SOLUTIONS – 7

PHYSICS BACKGROUND AND FORMULAE

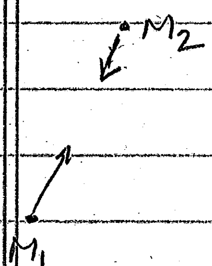
Now we know that matter has two intrinsic properties, Mass (M, always +ive), Charge Q [+ive and −ive]. Each of them brings into play a fundamental force/field

Mass

Newton's Law of Gravitation

$$\vec{F}_G = -\frac{GM_1M_2}{r^2}\hat{r}$$

$$G = 6.7\times10^{-11}\,\frac{N\text{-}m^2}{(kg)^2}$$



① Forces Act Along Line joining M_1 and M_2

② $\vec{F} \parallel -\hat{r}$, force is attractive, If you let $M_1 + M_2$ go they will come together, thus reducing r.

③ A special case of this is the force that a mass experiences when it is just outside the Earth

Charge

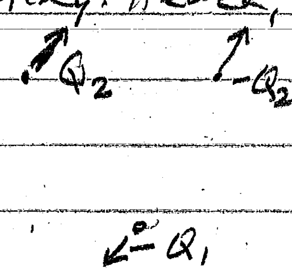
Coulomb's law (Electrostatic force)

$$\vec{F}_E = k\,\frac{Q_1Q_2}{r^2}\hat{r}$$

$$k = 9\times10^{9}\,\frac{N\text{-}m^2}{C^2}$$

① Force Acts Along Line Joining Q_1 and Q_2

② If Q_1, Q_2 have the same sign, both +ive or both −ive, Q_1Q_2 is a positive quantity. Hence,



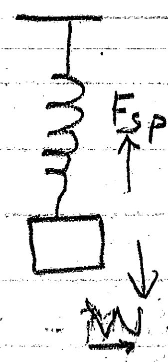
$\vec{F}_E \parallel +\hat{r}$

The forces are Repulsive, if you let Q_1, Q_2 go they will move apart, thereby

M

Near Earth a mass m experiences a force (weight)

$\vec{F}_G = \vec{W} = -Mg\hat{r}$



So if you suspend a mass from a spring the spring stretches to get equilibrium.

– To understand $\vec{W}$ we make two statements:

1 If a stationary mass m experiences a force when there is no visible agency applying the force, it must be located in a gravitational field

$\vec{F}_G = M\vec{G}_F$

2. A stationary mass generates a gravitational field.

$G_F = -g\hat{r}$

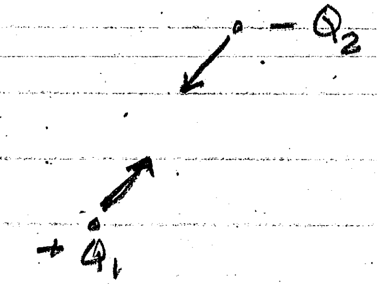
for Earth.

Q

increasing r.

③ If Q_1, Q_2 have opposite signs, one +ive, one –ive, product Q_1Q_2 is –ive, hence

$\vec{F}_G \parallel -\hat{r}$



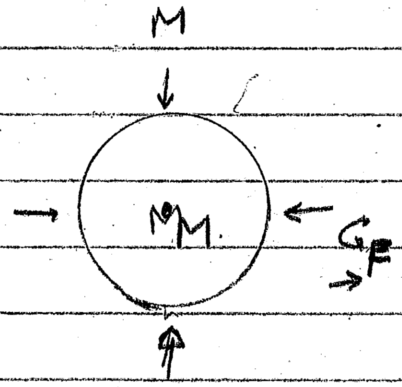
The forces are ATTRACTIVE. If you let Q_1, Q_2 go they will come together, thereby reducing.

By analogy with the Gravitational case we now assert

1. If a stationary charge q experiences a force when there is no visible agency applying the force it must be located in an $\vec{E}$ field

$$\vec{F}_E = q\vec{E}$$

Clearly, E will have the dimensions $MLT^{-2}Q^{-1}$ and the unit will be Newton/Coulomb



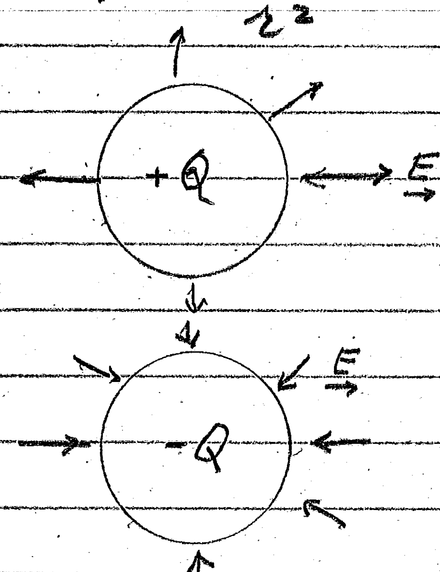
The $\vec{G}_F$ vectors are radii pointing toward the center

Remember! we discovered the $\vec{G}$-field by measing $\vec{F}_G$

Q

2. A stationary point charge Q sitting at $r=0$, Creates a Coulomb $\vec{E}$ field given by

$$\vec{E} = k\frac{Q}{r^2}\hat{r}$$



(SOURCE) Points away from $(r=0)$

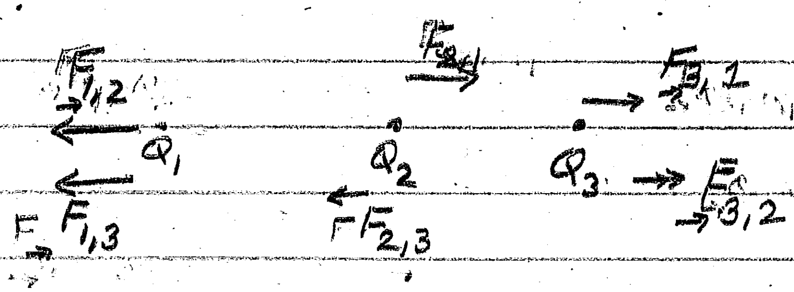
Points toward $r=0$ (SINK)

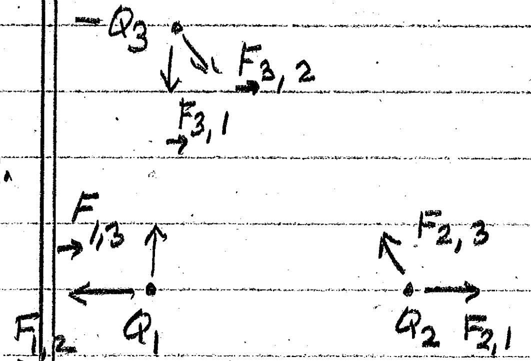
We discover the $\vec{E}$-field by measuring the force on a +ive test charge q and then write

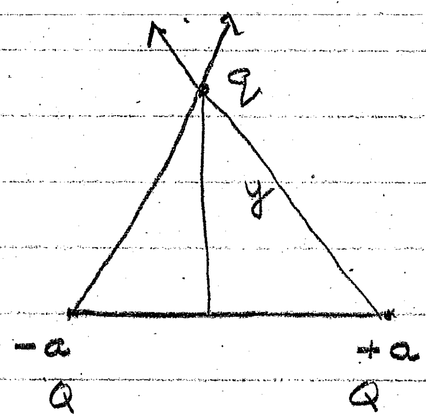
$$\vec{E} = \frac{\vec{F}_E(x,y,z)}{q}$$

FORMULAE

1 forces when 3 charges on x-axis



2. 

3. 

$$\vec{F}_q = \frac{2kqQy\,\hat{y}}{(a^2+y^2)^{3/2}}$$

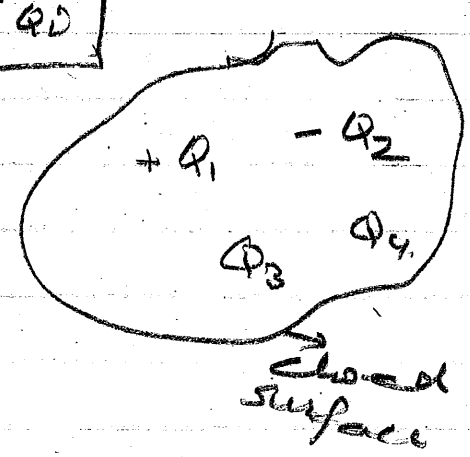
4 Flux of $\underline{E}$ $\quad \Delta\Phi_E = \vec{E}\cdot\vec{\Delta A}$

5. <u>Gauss' Law</u>. Total flux of $\underline{E}$ through a closed surface is determined solely by the sources (+ive charges) and sinks (−ive charges) enclosed by the surface.

$$\boxed{\sum_c \vec{E}\cdot\vec{\Delta A} = \frac{1}{\varepsilon_0}\Sigma Q_D}$$

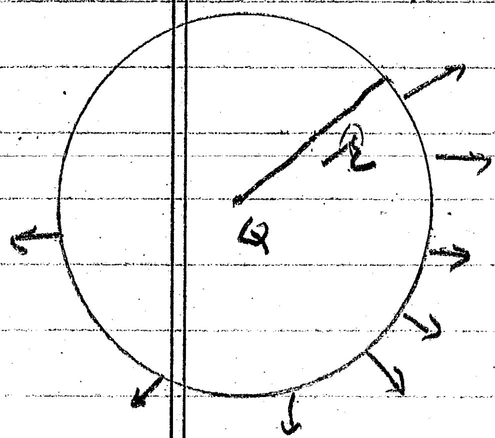
$$\frac{1}{4\pi\varepsilon_0} = k$$

$$\varepsilon_0 = 9\times10^{-12}\ \frac{C^2}{N\text{-}m^2}$$



Applications of Gauss's law:

1. single charge $+Q$ at $r=0$.



Gaussian surface, sphere of radius r

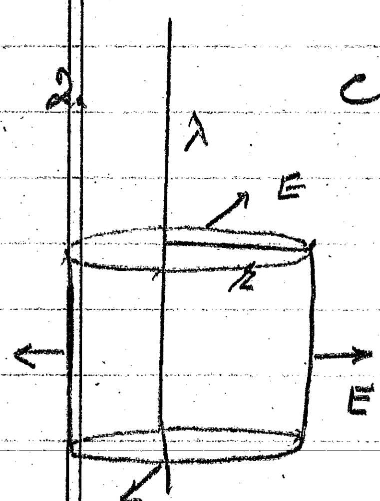
$\underset{\to}{E} \parallel \hat{r}$ and function of r only

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}.$$

$$\underset{\to}{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

for Q -ive $\quad \underset{\to}{E} = -\dfrac{|Q|}{4\pi\varepsilon_0 r^2}\hat{r}$

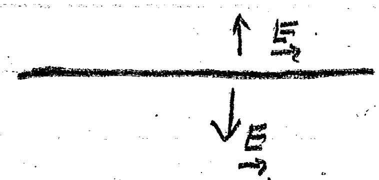
2. Charge λ C/m on line $\parallel \hat{y}$



$\underset{\to}{E} \parallel \hat{r}$ and function r only

$$\underset{\to}{E} = \frac{\lambda}{2\pi r}\hat{r}$$

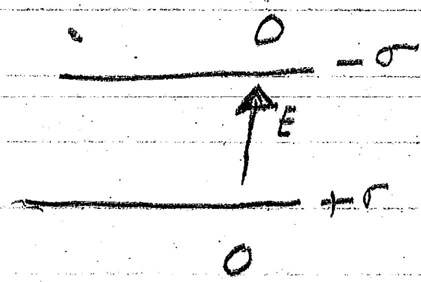
3. Sheet of charge: σ C/m^2 sheet parallel to xz-plane at $y=0$



$y>0 \quad \underset{\to}{E} = \dfrac{\sigma}{2\varepsilon_0}\hat{y}$

$y<0 \quad \underset{\to}{E} = -\dfrac{\sigma}{2\varepsilon_0}\hat{y}$

4. Two sheets parallel to xz-plane.
$+\sigma$ C/m² at $y=0$
$-\sigma$ C/m² at $y=d$.

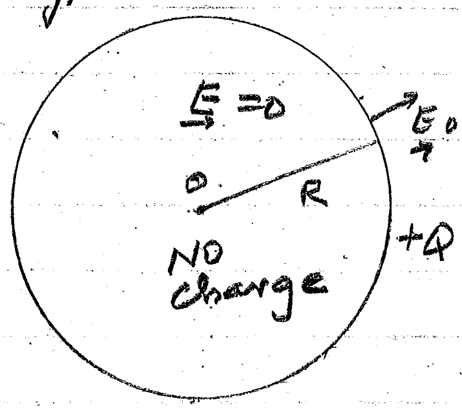


$0<y<d \quad \vec{E} = \dfrac{\sigma}{\epsilon_0}\,\hat{y}$.

$y>d \quad E=0$

$y<0 \quad E=0$.

5. Hollow sphere of charge Q, centered at $r=0$.
(Conducting)

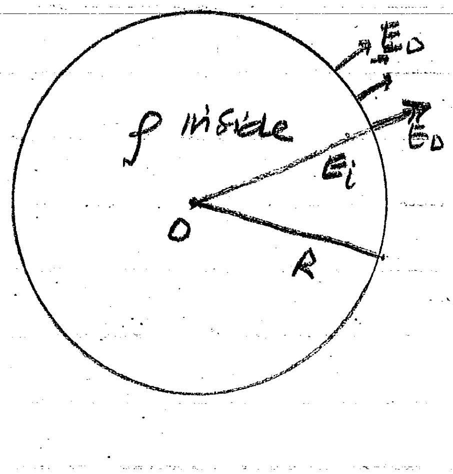


$r<R \quad \vec{E}=0$

$r>R \quad \vec{E} = \dfrac{Q}{4\pi\epsilon_0 r^2}\,\hat{r}$

6. INSULATING sphere centered at $R=0$, Q distribute-d uniformly defining charge density ρ.

$$\rho = \frac{Q}{\frac{4\pi}{3}R^3}$$



$r<R$

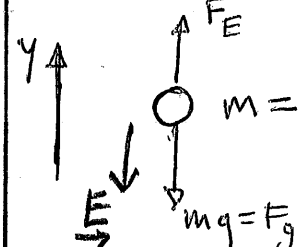
$$\vec{E}_i = \frac{\rho r}{3\epsilon_0}\,\hat{r}$$

$r>R$

$$\vec{E}_o = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{r}$$

20-28

$\vec{F}_E = q\vec{E}$, since q is -ive $\vec{F}_E$ is opposite to $\vec{E}$



$$m = 0.10 \times 10^{-3}\ \text{kg},\quad q = 1.0\times 10^{10}\ e^- \times \frac{-1.6\times 10^{-19}\ C}{e^-}$$

$$q = -1.6\times 10^{-9}\ C$$

Force due to gravity is $-mg\hat{y} = -(0.10\times 10^{-3}\ \text{kg})(9.8\ \frac{m}{s^2})\hat{y}$

To balance this, $= -(9.8\times 10^{-4}\ N)\hat{y}$

we need a force due to the electric field as $+(9.8\times 10^{-4}\ N)\hat{y}$

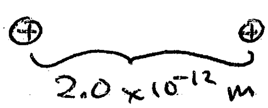
So,

$$\vec{E}q = \vec{F} = +(9.8\times 10^{-4} N)\hat{y}$$

$$\vec{E} = \frac{9.8\times 10^{-4}\ N}{-1.6\times 10^{-9}\ C}\hat{y} = -6.1\times 10^{5}\ \frac{N}{C}\hat{y}.$$

Thus, the electric field strength is $6.1\times 10^5\ \frac{N}{C}$; it points in the $-\hat{y}$ (downward) direction.

20-40



$$\vec{F}_E = \frac{1}{4\pi\varepsilon_0}\frac{qq}{r^2}\hat{r} \qquad q = +e = 1.6\times 10^{-19}\ C$$

$$= +\left(8.99\times 10^9\ \frac{Nm^2}{C^2}\right)\left(\frac{(1.6\times 10^{-19} C)^2}{(2.0\times 10^{-12}\ m)^2}\right)\hat{r}$$

$$= +\left(8.99\times 10^9\ \frac{Nm^2}{C^2}\right)\left(0.8\times 10^{-7}\ \frac{C}{m}\right)^2\hat{r}$$

$$= +(8.99\times 10^9 \cdot 0.64\times 10^{-14})\ N\hat{r}$$

(a) $\vec{F} = +5.8\times 10^{-5}\ N\hat{r}$

Repulsive

20-40 continued

$$\underset{\rightarrow}{F_g} = -G\,\frac{m_p m_p}{r^2}\,\hat{r} \qquad \text{attractive.}$$

$$= -\left(6.67\times10^{-11}\ \frac{\mathrm{N\,m^2}}{\mathrm{kg^2}}\right)\left(\frac{1.67\times10^{-27}\ \mathrm{kg}}{2.0\times10^{-12}\ \mathrm{m}}\right)^2 \hat{r}$$

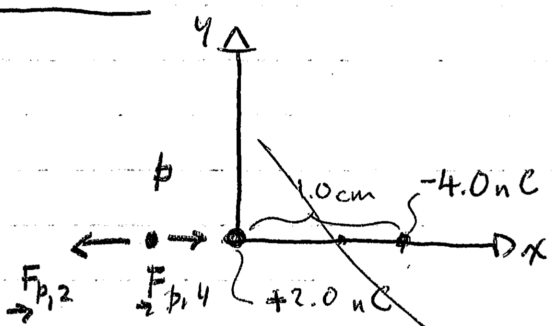
$$= -(6.67\times10^{-11})(0.835\times10^{-15})\ \mathrm{N}\,\hat{r}$$

(b) $$\underset{\rightarrow}{F_g} = -5.57\times10^{-26}\ \mathrm{N}\,\hat{r}$$

(c) $$\frac{F_E}{F_g} = \cancel{\frac{5.57\times10^{-26}\ \mathrm{N}}{5.8\times10^{-5}\ \mathrm{N}}} = \cancel{0.96\times10^{-21}} \cancel{\cong 10^{-21}}$$

$$= \frac{10^{-5}}{10^{-26}} \sim 10^{21}$$

20-52



(a) Force, in this case, is proportional to $\frac{1}{r^2}$; the force on the proton is repulsive from the charge at the origin, and it is attractive to the charge at (1,0) (cm).

Place the proton at x_0.

$$\underset{\rightarrow}{F} \propto \frac{2.0}{x_0^2}\hat{x} - \frac{4.0}{(x_0 - 1\,\mathrm{cm})^2}\hat{x}$$ We want $\underset{\rightarrow}{F} = 0$, so $\frac{2.0}{x_0^2} = \frac{4.0}{(x_0 - 1\,\mathrm{cm})^2}$.

$$\frac{(x_0-1)^2}{x_0^2} = 2.0$$

$$\frac{x_0^2 - 2x_0 + 1}{x_0^2} = 2.0$$

$$x_0^2 - 2x_0 + 1 = 2.0\,x_0^2$$

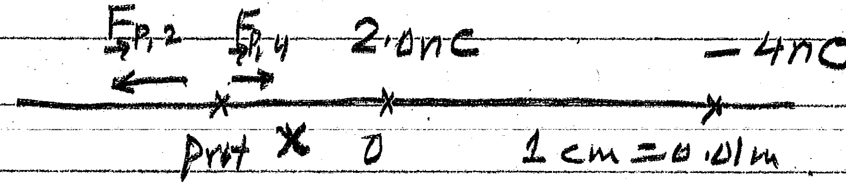
$$0 = 1.0\,x_0^2 + 2x_0 - 1$$

$$x_0 = \frac{-2 \pm \sqrt{4-4}}{2}$$ by quadratic formula

$\Rightarrow$ $\boxed{x_0 = -1.0\,\mathrm{cm}}$

By symmetry, the charge must lie on the x-axis. The position is (-1.0 cm, 0 cm).

20-52 Since $\vec{F}_E$ acts only along line joining charges proton must be placed on x-axis



and closer to the 2.0nC charge b/c force is proportional to Q. Also the two forces must be opposite so locate proton at −x.

Forces on proton are

$$\vec{F}_{p,2} = -k \frac{2\times10^{-9}\times1.6\times10^{-19}}{x^2}\hat{x}$$

$$\vec{F}_{p,4} = +k \frac{4\times10^{-9}\times1.6\times10^{-19}}{(0.01+x)^2}\hat{x}$$

and we want $\vec{F}_{p,2} + \vec{F}_{p,4} = 0$.

$$-k\times\frac{2\times10^{-9}\times1.6\times10^{-19}}{x^2} + \frac{k\times4\times10^{-9}\times1.6\times10^{-19}}{(0.01+x)^2} = 0$$

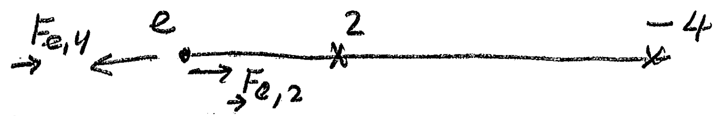
Remove all common factors & let us use cm

$$\frac{2}{x^2} = \frac{4}{(1+x)^2}, \quad \frac{1}{x^2} = \frac{2}{(1+x)^2}$$

Take sq. root $\frac{1}{x} = \frac{\sqrt{2}}{1+x}$ $\quad \sqrt{2}x = x+1$

$$x = \frac{1}{\sqrt{2}-1} = \frac{1}{0.414} = 2.415\text{ cm}$$

20-52

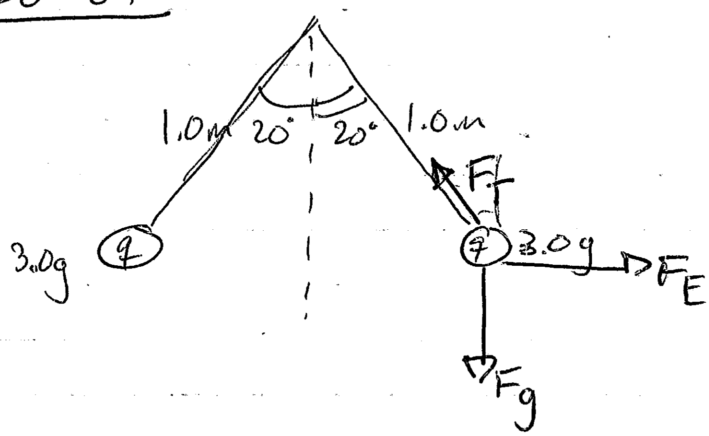


Consider the 0 net force condition for a charge q

$$\vec{F}_{net} = +K\frac{(2.0)\,q}{x_0^2}\hat{x} + K\frac{(-4.0)(q)}{(x_0+1)^2}\hat{x} = 0\,.$$

So, $\boxed{\text{yes,}}$ the electron would also experience 0 net force, since replacing q with $-q$ leaves $F_{net} = 0$.

20-64



$\vec{F}_g + \vec{F}_E + \vec{F}_T = 0$ EQUILIBRIUM

Since $\vec{F}_g \perp \vec{F}_E$, the vertical component of $\vec{F}_T$ must balance $\vec{F}_g$.

$$|\vec{F}_g| = (3.0\times10^{-3}\,\text{kg})(9.8\,\tfrac{\text{m}}{\text{s}^2})$$
$$= 29.4\times10^{-3}\,\text{N}$$

Trigonometry shows

$$\cos(20°) = \frac{F_T}{29.4\times10^{-3}\,\text{N}}$$

$$(0.940)(29.4\times10^{-3}\,\text{N}) = F_T$$
$$27.6\times10^{-3}\,\text{N} = F_T$$

And $F_T\sin(20°)$ is the horizontal component of F_T, which is balanced exactly with F_E.

$$(27.6\times10^{-3}\,\text{N})(\sin(20°)) \doteq 9.4\,\text{N}$$

The charges are $2\cdot(1.0\,\text{m})(\sin 20°) = 0.68$ m apart.

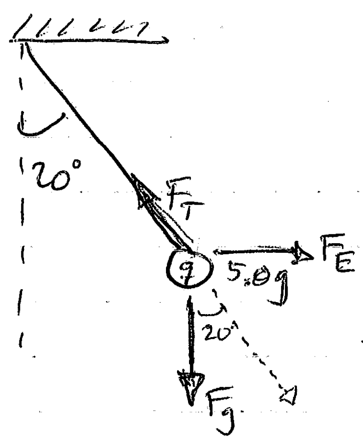
20-64 continued

So: $$9.4\,N = K\frac{(q)^2}{(0.68)^2\,m^2}$$

$$q^2 = \frac{(9.4\,N)(0.46\,m^2)}{K} = \frac{4.3\ N\ m^2}{8.99\times10^9\,\frac{Nm^2}{C^2}} = 0.48\times10^{-9}\,C^2$$

$$q^2 = 4.8\times10^{-10}\,C^2 \rightarrow \boxed{q = 2.2\times10^{-5}\ C}$$

20-65



$$\tan(20°) = \frac{F_E}{F_g}$$

$$F_g \tan 20° = F_E$$

$$mg \tan 20° = qE$$

$$\frac{(5.0\times10^{-3}\,kg)(9.8\,\frac{m}{s^2})}{10^5\,N/C}\tan 20° = q$$

$$(49\times10^2\ C)(\tan 20°) =$$

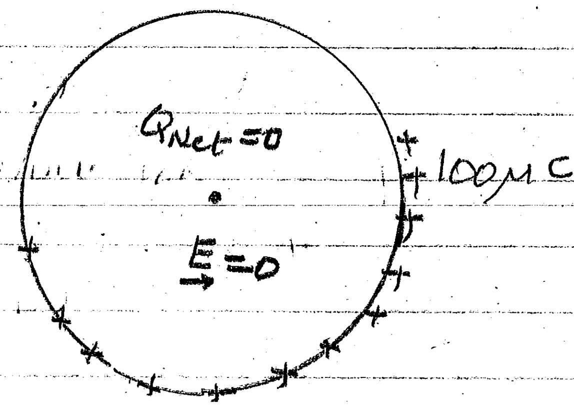
$$\boxed{1.7\times10^2\ C = q}$$

S-11 $\underset{\rightarrow}{E} = \dfrac{\underset{\rightarrow}{F}_E}{q}$ We know that if a stationary charge (neglect gravity) experiences a force it will be located in an E-field. So if you are a "surveyor", given a charge q and a force measuring device you can discover the presence of $\underset{\rightarrow}{E}$ at any location (x, y, z) if you attach the charge to the device and the device registers a non-zero $\underset{\rightarrow}{E}(x, y, z)$.

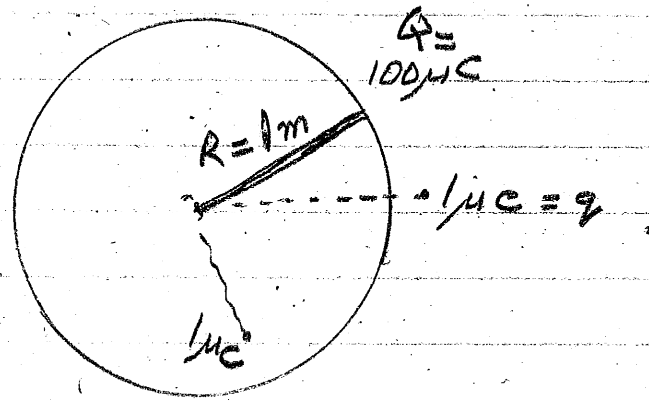
S-12 A conductor has mobile electrons so if there was any E-field inside it the electrons would not be stationary. So for stationary conditions $\underset{\rightarrow}{E} = 0$ everywhere inside. Now Gauss' law tells us that if there is any charge inside the conductor it must create an E-field because $\sum_c \underset{\rightarrow}{E} \cdot \Delta \underset{\rightarrow}{A} = \dfrac{1}{\varepsilon_0} \sum Q_i$ so if $\underset{\rightarrow}{E} = 0$

there can be no *net* charge at any point inside the conductor. Whatever charge you put on it must "sit" on the surface.

If it's +ive Q you get



5-13 $R = 1m$



(i) $r = 0.49m$

$1\mu C$ is inside sphere

$\vec{E} = 0$

so $\vec{F}_E = 0$

(ii) $r = 0.51cm$

$1\mu C$ outside

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{r}$$

$$\vec{F}_E = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\hat{r}$$

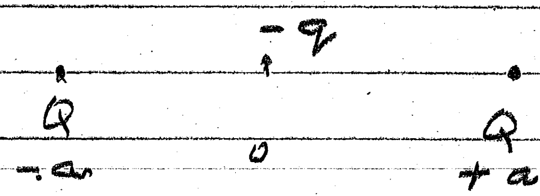
$$= \frac{9\times10^{9}\times10^{-4}\times10^{-6}}{(0.51)^2} N\hat{r}$$

$$= 3.46\ N\hat{r}$$

S-14

Force on $-q$

$$\vec{F} = -\frac{2kQq\,y}{(a^2+y^2)^{3/2}}\,\hat{y}$$



here $y = 0.001\,m$

$a = 0.5\,m$

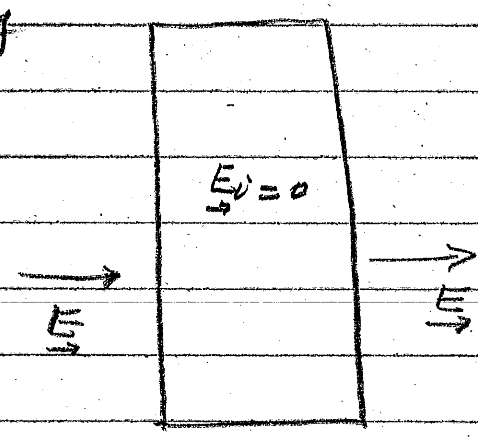
$y \ll a$

$$\vec{F} = -\frac{2kQq\,y}{a^3}\,\hat{y}$$

The force is proportional to the displace-ment and opposite to it so q will exhibit linear harmonic oscillations.

S-15

Under stationary conditions $\vec{E}$ inside the conductor must be zero so conductor must create an $\vec{E}$ field given by



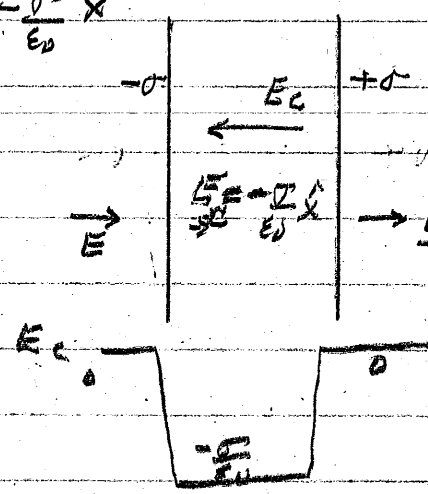
$$\vec{E}_c = -100\,N/C\,\hat{i}$$

so that

$$\vec{E}_v = \vec{E} + \vec{E}_c = 0$$

To do so conductor will need two sheets of charge density $\pm\sigma$, $+\sigma$ on right face $-\sigma$ on left face to make

$$\underset{\to}{E_c} = -\underset{\to}{E} = -\frac{\sigma}{\epsilon_0}\hat{x}$$



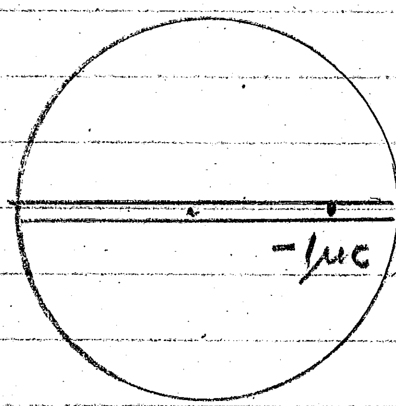
$$\frac{\sigma}{\epsilon_0} = 100 N/c$$

$$\sigma = \epsilon_0 \times 100 N/c$$

$$= 9 \times 10^{-10}\ c/m^2$$

5-16 The charge density on the sphere is

$$\rho = \frac{50 \times 10^{-6}}{\frac{4\pi}{3} \times 1^3}\ c/m^3$$



The $\underset{\to}{E}$ - field due to this at the $-1\mu c$ charge is

$$\underset{\to}{E} = \frac{\rho}{3\epsilon_0} r\hat{r}$$

so force

$$\underset{\to}{F_E} = -\frac{\rho}{3\epsilon_0} \times 10^{-6} r\hat{r}$$

which is a force proportional to displacement and opposite to it so the $-1\mu c$ charge will be a linear harmonic oscillator