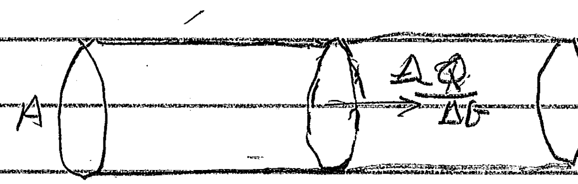
Week 9 – SOLUTIONS

Now we give up the so-called "stationary conditions", that is we allow the mobile electrons inside the conductor to move when an E-field is applied to it. The first concept we need is current

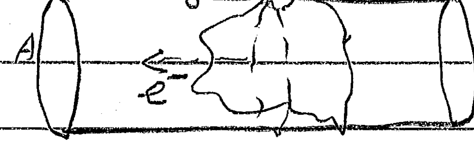
CURRENT Imagine a cylindrical conductor (wire) in which charge is flowing. If a charge Δq flows through any cross-section in a time interval Δt we will define a current



$$I = \frac{\Delta Q}{\Delta t}.$$

LT^{-1} $\quad$ C/S $\quad$ Scalar.

Now what "flows" are the Electrons. so we think of electron cloud drifting through the conductor toward the left with speed $+v_D$ and since electrons have charge negative charge I is to the right

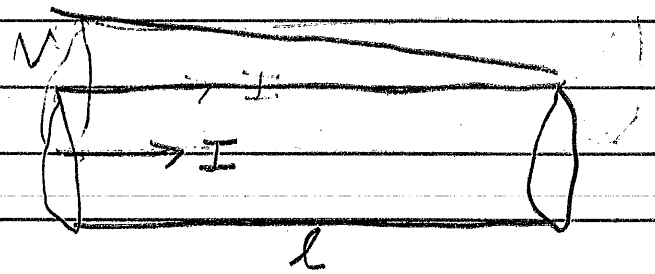


$$I = n_e(-e)(-v_D)A$$
$$= n_e e A v_D$$

where

n_e = No. of mobile electrons/m^3

Next we have established that it costs energy to transport charge through a conductor



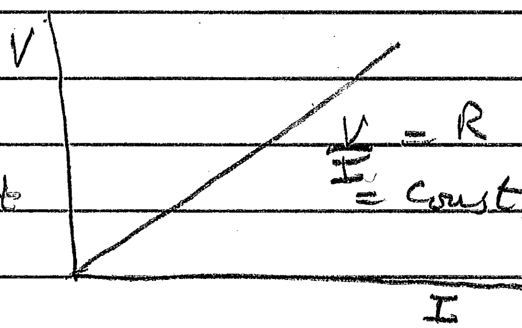
$$\vec{E} = -\frac{V}{l}\hat{x}$$

which Implies that as we go from left to right the potential must drop.

The ratio of V to I is called resistance

$$R = \frac{V}{I} \text{ (ohm)}$$

$ML^2T^{-2}Q^{-1}T$



and if R is independent of I, V vs I is a straight line and the Resistance is said to be "ohmic"

By varying the length (l) and the cross-sectional area (A) we learn that

$$R = \frac{\rho l}{A}$$

where ρ = Electrical Resistivity and we can write

$$I = \frac{V}{R} = \frac{A}{\rho \ell} V = \frac{\sigma A V}{\ell}$$

[Please compare this with equation for heat current in a conductor

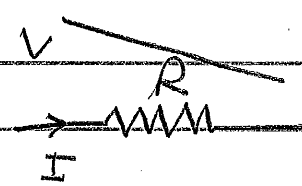
$$\frac{\Delta Q}{\Delta t} = -kA\frac{\Delta T}{\Delta X}$$]

σ = Electrical Conductivity.

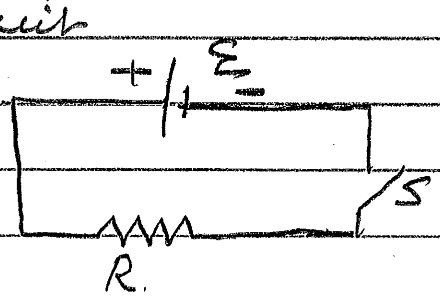
So now we have another device

Resistor

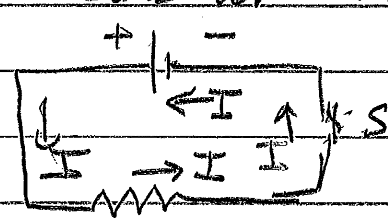
$$R = \frac{V}{I}$$



and to drive a current through it we take our generator and make the circuit



When you close the switch S the battery drives a current through R



To push charge through a potential difference costs energy

$$\Delta U = \Delta q \, V$$

so per second energy cost

$$\text{Power} = \frac{\Delta U}{\Delta t} = \frac{\Delta q}{\Delta t} V = I V$$
$$= I^2 R = \frac{V^2}{R}.$$

ML^2T^{-3} Joule/sec = Watt Scalar

Supplementary Problems - Wk 9

S-21 1 free electron per atom of Cu

$64\ g/mol$

$\rho = 8.9\ g/cm^3$

1 free e^- per atom means # of atoms = # of free e^-'s

just find # of atoms of Cu in $1\ m^3$

$$n_e = 1\,\cancel{m^3}\left(\frac{1\times10^6\ \cancel{cm^3}}{1\ \cancel{m^3}}\right)\left(\frac{8.9\ \cancel{g}}{\cancel{cm^3}}\right)\left(\frac{\cancel{mol}}{64\ \cancel{g}}\right)\left(\frac{6.02\times10^{23}\ atoms}{\cancel{mol}}\right)$$

$n_e = 8.37\times10^{28}$ free electrons available to carry current

S-22 drift velocity is given by

$$V_d = \frac{I}{n_e q A}$$

where n_e is # of free e^-'s per unit volume, $q = |e| = 1.6\times10^{-19}\ C$, and A is cross sectional area

for $I = 1$ Amp, n_e from S-21, A is given by

$A = \pi\left(\frac{d}{2}\right)^2$, for diameter $d = 1\ mm = 0.001\ m$

$$A = \pi\left(\frac{0.001\ m}{2}\right)^2 = \frac{\pi}{4}\times10^{-6}\ m^2$$

$$\Rightarrow V_d = \frac{1\ A\ m^3}{(8.37\times10^{28})(1.6\times10^{-19}\ C)(\frac{\pi}{4}\times10^{-6}\ m^2)} \qquad (1\ A = 1\ C/s)$$

$$V_d = 9.5\times10^{-5}\ m/s = 0.095\ mm/s$$

S-23

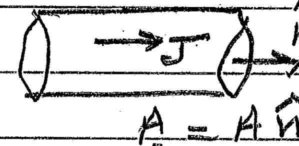
if $d \to 2d$, then $A = \pi\left(\frac{d}{2}\right)^2 \to \pi\left(\frac{\cancel{2}d}{\cancel{2}}\right)^2 = \pi d^2$

so cross sectional area increases by a factor of 4

since $v_d = \frac{I}{n_e q A}$ is inversely proportional to A

$\Rightarrow$ v_d will decrease by a factor of 4

S-24 we know $V = IR$ and $I = \vec{J}\cdot\vec{A}$



and $R = \frac{l}{\sigma A} \Rightarrow l = R\sigma A$

recall also that $V = -E\Delta l$ or $V = \vec{E}\cdot\vec{l}$

therefore

$$V = \vec{E}\cdot\vec{l} = (\vec{J}\cdot\vec{A})R$$

$$= \frac{(\vec{J}\cdot\vec{A})l}{\sigma A}$$

$$\Rightarrow \sigma\vec{E}\cdot\vec{l} = \frac{\vec{J}\cdot\vec{A}\,l}{A}$$

since direction of current and "direction" of cross sectional area are the same, we can say

$$\vec{A}l = A\vec{l}$$

$$\Rightarrow \sigma\vec{E}\cdot\vec{l} = \frac{\vec{J}\cdot\vec{l}\cancel{A}}{\cancel{A}}$$

$$\Rightarrow \sigma\vec{E}\cdot\vec{l} = \vec{J}\cdot\vec{l}$$

$$\Rightarrow \sigma\vec{E} = \vec{J}$$

Chapter 22

22-10 since $I = \frac{\Delta Q}{\Delta t}$, then a current multiplied by a time is a charge

So to kill the battery, we must simply use up the amount of charge given by (90 A)(1 hr)

$$\Rightarrow Q = I t = (90 \tfrac{C}{s})(1 \cancel{hr})\left(\frac{3600 s}{\cancel{hr}}\right)$$

$$Q = 3.24 \times 10^5 C = 324 kC$$

22-13 1.5 A current means 1.5 C are transported per second. How many ions is this? It is the number of charges e that are required to make 1.5 C

i.e., $n_{ion}\, e = 1.5\, C$

$$\Rightarrow n_{ion} = \frac{1.5\, C}{e} = \frac{1.5 \cancel{C}}{1.6 \times 10^{-19} \cancel{C}}$$

$$n_{ion} = 9.375 \times 10^{18} \text{ ions/sec.}$$

22-20 wire has $R = 0.010\,\Omega$

if we stretch the wire w/o changing the volume (i.e. the amount of material), we will decrease its cross sectional area A

So $L' = 2L$ after the stretching

$A' = ?$

$V = AL$, also $V = A'L'$

$$\Rightarrow V = A'(2L) \Rightarrow A' = \frac{V}{2L} = \frac{1}{2}\left(\frac{V}{L}\right) = \frac{1}{2}A$$

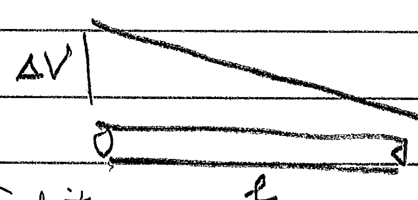
so since $R = \frac{\rho L}{A}$ and $R' = \frac{\rho L'}{A'}$

$$\Rightarrow R' = \frac{\rho(2L)}{(A/2)} = 2\rho L\left(\frac{2}{A}\right) = 4\frac{\rho L}{A} = 4R$$

so the resistance goes up by a factor of 4.

22-31 60 W, 240Ω bulb, filament length is $\ell = 60$ cm

(a) $\Delta V = 120\,V$



recall $|\Delta V| = \underbrace{E\Delta x}_{\text{distance}} = E\ell$ ← distance

So the field in the bulb is:

$$E = \frac{V}{\ell} = \frac{120\,V}{0.60\,m} = 200\,V/m \text{ (or N/C)}$$

(22-31) (b) if $l \to 2l$ w/o changing area or V

$$\Rightarrow E = \frac{V}{l} \to E' = \frac{V}{2l} = \frac{120\,V}{2(1.60\,m)} = 100\ V/m$$

(c) the old current is (assuming ohmic):

$$I = \frac{V}{R} = \frac{120\,V}{240\,\Omega} = 0.5\ A$$

now since $I \propto E$, if $E \to \frac{E}{2}$

then $I \to \frac{I}{2}$

$$\Rightarrow I_{new} = 0.25\ A$$

(d) two ways:

$$R_{new} = \frac{V}{I_{new}} = \frac{120\,V}{0.25\,A} = 480\,\Omega$$

or

$$R = \frac{\rho L}{A} = 240\,\Omega \text{ (given)}$$

$$\Rightarrow R_{new} = \frac{\rho(2L)}{A} = 2R = 480\,\Omega$$

22-33 if $P = 1000\,W$ when $V = 120\,V$, then using

$P = \frac{V^2}{R}$, we see that

$$R = \frac{V^2}{P} = \frac{(120\,V)^2}{1000\,W} = 14.4\,\Omega$$

R will always be the same, so for $V = 220\,V$, using Ohm's law:

$$I = \frac{V}{R} = \frac{220\,V}{14.4\,\Omega} = 15.3\,A > 15\,A$$

therefore the current would be too much.

22-38 the idea here is we can relate the thickness to the charge required, and then get a time from the charge and the current.

to get the charge, we need the volume of the ionized zinc in question, then we can count atoms to get the charge, much like in supp prob S-21

$V = A \Delta y \times 2$ w/ area A, thickness Δy
$\times 2$ because we're doing both sides

$A = (2.0\text{ cm})(2.0\text{ cm}) = 4.0\text{ cm}^2 = 4.0\times10^{-4}\text{ m}^2$

$\Rightarrow V = 2(4.0\times10^{-4}\text{ m}^2)(100\times10^{-9}\text{ m}) = 8.0\times10^{-11}\text{ m}^3$

the mass of this is density of Zn = 7140 kg/m^3

$M = \rho V = 7140\text{ kg/m}^3 (8.0\times10^{-11}\text{ m}^3) = 5.71\times10^{-7}\text{ kg}$

zinc has a molar mass of 65.4 g/mol
So the number of atoms is

$n = 5.71\times10^{-7}\text{ kg}\left(\frac{1000\text{ g}}{\text{kg}}\right)\left(\frac{\text{mol}}{65.4\text{ g}}\right)\left(\frac{6.02\times10^{23}}{\text{mol}}\right) = 5.26\times10^{18}$ atoms

each has a charge of $2e$, so total charge is

$Q = 2ne = 2(5.26\times10^{18})(1.6\times10^{-19}\text{ C}) = 1.68\text{ C}$

at a current of 1.0 mA, the time required is

$\Delta t = \frac{\Delta Q}{I} = \frac{1.68\text{ C}}{1.0\times10^{-3}\text{ A}} = 1680\text{ s} = 28\text{ min}$

22-50 since the same current flows through each half, we can say by ohm's law that

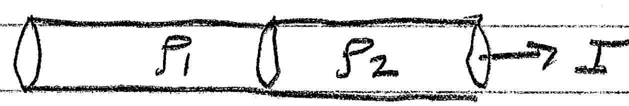
$$I_1 = I_2 \Rightarrow \frac{\Delta V_1}{R_1} = \frac{\Delta V_2}{R_2}$$

since $R = \frac{\rho L}{A}$, and $L_1 = L_2$ and $A_1 = A_2$

$$\Rightarrow \frac{\Delta V_1 \cancel{A_1}}{\rho_1 \cancel{L_1}} = \frac{\Delta V_2 \cancel{A_2}}{\rho_2 \cancel{L_2}} \Rightarrow \frac{\Delta V_1}{\rho_1} = \frac{\Delta V_2}{\rho_2}$$

$$\Rightarrow \frac{\Delta V_1}{\Delta V_2} = \frac{\rho_1}{\rho_2}$$; since $\frac{\rho_2}{\rho_1} = 2$, then clearly

$$\frac{\Delta V_1}{\Delta V_2} = \frac{1}{2}$$



$\rho_2/\rho_1 = 2$.

22-60 we will use the work done by the motor to connect the ideas of mechanical energy (Kinetic here) and electric potential energy,

i.e.: $W = \Delta KE = \frac{1}{2} m v_f^2 - 0 = \frac{1}{2}(0.120\,kg)(1.5\,m/s)^2$

$W = 0.135\,J$

now taking the efficiency into account, i.e. W done on car is 0.8 W done by battery:

$$W_{bat} = \frac{W}{0.8} = 0.169\,J$$

this work comes from the electric potential energy of the battery, which is related to charge passing through motor by $U = W_{bat} = qV$; therefore:

$$q = \frac{W_{bat}}{V} = \frac{0.169\,J}{2(1.5\,V)} = 5.6 \times 10^{-2}\,C$$