Week 8 – SOLUTIONS

ENERGY CONSERVATION

IN Phys 121 we discussed the Conservation of mechanical Energy. The main "actors" were

Mechanical Work

$$\Delta W = \vec{F} \cdot \vec{\Delta s} = F \Delta S \cos(\vec{F}, \vec{\Delta s})$$

$\vec{F}$ → force Vector

$\vec{\Delta s}$ – Displacement Vector

Kinetic Energy

Work stored in motion

$$K = \frac{1}{2} M V^2$$

M → Mass

V → speed.

Potential Energy

Work stored in assembling a system in presence of a conservative force.

Change of potential Energy

$$\Delta U = - \vec{F}_{cons} \cdot \vec{\Delta S}$$

$\vec{F}_{cons}$ is the prevailing conservative force and a force is said to be conservative if the work done is independent of the path and is determined only by the end points of the displacement.

The minus sign on the right side of this equation arises because in order to do the work (without changing the speed) the force doing the work must be exactly equal but opposite to the prevailing force.

The conservation of energy was expressed as

$$K_f + U_f = K_i + U_i$$

final   initial

for an isolated system (no external force, no friction) and

$$K_f + U_f = K_i + U_i + W_{NCF}$$

where

W_{NCF} = work done by non-conservative force

The conservative forces available in 121 were:

1. Near Earth, weight, $\vec{W} = -Mg\hat{y}$
   yielding
   $$U_g(y) = Mgy \quad \text{assuming } U_g = 0 \text{ at } y = 0.$$
2. Spring force $\vec{F}_{sp} = -kx\hat{x}$
   giving
   $$U_{sp}(x) = \frac{1}{2}kx^2$$

Now we have a new force, Coulomb force.

$$\vec{F}_E = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2}\hat{r} = \frac{kQ_1Q_2}{r^2}\hat{r}$$

Like the force of Gravitation this is also a conservative force so the change of of potential energy for the force can be written as

$$\Delta U_E = -\vec{F}_E \cdot \Delta \vec{S} \quad \text{(Potential Energy change)}$$

Here, we take potential energy to be zero at $r \to$ Infinity and (without evaluating the integral) write down that if you have a charge Q sitting at $r=0$ and you bring a charge q from Infinity to the point r, the work stored in the system is

$$U_E(r) = \frac{Qq}{4\pi\varepsilon_0 r} = \frac{kQq}{r} \text{ (scalar as before)}$$

But here we define another Quantity called Electric potential whose change is written as

$$\Delta V = \frac{\Delta U_E}{q} = -\frac{\vec{F}_E}{q} \cdot \Delta \vec{S} = -\vec{E} \cdot \Delta \vec{S}$$

V $ML^2T^{-2}Q^{-1}$ Volt scalar

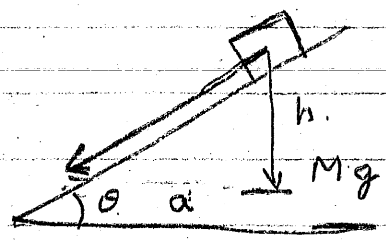
and write for potential due to Q at point r

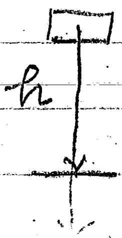
$$V(r) = \frac{Q}{4\pi\varepsilon_0 r} = \frac{kQ}{r}$$

so conservation principle now includes mechanical energy and Electrical energy

$$K_f + U_f(g) + U_f(sp) + U_f(E) = K_i + U_i(g) + U_i(sp) + U_i(E)$$

S-17 A force is said to be conservative if the work done by it is independent of the path and determined only by the end points of the displacement. Example.





$\underset{\rightarrow}{W} = -Mg\hat{y}$

$\underset{\rightarrow}{\Delta S} = -h\hat{y}$

$\hat{y}\cdot\hat{y} = 1$

$\underset{\rightarrow}{W} = -Mg\hat{y}$

$\underset{\rightarrow}{\Delta S} = -a\hat{x} - h\hat{y}$

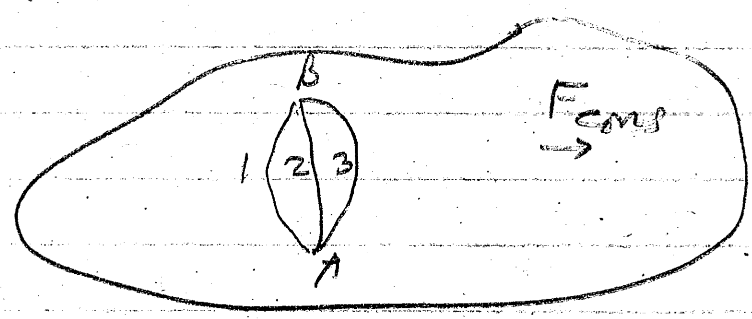
$\hat{x}\cdot\hat{y} = 0$

$\Delta U = Mgh$

$\Delta U = Mgh.$

So

ΔW_{AB} is same for paths 1, 2, 3.

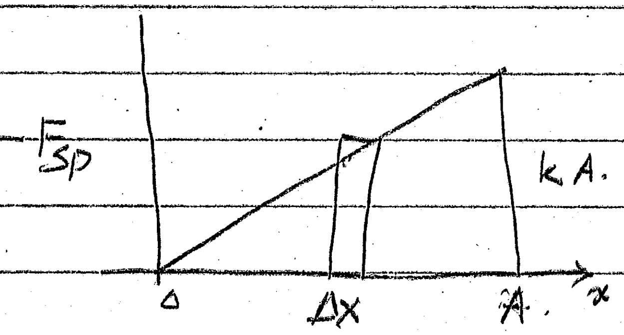


S-18 Potential Energy is the work stored in a system when it is assembled in the presence of a Conservative force. Once stored it can be used to convert to other forms of energy.

Example $\underset{\rightarrow}{F}_{sp} = -kx\hat{x}$

$\Delta U_{sp} = -\underset{\rightarrow}{F}_{sp}\cdot\underset{\rightarrow}{\Delta x}$

ΔW for small change in x is



$$\Delta W = k x \Delta x$$

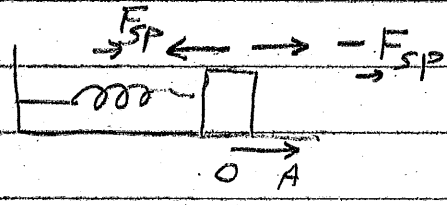
Work stored in going from 0 to A is area of triangle

$$U_{sp}(A) = \frac{1}{2} k A^2$$

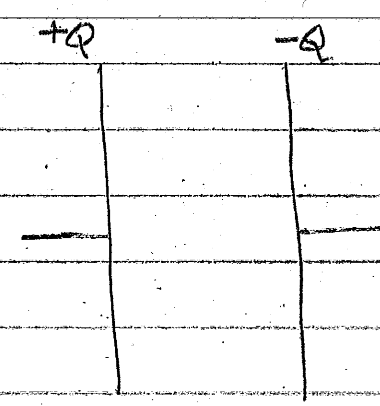
<u>S-19</u> The minus sign in the equation

$$\Delta U = -\vec{F}_E \cdot \vec{\Delta s}$$

arises because to store work (without allowing change in velocity) you must apply a force which is opposite to conservative force. Look at answer to S-18. You need $-\vec{F}_{sp}$ otherwise mass will accelerate.



<u>S-20</u> A capacitor consists of two parallel plates which carry charges $+Q$ and $-Q$. If the plate area is A they have charge densities $\pm\sigma = \pm Q/A$



and therefore there is an $\underline{E}$-field between the plates

$$\underset{\rightarrow}{E} = \frac{\sigma}{\epsilon_0}\hat{x}$$

So a charged capacitor is not "empty" electrically. It has an $\underline{E}$-field in it and the energy expended to place charges $\pm Q$ on it gets stored in the $\underline{E}$-field.

Chap. 21

8 a) $v_i = 8\times10^5$ m/s

$$K_f + U_f = K_i + U_i$$

IF THE PROTON MOVED TO A LOWER potential ($U_f < U_i$) PROTON WOULD ACCELERATE ($K_f > K_i$). To slow DOWN, IT MUST BE MOVING TO A REGION OF HIGHER POTENTIAL.

b) $\frac{1}{2}mv_i^2 = q\Delta V$ $m = 1.6\times10^{-27}$ kg

$q = 1.6\times10^{-19}$ C

$$\frac{mv_i^2}{2q} = \Delta V \approx 3300\text{V}$$

$K_f = 0$, $U_f = 3000$ eV

c) WE KNOW THAT THE P.E. AT THE END WAS 3000 eV. THEN, SINCE ENERGY IS CONSERVED $U_f = K_i = 3000$ eV.

15 15) A BALL BEARING IS A SPHERE. The $\vec{E}$-field outside is $\vec{E} = (kQ/r^2)\hat{r}$ exactly the same as that due to a point charge Q at r=0. Hence,

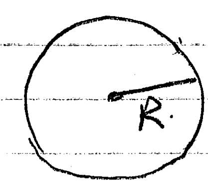
$$V = k\frac{Q}{r} \quad r \geq R \quad (R \equiv \text{RADIUS})$$

THEN,

$$Q = e(2\times10^9) = -1.6\times10^{-19} \times 2\times10^9 \text{ C}$$

$$R = 0.5 \text{ mm}$$

$$V \approx -5800\text{V}$$



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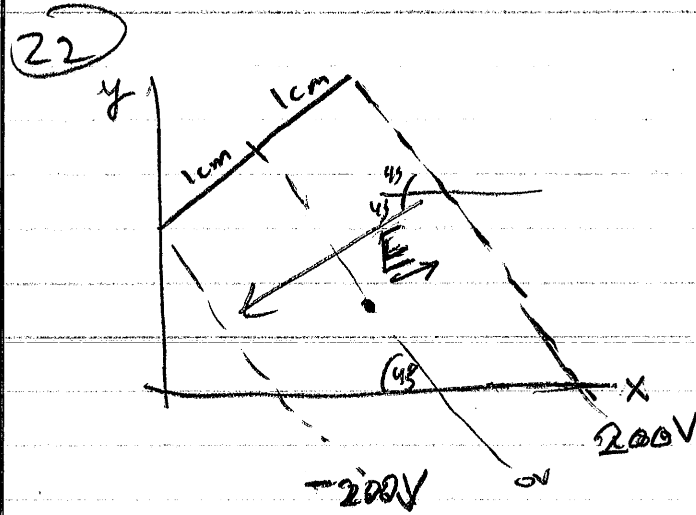


NOW WE KNOW THE DIRECTION SHOULD BE $\perp$ TO THE EQUIPOTENTIAL LINES. THUS, SINCE IT RUNS FROM $+V \Rightarrow -V$ IT HAS DIRECTION OF $\frac{5\pi}{4}$.

$$|E| = \frac{\Delta V}{\Delta d} = 20{,}000 \ V/m$$

$$\underline{E} = 20{,}000 \ V/m \left[-\cos 45\hat{x} - \sin 45\hat{y}\right)$$

(27)

$$C = \varepsilon_0 \frac{A}{d} = \varepsilon_0 \frac{L^2}{d} \qquad \varepsilon_0 = 9 \times 10^{-12} \ F/m$$

$$\Rightarrow \frac{dC}{\varepsilon_0} = L^2 \Rightarrow L = \sqrt{\frac{dC}{\varepsilon_0}}$$

$C = 100\,pF = 10^{-10}\,F$ $\qquad L \approx 4.8\,cm$

$d = 0.2\,mm$

(33) $l = 35\,cm = 35\times10^{-2}\,m$ $d = 0.25\,mm = 0.25\times10^{-3}\,m$

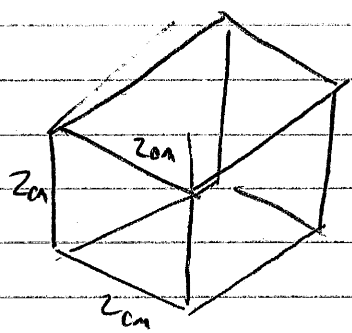
$A = l^2$

$C = \epsilon_0 K \frac{A}{d}$ $\epsilon_0 = 9\times10^{-12}\,F/m$

$K_{PAPER} = 3.0$, $C = 3\epsilon_0 \frac{A}{d} = 3\epsilon_0 \frac{l^2}{d}$

$\approx 13nF$

(41)



$u_E = \frac{1}{2}\epsilon_0 E^2 \leftarrow$ Energy density

WE KNOW $u_E = \frac{u}{Vol}$

$u = 50\,pJ$, $Vol = (2\,cm)^3$

THEN, $\sqrt{\frac{2u_E}{\epsilon_0}} = E$

$\Rightarrow E \approx 1200\,V/m$

5)

$q_1 = -10nC$ $\qquad q_2 = +20nC$

$\xleftarrow{E_2}$ $\xleftarrow{E_1}$

$x=0$ $\quad x \quad$ $x = 15cm$

1) To make the $\vec{E}$ field on the x-axis equal to zero you need to go to a point outside.

2) point must be closer to q_1.

$E_2 \leftarrow \quad \vec{E_1} \rightarrow \quad -10nC \qquad +20nC$

$x \qquad x=0 \qquad x=15cm$

$$\vec{E_1} = \frac{k q_1}{x^2}\hat{x} \qquad \vec{E_2} = -\frac{k q_2}{(x+0.15)^2}\hat{x}$$

$$\vec{E} = (\vec{E_1}+\vec{E_2}) = 0 = \frac{k q_1}{x^2} - \frac{k q_2}{(x+0.15)^2}$$

so

$$\frac{10}{x^2} = \frac{20}{(x+0.15)^2}$$

$$\frac{1}{x} = \frac{\sqrt{2}}{x+0.15}$$

$$\sqrt{2}x = x+0.15 \qquad x = \frac{0.15}{\sqrt{2}-1} = 0.36m$$

Potential at $x = 0.36\,m$

$$V = V_1 + V_2 = \frac{k q_1}{r_1} + \frac{k q_2}{r_2}$$

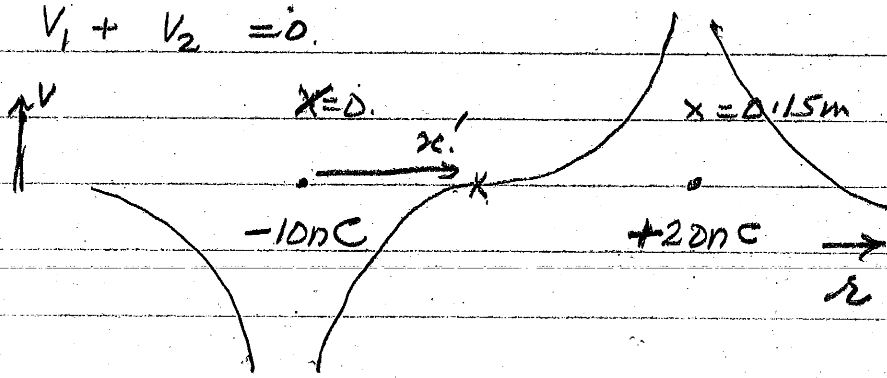
$$= 9\times10^9\left[-\frac{10\times10^{-9}}{0.36} + \frac{20\times10^{-9}}{0.51}\right]$$

$= 103$ Volts

To make $V=0$.

$$V = V_1 + V_2 = 0.$$

you need a point between 0 and 0.15m



$$-\frac{k \times 10 \times 10^{-9}}{x'} + \frac{k \times 20 \times 10^{-9}}{(0.15 - x')} = 0$$

$$\frac{1}{x'} = \frac{2}{0.15 - x'}$$

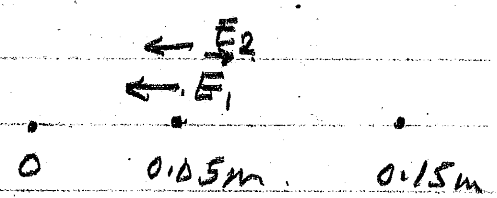
$$2x' = 0.15 - x'$$

$$x' = \frac{0.15}{3} = 0.05\,m.$$

at that pt.

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \left(-\frac{k \times 10 \times 10^{-9}}{(0.05)^2} - \frac{k \times 20 \times 10^{-9}}{(0.10)^2} \right) \hat{x}$$

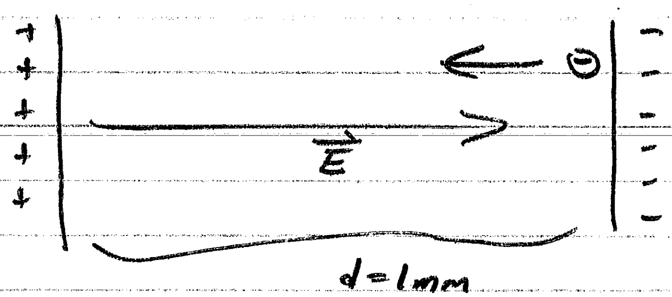


$$= -54000 \text{ V/m } \hat{x}$$

(Incidentally, if you put $-10nC$ at $x = 0.15m$ and $20nC$ at $x=0$, $\vec{E}$ will be reversed and V will be zero at $x = 0.1m$).

(62) $\vec{E} = \dfrac{20{,}000\ V}{m}$

$$K_f + U_f = K_i + U_i \qquad K_i = 0$$



Notice charge is -ive so as Electron goes from – to + its potential energy reduces!

WE CAN DO THIS 2 WAYS. WE CAN SOLVE $F = qE$ AND USE THIS TO FIND $\vec{v}$, OR USE ENERGY. ENERGY IS EASIER.

$$\Delta V = -dE$$

$$U_f - U_i = q\Delta V = -1.6 \times 10^{-19} \times 10^{-3} \times 20{,}000 \text{ Joules.}$$

THEN,

$$K_f = \tfrac{1}{2}mv^2 = q\Delta V = 1.6 \times 10^{-19} \times 10^{-3} \times 20{,}000. \text{ Joules.}$$

$$v^2 = \frac{2qEd}{m}$$

$$v = \sqrt{\frac{2qEd}{m}} \approx 2.7 \times 10^{6}\ m/s$$

$m = 9 \times 10^{-31}\ kg$

(69)

$K_f + U_f = K_i + U_i$

$e^+ \longrightarrow$ (NUCLEUS) $q = 26e^+$

$U_i = 0$

$K_i = ?$

9 fm = r

$K_f = 0$, $U_f = \frac{k\,qQ}{r}$

INITIALLY WE HAVE ONLY KE.

$q = 1.6 \times 10^{-19}\,C$

$Q = 26 \times 1.6 \times 10^{-19}\,C$

$r = 9 \times 10^{-15}\,m.$

$m_p = 1.6 \times 10^{-27}\,kg$

$K_i = \frac{1}{2} m_p v_i^2$

FINALLY WE HAVE ONLY PE!

$U_f = \frac{e\,k\,Q}{r} = \frac{1}{2} m_p v_i^2$

THEN,

$$v_i^2 = \frac{2(26)e^2}{m_p r} k$$

$$v_i = e\sqrt{\frac{52}{m_p r} k} \approx 4 \times 10^7\ m/s$$