EXERCIZES - 10.

FORMULAE

Faraday- Lenz law: If flux of $\underset{\rightarrow}{B}$ varies as a function of time, a non-Coulomb $\underset{\rightarrow}{E}$ appears in every loop surrounding the region where flux of $\underset{\rightarrow}{B}$ is changing. Hence, circulation of $\underset{\rightarrow}{E}_{NC}$ around a closed loop is given by the equation

$$\sum_c \underset{\rightarrow}{E}_{NC} \cdot \underset{\rightarrow}{\Delta l} = -\frac{\Delta \phi_B}{\Delta t}$$

Note: MINUS SIGN ON THE RIGHT SIDE ENSURES That the SENSE OF $\underset{\rightarrow}{E}_{NC}$ is such as to OPPOSE the change in the flux of $\underset{\rightarrow}{B}$.

EMF = Work done by $\underline{E}$ on a unit charge

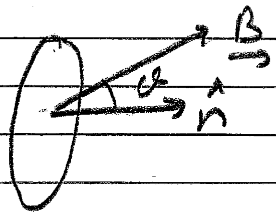
So

$$\sum_c \underset{\rightarrow}{E}_{NC} \cdot \underset{\rightarrow}{\Delta l} = \mathcal{E},$$

the Emf in the loop

FLUX OF $\underset{\rightarrow}{B}$ is $\Delta \phi_B = \underset{\rightarrow}{B} \cdot \underset{\rightarrow}{\Delta A}$

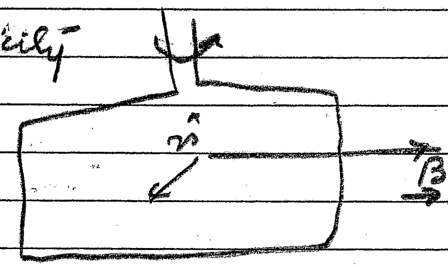
$$= B \Delta A \cos(\hat{n}, \underset{\rightarrow}{B})$$



so flux can change in 3 ways:

1. change $\underline{B}$ as a function of time
2. change Area as a function of time
3. change angle between $\hat{n}$ and $\underline{B}$ as a function of time.

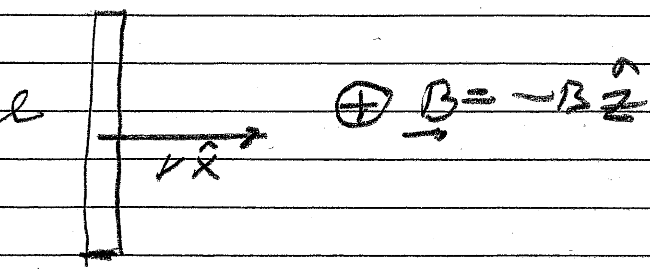
No (3) is used to generate alternating current electricity



A coil of area A is rotated about the y-axis at a constant angular velocity ω in the presence of a constant $\underline{B}$ field, $\underline{B}\hat{x}$. It will generate an EMF

$$\mathcal{E} = BA\omega \sin \omega t.$$

<u>Motional EMF</u>



A conducting bar of length l moves at constant velocity $v\hat{x}$ in a constant $\underline{B} = -B\hat{z}$ field. Because of the force $\underline{F}_B = q[\underline{v} \times \underline{B}]$

On the electrons inside the bar an Emf will develop between its ends

$$\mathcal{E} = BVl$$

Minus at the bottom

+ at the top.

$\vec{F}_E = eVB\hat{y}$

$\vec{F}_B = -eVB\hat{y}$

$E = VB$

$\mathcal{E} = VBl.$

Inductor $\quad L = -\dfrac{\mathcal{E}}{\frac{\Delta i}{\Delta t}}$

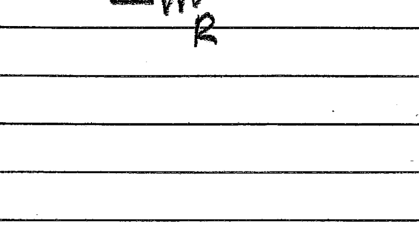
Self Inductance of Solenoid $L_S = \mu_0 n^2 V$

n = # of turns/meter, V = volume

Work done to establish I in L

$$U_B = \frac{1}{2} L I^2$$

apply to solenoid $\quad \eta_B = \dfrac{B^2}{2\mu_0}$ energy stored per m^3 in $\vec{B}$-field



LR circuit Connect 1→2 at $t=0$, Battery establishes current.

$$i = \frac{\mathcal{E}}{R}\left[1 - e^{-\frac{Rt}{L}}\right]$$

$$V_L = \mathcal{E} e^{-Rt/L}.$$

Time Const

$$\tau = \frac{L}{R}$$

depends on both R and L.

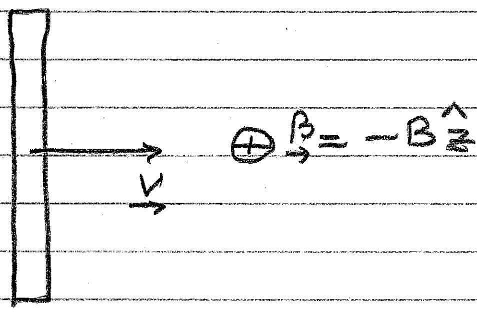
Process involves establishing a current,
L resists it so larger L is, longer it
will take. i is controlled by $\frac{1}{R}$, larger
R, smaller i, quicker it will get there.

EXERCIZES – 10.

E10-1 A copper bar of length l moves at a velocity of $V\hat{x}$ in a constant $\underset{\to}{B}$-field, $\underset{\to}{B} = -B\hat{z}$. First, let us show that an EMF or BVl appears between the ends of the bar. Next, we must ask, since a $\underset{\to}{B}$ field does not do any work on the charge ($\underset{\to}{F_B} \perp \underset{\to}{V}$) where does the energy to set-up $\mathcal{E}$ come from?

$\underset{\to}{F_B} = q\,[\underset{\to}{V} \times \underset{\to}{B}]$

the electrons in the bar feel the force



$\underset{\to}{F_B} = -eVB\hat{y}$ and move down creating

+ charges on top, – at bottom

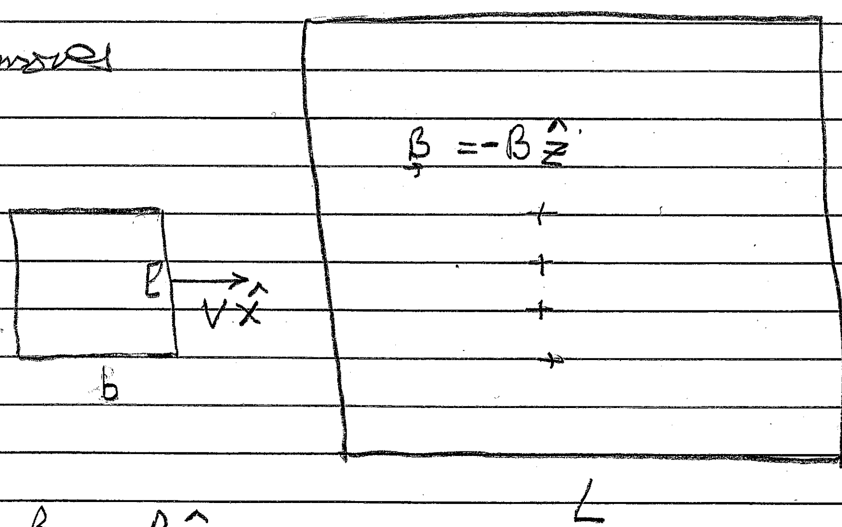
and producing an $\underset{\to}{E} = -E\hat{y}$. The experience a force $\underset{\to}{F_E} = +eE\hat{y}$ for $\equiv m$

$$\underset{\to}{F_B} + \underset{\to}{F_E} = 0 \quad \text{creating } E = VB$$

and $\mathcal{E} = \vec{E} \cdot \vec{\Delta l} = vBl$.

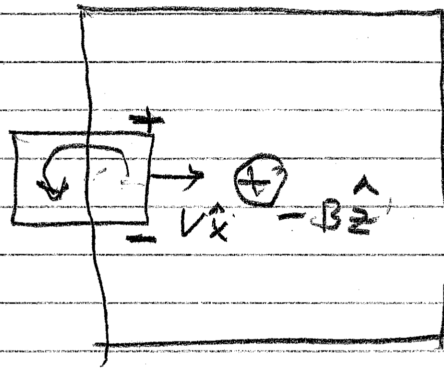
To understand the origin of the energy we need to realize that on the way to em there is flow of charge (current) in the bar so the force $I[\vec{\Delta l} \times \vec{B}]$ will come into play and it will be $-IlB\hat{x}$ so to move bar at constant velocity a force $IlB\hat{x}$ must be applied. This force will do the work that goes to establish the eventual $\vec{E}$-field.

E10-2 A rectangular conducting loop ($l \times b$) moves at constant velocity $v\hat{x}$ through a region of constant $\vec{B} = -B\hat{z}$



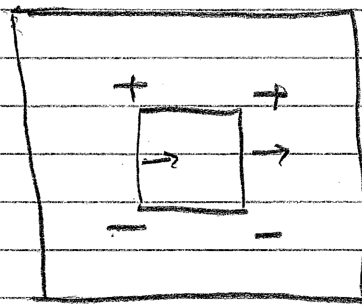
of width L. Starting when $t=0$ when the loop enters $\vec{B}$ sketch the Emf in the loop until it leaves $\vec{B}$.

Just after Entry emf will VBl, − at bottom + at top.

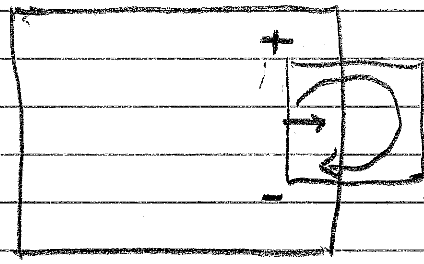


current ccw to oppose increase of Φ_B down.

Emf is zero.

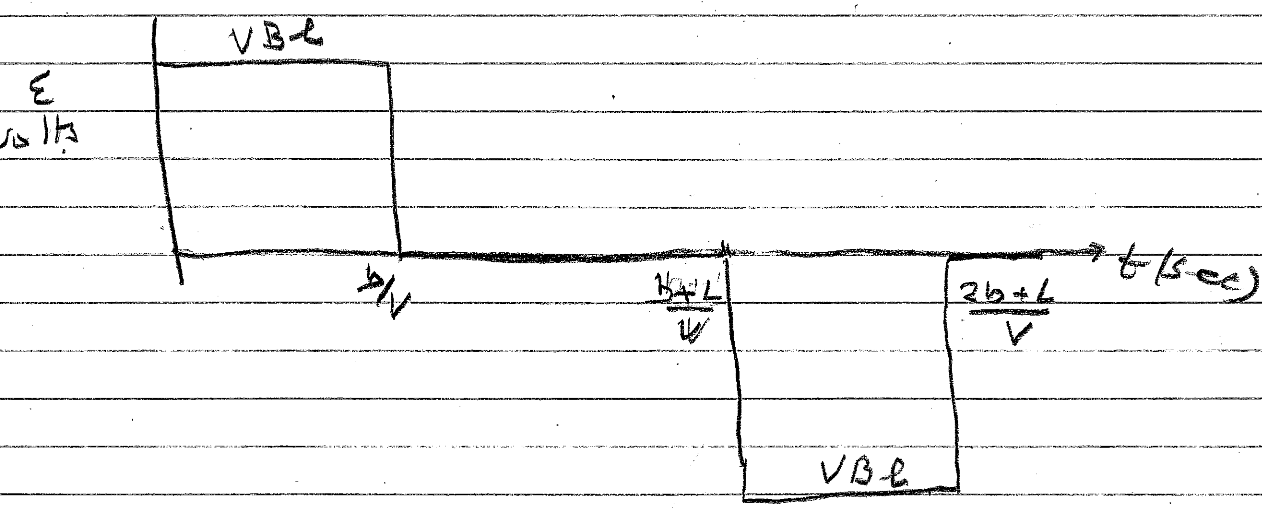


Partially out

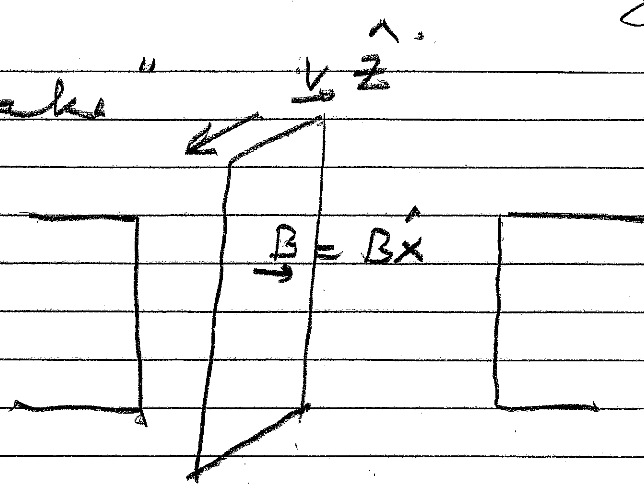


current cw to oppose decrease of Φ_B down.

Totally out emf is zero.



E10-3: "Magnetic Brake"



Given a $\underrightarrow{B} = B\hat{x}$

Take a metal plate, put it $\perp \underrightarrow{B}$ [yz-plane] and try to move it along $\hat{z}$. You will notice that the plate experiences a "Drag": a force appears which opposes the motion. Why?

When you move the plate with vel. $v\hat{z}$ the electrons inside it feel the force [charge −ve]

$$\underrightarrow{F_B} = q[\underline{v} \times \underrightarrow{B}] = -evB\hat{y}$$

which sets up a "current" along $+\hat{y}$. This current feels a force

$$\underrightarrow{F_I} = I[\underrightarrow{\Delta \ell} \times \underrightarrow{B}] = -I\ell B\hat{z}$$

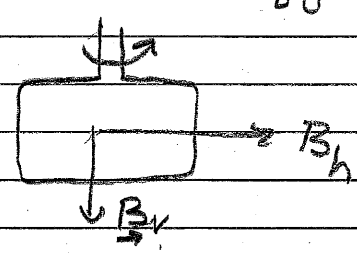
which opposes the motion!

E10-4 A 100 turn coil of diameter 2m is being rotated about the vertical at 1800 rpm. The only $\underline{B}$ field is that due to Earth: vertical component $2.85 \times 10^{-5}\,T\,(B_v)$ horizontal component $3.8 \times 10^{-5}\,T\,(B_H)$

a) which component is responsible for producing the emf in this generator? why?

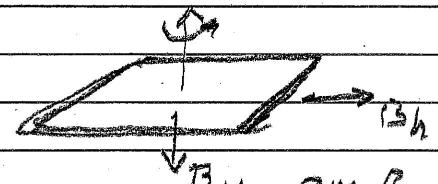
b) what is the maximum emf generated?

a) Only the horizontal component is effective because there is no flux due to B_v. If



b) $\mathcal{E}_m = B A \omega N$

Note if rotation is



emf is zero!

here $B_h = 3.8 \times 10^{-5}\,T$

$A = \pi \times 1^2 = \pi\,m^2$

$N = 100$

$\omega = \dfrac{2\pi}{T} = 60\pi$ rad/s.

1800 rpm = 30 rps

$T = \frac{1}{30}$ sec

$\mathcal{E}_m = 3.8 \times 10^{-5} \times \pi \times 100 \times 60 \times \pi$

$= 2.25$ Volts

E10-5 In modern medical devices the patient is often placed in a $\vec{B}$-field. An significant concern is what kinds of emf/current will be generated if the power goes off. Supposing the field is 1.5T and the area it covers is $0.033\,m^2$. Determine the shortest time in which the field can be allowed to collapse if the emf is to be less than 10mV.

$$\mathcal{E} = -\frac{\Delta \phi_B}{\Delta t}$$

$$10^{-2} = -\frac{\Delta B}{\Delta t} \cdot A$$

$$\frac{\Delta B}{\Delta t} = -\frac{10^{-2}}{3.3 \times 10^{-2}} = -0.3\ T/sec.$$

$$\Delta t = \frac{1.5}{0.3} = 5\,sec.$$

[Since magnets have a large Inductance it is best to depend on $\frac{L}{R}$ to slow down the change in $\vec{B}$]