Solutions - 6.

Geometrical optics: when the openings and obstructions are large compared to the wavelength of light, diffraction effects become negligible and the path of light is as if it is travelling along straight lines - called rays and one can use geometry to describe the propagation of light.

The fundamental principle is due to Fermat and asserts that light follows a path which takes the least time.

We know that speed of light in vacuum is $c = 3 \times 10^8 m/s$, same for all wavelengths. However, in a medium speed

$$v = \frac{c}{n}, \quad f_0 = f_n, \quad \lambda_n = \frac{\lambda_0}{n}$$

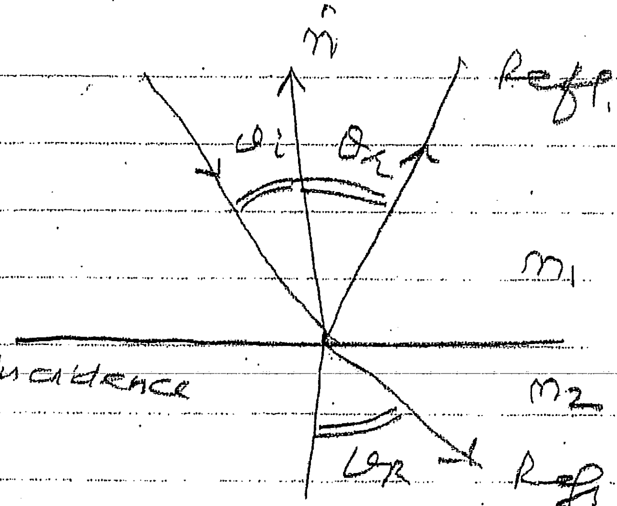
$n = 1$ Vac/air

where n is the refractive index which is a function of the wavelength and therefore gives rise to dispersion.

When a light ray hits surface of separation of two media it gives rise to a reflected ray and a refracted.

In all cases, the relevant directions are defined with respect to the normal (direction perpendicular to

surface and
Fermat's principle
Tells us that for
Reflection

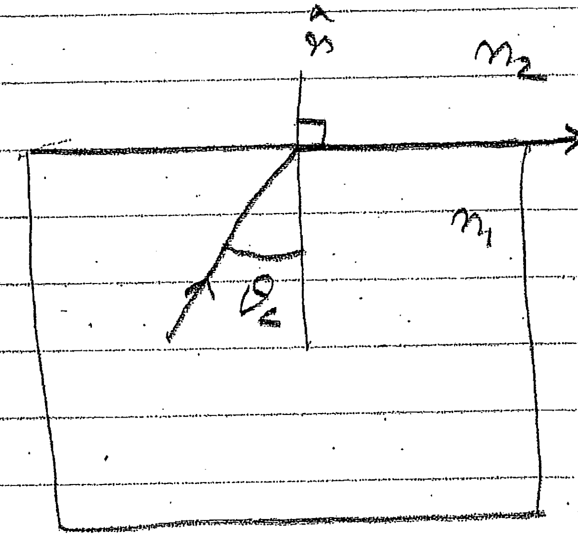


$\theta_R = \theta_i$ ← angle of incidence
↙ angle of reflection

Refraction

$n_2 \sin\theta_R = n_1 \sin\theta_i$

When $n_2 < n_1$, $\theta_R > \theta_i$. Interesting case - Critical angle
on refraction
light travels
parallel to
Surface



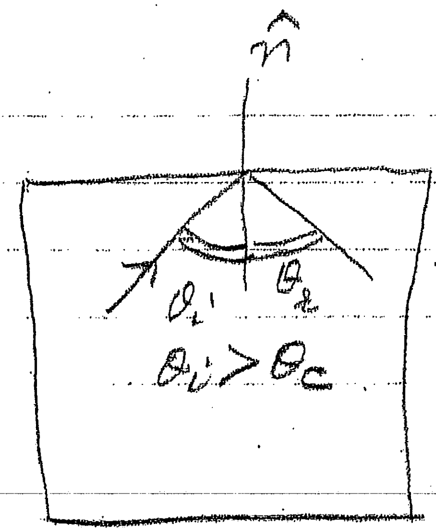
$\theta_R = \pi/2$

$n_1 \sin\theta_c = n_2 \sin\frac{\pi}{2}$

So $\sin\theta_c = \frac{n_2}{n_1}$

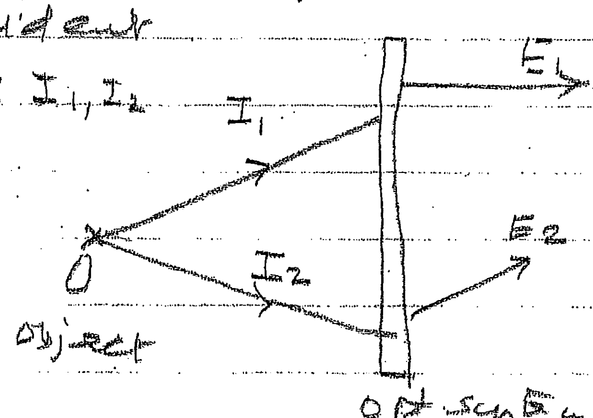
If angle of incidence becomes larger than θ_c no light leaves the first medium, you get total Internal reflection

$\hat{n}$



Once we know the method to trace path of light we can use it to discuss formation of Images by optical systems – Mirrors (reflection only) and Lenses (refraction)

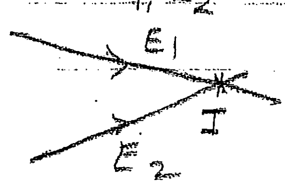
To locate image proceed as follows: Two incident rays I_1, I_2 from O



Give rise to two emergent rays after passage through opt. system.

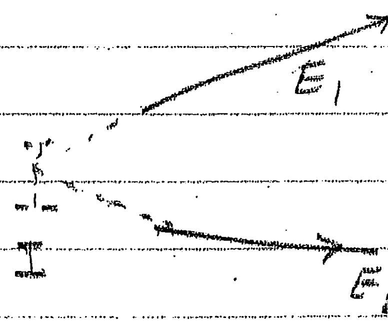
Two Cases arise

I E_1, E_2 Converge



Light actually goes through I – Real Image of O. and can be projected on a screen

II E_1, E_2 diverge



E_1, E_2 donot intersect, we extrapolate to locate point of "intersection" NO LIGHT actually goes through I, it only appears to come from I, I is A VIRTUAL IMAGE It Cannot be projected on a Screen.

Spherical MIRRORS: Applying the above to Sign Convention: Lighted Side +ive
Dark Side -ive.

All mirrors are represented by equations

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{r}, \quad m = -\frac{q}{p}$$

here p = object distance
q = Image distance
m = magnification $\left(\frac{\text{size of Image}}{\text{size of object}}\right)$

PLANE MIRROR $r \to \infty$ $q = -p$, $m = 1$.

CONVERGENT → Concave MIRROR r is +ive, many cases

DIVERGENT → Convex Mirror r is -ive, q is -ive all images, virtual, upright, reduced $(m < 1)$

Thin lenses Two spherical Surfaces

Lens makers formula

Sign - Convention - +ive along path of light
-ive against path of light

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_F} - \frac{1}{R_B}\right]$$

<u>Convergent</u> R_F +ive, R_B −ive, f +ive.

<u>Divergent</u> R_F −ive, R_B +ive f −ive.

Eqns. are $$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$m = -\frac{q}{p}$$

<u>Convergent</u> : Many Cases

<u>Divergent</u> All images, virtual, upright, reduced.

CHARGE (Q)

Charges can be +ive or −ive.

Charge is quantized. All charges are integer multiples of 1.6×10^{-19}C

$$Q = (N_+ - N_-) \times 1.6 \times 10^{-19} C$$

N_+ = # of positive charges

N_- = # of negative charges

electron $N_- = 1$, $N_+ = 0$ $Q_e = -1.6 \times 10^{-19} C$

proton $N_- = 0$, $N_+ = 1$ $Q_p = +1.6 \times 10^{-19} C$

Atoms are Neutral $N_+ = N_-$

No. of atoms in one mol is 6×10^{23}

FORCE BETWEEN TWO POINT CHARGES

COULOMB FORCE

$$\underset{\rightarrow}{F} = \frac{K q_1 q_2}{r^2}\hat{r} \qquad k = 9\times10^{9}\frac{N\text{-}m^2}{C^2}$$

So if q_1, q_2 have same sign

$$\underset{\rightarrow}{F} \parallel +\hat{r} \quad \text{Repulsive}$$

If they have opposite signs

$$\underset{\rightarrow}{F} \parallel -\hat{r} \quad \text{attractive}$$

cf. Grav. force

$$\underset{\rightarrow}{F_G} = -\frac{G M_1 M_2}{r^2}\hat{r}, \quad G = 6.7\times10^{-11}\frac{N\text{-}m^2}{(kg)^2}$$

$\underset{\rightarrow}{E}$-field: A stationary charge q placed in an $\underset{\rightarrow}{E}$-field experiences a force

$$\underset{\rightarrow}{F_E} = q\underset{\rightarrow}{E}$$

You can map $\underset{\rightarrow}{E}$ by measuring $\underset{\rightarrow}{F_E}$ at many position

Coulomb $\underset{\rightarrow}{E}$-field

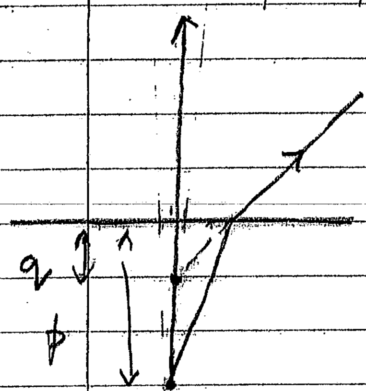
A stationary charge creates a Coulomb $\underset{\rightarrow}{E}$-field, q at $r=0$

$$\underset{\rightarrow}{E} = \frac{kq}{r^2}\hat{r}$$

P 18.24

As proved in class we $\frac{q}{p} = \frac{n_2}{n_1}$

Here image distance is $q = 2.0$ cm.



$q = \frac{n_2}{n_1} p = \frac{n_{air}}{n_{plastic}} p$, $q = 2$ cm.

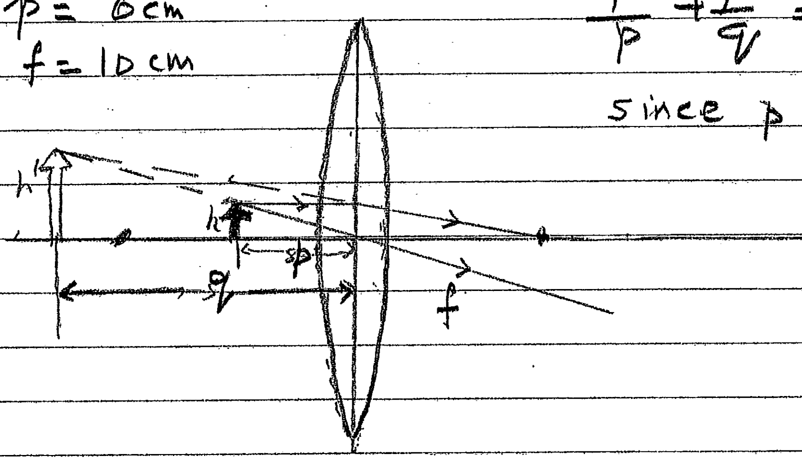
$\Rightarrow p = 3.2$ cm.

$n_{pl} = 1.6$
(p 589)

P 18.28 $p = 6$ cm
$f = 10$ cm

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

since $p < f$, q must be negative. $m = -\frac{q}{p}$.
Image is
virtual
upright
enlarged.



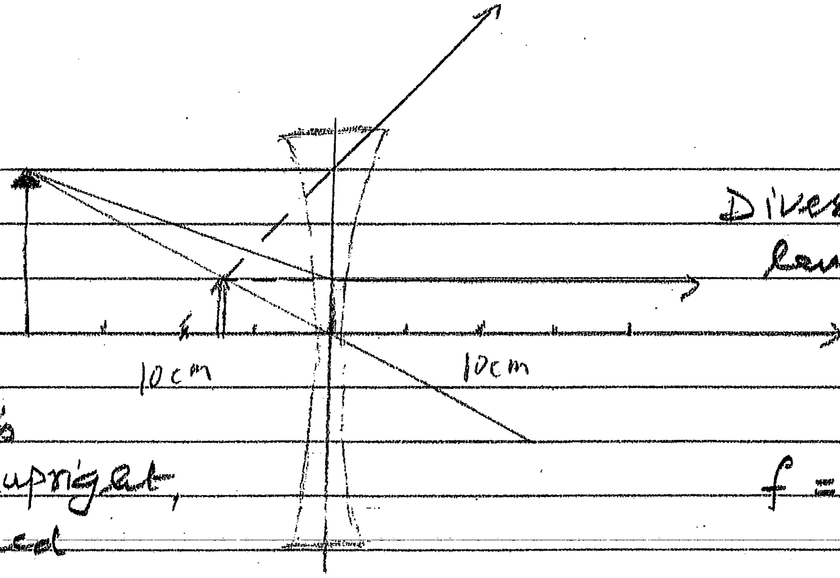
The above graph isn't very carefully drawn to scale.
But if you draw it carefully with a ruler, you'll see that the two extrapolated rays meet 15 cm from the lens.

That is, $q = -15$ cm.

$\frac{1}{6} + \frac{1}{q} = \frac{1}{10}$

$\frac{1}{q} = -\frac{4}{60}$ cm^{-1}

P 18.29



Divergent lens f is $-$ive
q must be $-$ive

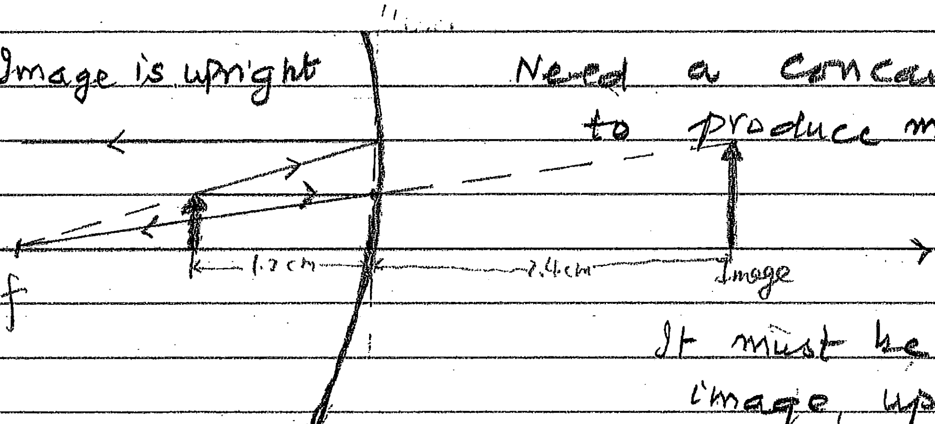
Image is virtual, upright, reduced

$f = -10$ cm

$q = -6.7$ cm to the left of the lens, on the same side as the object.

$$\frac{1}{20} + \frac{1}{q} = -\frac{1}{10} \ \mathrm{cm}^{-1}.$$

P 18.34 Image is upright



Need a concave mirror to produce $m = 2$, upright

$p = 1.2$ cm.

It must be a virtual image, upright and enlarged

$m = -q/p$

q is $-$ive

Given $m = 2$ and $p = 1.2$ cm.

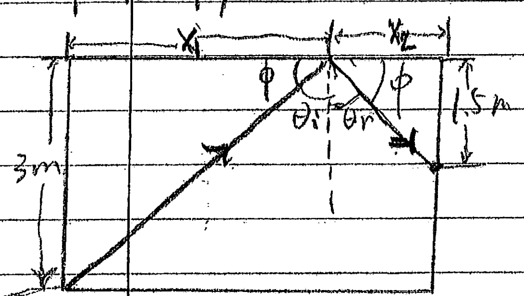
$\Rightarrow q = -2.4$ cm

The image is 2.4 cm behind the mirror and is virtual.

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{1.2} - \frac{1}{2.4}$$

$f = 2.4$ cm.

P 18.41



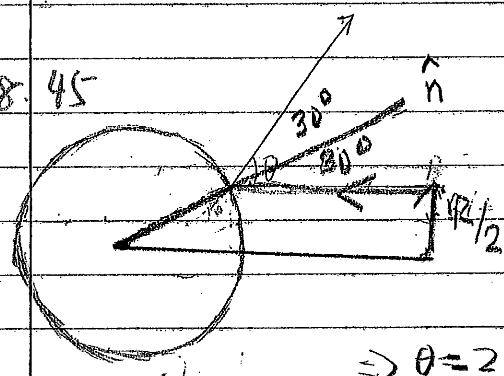
Use geometry

$\tan\phi = \frac{1.50m}{x_2} = \frac{3.00m}{x_1}$ $x_1 + x_2 = 5.00m$

$\Rightarrow \frac{x_1}{x_2} = \frac{2}{1}$ $x_1 + x_2 = 5.00m$

$\Rightarrow x_1 = \frac{10}{3} m \Rightarrow \phi = \tan^{-1}\left(\frac{3m}{\frac{10}{3}m}\right) \Rightarrow \phi = 42^\circ$

P 18.45

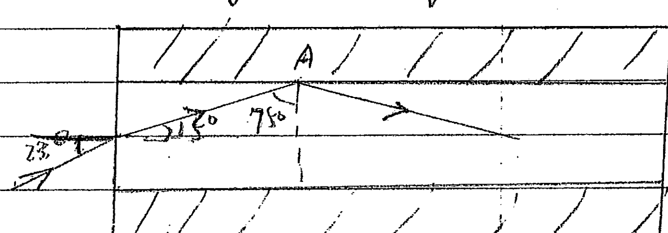


Use the law of reflection,
the angle the normal makes with the horizontal is
$\sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$

$\Rightarrow \theta = 2 \cdot 30^\circ = 60^\circ$

18.51 To solve the problem you need both Snell's law and the formula to calculate the critical angle of total internal reflection.

The critical angle $\theta_c = \sin^{-1}\left(\frac{1.45}{1.50}\right) = 75^\circ$ so incidence angle on top surface must be larger than 75°.
This means the angle of refraction from the air-glass interface is $90^\circ - 75^\circ = 15^\circ$

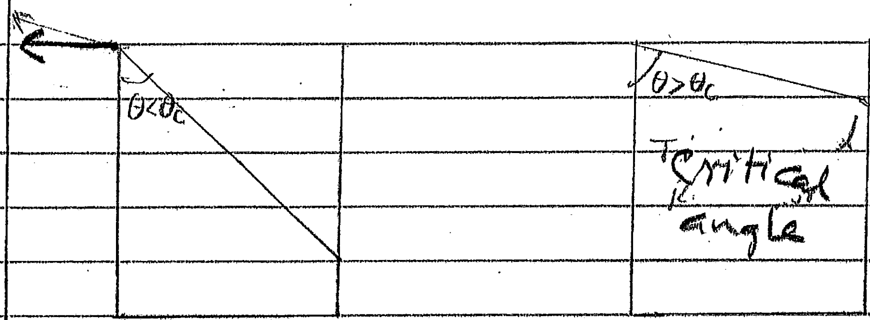


$\Rightarrow \theta = \sin^{-1}\left(\frac{1.50 \cdot \sin 15^\circ}{1.00}\right) = 23^\circ$

If $\theta > 23^\circ$ the incidence at A becomes less than $\theta_c = 75^\circ$

P 18.57

(a)



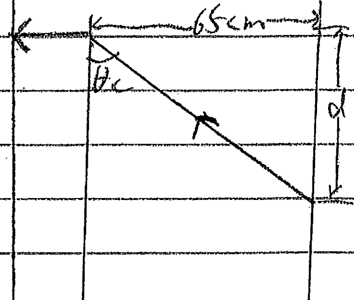
Refracted ray parallel to water surface.

To reach your eye, the ray must refract through the top surface of water and travel parallel to it.

Rays coming from the bottom of the tank are incident on the top surface at fairly small angles, but rays from marks near the top are incident at very large angles — larger than the critical angle θ_c. These rays undergo total internal reflection and do not exit into air.

Thus, you can only see the marks from bottom upward.

(b)

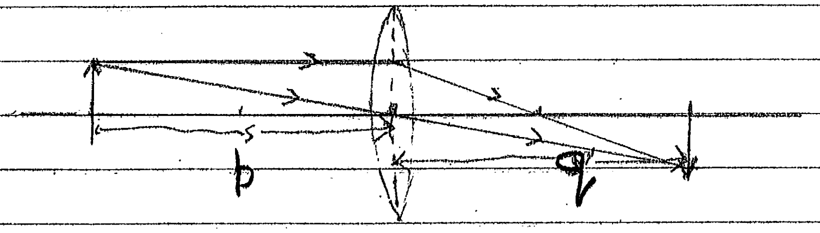


$L/d = \tan\theta_c$ $\qquad \theta_c = \sin^{-1}\left(\frac{1}{1.33}\right) = 48.75°$

$\Rightarrow d = \frac{L}{\tan\theta_c} = 57\text{cm}$

The highest mark you can see is 60cm

P 18.65



$m = -1$

$q = p$.

Obviously since $h = h'$, s must be equal to s'

If the object is too far away from the lens then the image will be small.

So far without thin lens equation there's no way we can solve the problem quantitatively other than ray-tracing (You can analyze the geometry and that's equivalent to deriving the thin lens equation,

If you set $s = 2f$ and do ray-tracing you'll see that $s' = 2f$ and that satisfies the condition.

If you prefer to use the thin lens equation,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \Rightarrow \frac{2}{p} = \frac{1}{f} \Rightarrow p = 2f$$

P 20.3

Since each oxygen molecule, composed of two oxygen atoms ; contain $8 \times 2 = 16$ protons and that

$1 \text{ mol} = 6.02 \times 10^{23}$

$\Rightarrow$ The amount of positive charge in 1 mole of oxygen is

$$6.02 \times 10^{23} \times 16 \times \underbrace{(1.6 \times 10^{-19} C)}_{\text{charge of one proton}} = 1.5 \times 10^{6} C$$

II

P 20.12

The first point from the graph gives $(r, F) = (1cm, 0.8mN)$

Since we also know the magnitude of q_1 and Coulomb's law:

$$\vec{F} = \frac{k q_1 q_2}{r^2} \hat{r} \Rightarrow q_2 = \frac{F r^2}{k q_1} = 8.1 \times 10^{-8} C$$

Since the force is attractive and q_1 is positive, q_2 must be negative.

P 20.15

Since charge q_2 is in static equilibrium, the net force exerted by the two charges must be zero.

$\Rightarrow \vec{F}_{net\,on\,q_2} = \vec{F}_{q_1\,on\,q_2} + \vec{F}_{2nC\,on\,q_2}$

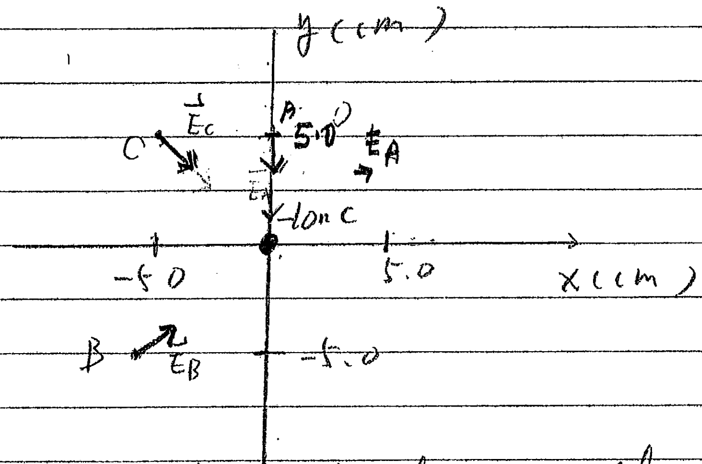
$= \left[k \cdot \frac{|q_1| \cdot |q_2|}{(0.2m)^2}, \hat{x} \right] +$

$\left[-k \cdot \frac{(2\times10^{-9}C)|q_2|}{(0.10m)^2}, \hat{x} \right]$

$= 0$ N/C

$\Rightarrow \frac{q_1}{(0.2m)^2} = \frac{2\times10^{-9}C}{(0.10m)^2} \Rightarrow q_1 = 8$ nC

P. 20.23



Above is the graph of the electric field generated by a negative charge located at origin.

(a) $E = k \cdot \frac{|q|}{r^2}$ is the strength of electric field.

in which $k = 9.0\times10^9\ N\ m^2/C^2$

$\Rightarrow$ $E_A = 3.6 \times 10^4 N/c$

$E_B = 1.8 \times 10^4 N/C$ are the strength of electric fields at point A, B and C respectively

$E_C = 1.8 \times 10^4 N/C$

(b) The three vectors are shown in the graph and are written as

$$\underline{E}_A = -3.6 \times 10^4 N/c \, \hat{y}$$

$$\underline{E}_B = 1.8 \times 10^4 N/c \left[-\cos 45 \, \hat{x} - \sin 45 \, \hat{y} \right]$$

$$\underline{E}_C = 1.8 \times 10^4 N/c \left[\cos 45 \, \hat{x} + \sin 45 \, \hat{y} \right]$$