4\pi r_0^2}$

I at end

$$I = \frac{P_W}{4\pi(r_0 - 2)^2} = \frac{2P_W}{4\pi r_0^2}$$

so

$$2(r_0-2)^2 = r_0^2$$

$$1(r_0^2 - 4r_0 + 4) = \frac{r_0^2}{2}$$

$$\frac{r_0^2}{2} - 4r_0 + 4 = 0$$

$$r_0 = \frac{4 \pm \sqrt{16-8}}{1} = 4 \pm \sqrt{8}$$

$$= 4 \pm 2\sqrt{2}$$

$$= 4 \pm 2.8 = 6.8 \text{ or } 1.2 m$$

$r_0 = 6.8m$

E11-4 Water and Glass have refractive indices of 1.33 and 1.5 respectively. Red light of wavelength 700nm in vacuum enters water and glass. Where will the wavelength be shortest

$n = \frac{c}{v}$ $\qquad\qquad$ $\lambda_0 f = c$

$v = \lambda f$ $\qquad\qquad$ $\lambda_n f = v$

because f does not change so $\lambda_n = \frac{c}{n}$

$$\lambda_n = \frac{\lambda_0}{n}$$

In water $\lambda_n = \frac{700}{1.33}$ nm $= 525$nm [Wavelength of Green light in air]

glass $\lambda_n = \frac{700}{1.5}$ nm $= 467$nm [Wavelength of Blue light in air]