SOLUTIONS — 5

FORMULAE :

Light is a transverse Electromagnetic wave whose speed in vacuum is 3×10^8 m/s and wavelengths in vacuum are $400\text{nm} \le \lambda_0 \le 800\text{nm}$

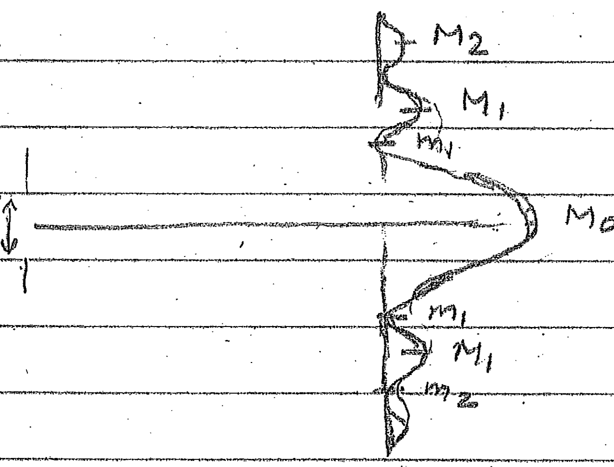
In a medium speed is

$$V = \frac{c}{n}, \qquad \lambda_n = \frac{\lambda_0}{n}$$

where n is the refractive index, $n > 1$ so $V < C$

Single Slit diffraction

When light of wavelength λ passes through a narrow slit of width a, it produces a diffraction pattern in which minima (Dark spots) are located at angles



$$\sin\theta_m = \frac{m\lambda}{a} \quad m = 1, 2, 3$$

and Maxima (bright spots) have intensities

$$I_0, \ \frac{4}{9\pi^2} I_0, \ \frac{4}{25\pi^2} I_0, \ \ldots$$

because diffraction involves superposition of very large number of waves.

Thin film Interference

By now we know that interference will be observed only for coherent sources. That is, sources where the two waves start in step (phase difference zero), one travels d_1 to get to the detector (screen) and the other d_2. If so, we get

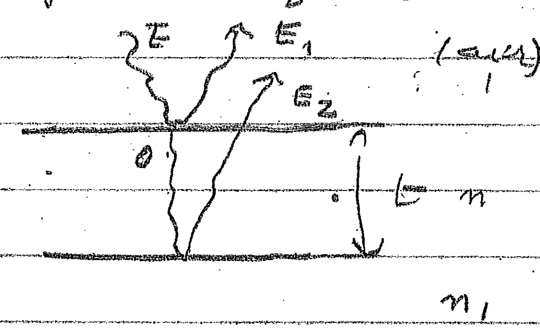
Maxima when $(d_1 - d_2) = m\lambda$; $m = 0, \pm 1, \pm 2$

and

Minima when $(d_1 - d_2) = (m + \frac{1}{2})\lambda$

$m = 0, \pm 1, \pm 2, \ldots$

In thin film interference the two waves are produce when the incident light wave is reflected from the front surface producing one wave (E_1) and the transmitted wave reflects from the back surface of the film of thickness t producing the second wave E_2.



Both are derived from so when they leave O they are in step. We need to know the path difference but we also recall that during reflection there can be a phase change if the velocity in the second medium is lower and there will be no phase change

If that velocity is higher. So in the picture:

Case I $n > 1$ phase change for E_1

$n_1 > n$ " " " E_2

Case II $n > 1$ phase change for E_1

$n_1 < n$ no phase change for E_2

<u>Case I</u> Constructive Interference if

$$2t = \frac{m\lambda_0}{n} \qquad m = 1, 2,$$

Destructive Interference if

$$2t = \frac{\lambda_0}{2n}(2m-1) \qquad m = 1, 2, 3$$

<u>Case II</u> Constructive Interference if

$$2t = \frac{\lambda_0}{2n}(2m-1) \qquad m = 1, 2, 3$$

Destructive interference if

$$2t = \frac{m\lambda_0}{n} \qquad m = 1, 2, 3$$

Geometrical optics

As we learnt when light passes through a slit of width a, it spreads by an angle

$$\sin\theta_1 = \frac{\lambda}{a}$$

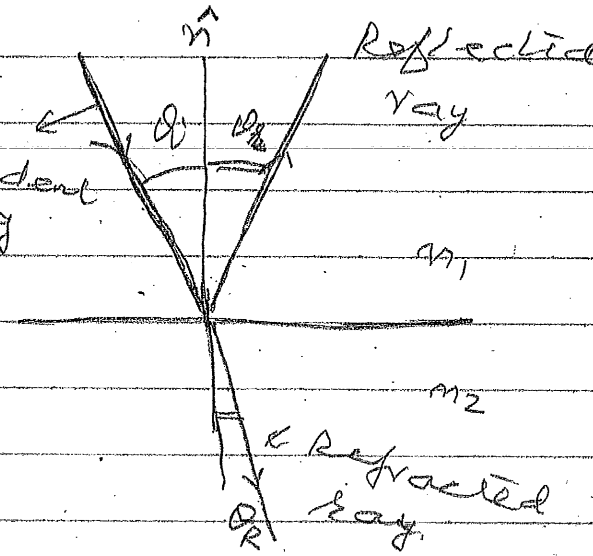
If $w \gg \theta$, the spread goes to zero and the path of light is as if it is travelling in straight lines.

F. Hewels' the name

Geometrical optics

Fermat's theorem asserts that light follows a path which takes the least amount of time.

Note All angles are measured with respect to the normal which is perpendicular to the surface.



In *Reflection*

$$\theta_r = \theta_i$$

Angle of reflection = angle of incidence

In *Refraction*

Snell's law holds

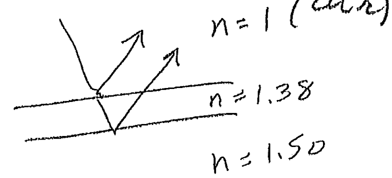
$$n_2 \sin \theta_R = n_1 \sin \theta_i$$

[illegible]

Chapter 17

17.26 Case I Destructive $2t = \frac{\lambda_0}{2n}(2m-1)$

use $m = 1$ for thinnest coating



since $n_{air} < n_{MgF_2} < n_{glass}$,

we will have 180° phase change at both interfaces ;

destructive interference of 500 nm light w/ 2 phase changes:

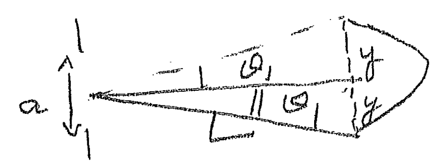
$2t = (2m - 1)\frac{\lambda_0}{2n}$ w/ t thickness

using λ_0 (in air) and $n = 1.38$:

$t = \frac{1}{2}\left(\frac{1}{2}\right)\frac{500\,nm}{1.38}$

☆ $t = 90\ nm$

17.29 Minim at $\sin\theta_m = \frac{m\lambda}{a}$



single slit diffraction

width of center max is the difference in positions of the $m = \pm 1$ first minima;

from $m\lambda = a \sin\theta_1 \approx \frac{ay}{L}$ (small angle approx)

the width w of the max is $2y$ w/ $m = 1$

$y = \frac{\lambda L}{a} \Rightarrow w = \frac{2\lambda L}{a}$

$\Rightarrow a = \frac{2\lambda L}{w} = \frac{2(680\times10^{-9}\,m)(5.5\,m)}{8.0\times10^{-2}\,m} = 9.4\times10^{-5}\,m$

☆ $a = 94\,\mu m$

17.31 $\quad \sin\theta_m = \frac{m\lambda}{a}$

using single slit formula 2nd minimum needs

$\sin\theta_2 = \frac{2\lambda}{a}$

with $m=2$, $a = 0.10 \times 10^{-3}$ m, and $\theta = 0.7° = 0.0122$ rad

and using $\sin\theta \simeq \theta$

we see

$$\lambda = \frac{a}{2}\theta_2 = \frac{0.10\times10^{-3}\,\text{m}\,(0.0122\,\text{rad})}{2} = 6.10\times10^{-7}\,\text{m}$$

$\star \quad \lambda = 610$ nm

17.32 $\quad \sin\theta_1 = \frac{\lambda}{a}$

Similar to previous problem, except can't use small angle approximation

$\because \lambda = a\sin\theta_1 \Rightarrow a = \frac{\lambda}{\sin\theta_1}$

for $n=1$, $\lambda = 633$ nm, $\theta = 45°$ $\quad (\sin 45° = \frac{\sqrt{2}}{2})$

$$\Rightarrow a = \frac{6.33\times10^{-7}\,\text{m}}{(\sqrt{2}/2)} = 6.33\times10^{-7}\,\text{m}\left(\frac{2}{\sqrt{2}}\right) = 8.95\times10^{-7}\,\text{m}$$

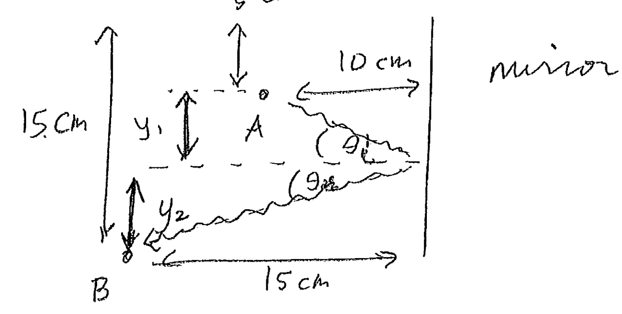
$\star \quad a = 895$ nm

Chapter 18

18.8

angle of reflection (θ_r)
angle of incidence (θ_i)

if we find y_1, we can add 5.0 cm to get the desired result.



(18.8) continued.

since θ's are equal, we know $\tan\theta_1 = \tan\theta_2$

$$\Rightarrow \quad \frac{y_1}{10\text{ cm}} = \frac{y_2}{15\text{ cm}}$$

also, from the picture we can see

$y_1 + y_2 = 10\text{ cm}$

$$\Rightarrow \quad \frac{y_1}{10\text{ cm}} = \frac{10\text{ cm} - y_1}{15\text{ cm}}$$

cross multiply:

$15 y_1 = 10(10 - y_1) \quad \Rightarrow \quad 15 y_1 = 100 - 10 y_1$

$\Rightarrow \quad 25 y_1 = 100$

$y_1 = 4\text{ cm}$

$\therefore$ the ray strikes the mirror $\boxed{9\text{ cm}}$ from the top.

18.11

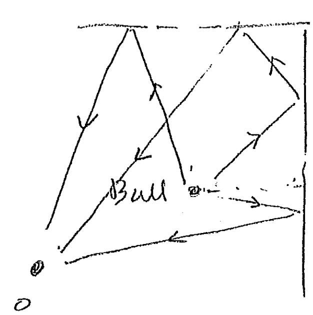
(treating the ball as a point)

(i) of course he will see one image on each mirror, but is there an additional image from the reflection of one mirror in the other?

using the law of reflection ($\theta_R = \theta_i$) and the diagram,

the answer is yes

So there are 3 images



(18.11) continued.

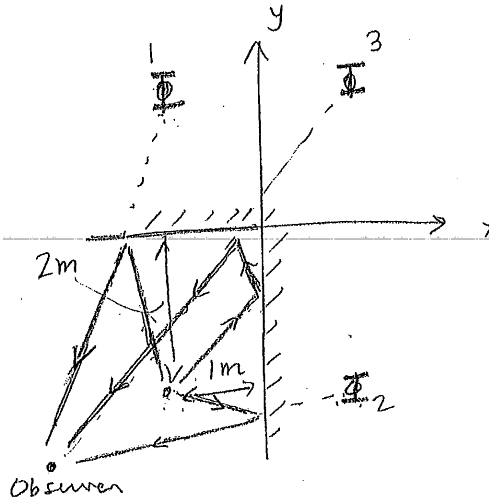
(ii) the images from flat mirrors are all located equally as far from the mirrors as the object; if we place an origin at the mirror intersection:

object is at (−1 m, −2 m)

image 1 is at (−1 m, + 2m)

image 2 is at (+1 m, −2 m)

image 3 is at (+1m, + 2m)

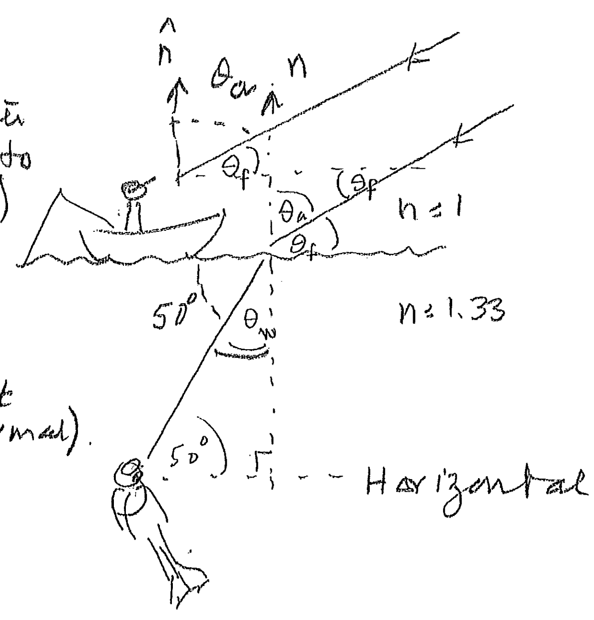


(iii) see part (ii) diagram
(Next page)

18.12 ALL ANGLES MEASURED FROM NORMAL.

diver will see a refracted image of the sun

fisherman sees no refraction



use trig to get θ_w (angle with ~~respect to~~ normal)

$\theta_w + 50° = 90°$

$\Rightarrow \theta_w = 40°$

use Snell's law to get θ_a (w.r.t normal).

$n_a \sin\theta_a = n_w \sin\theta_w$

$n_a = 1$; $n_w = 1.33$

$\sin\theta_a = \frac{1.33}{1} \sin 40° = 58.7°$

use trig again to get θ_f (notice = angles in diagram from geometry)

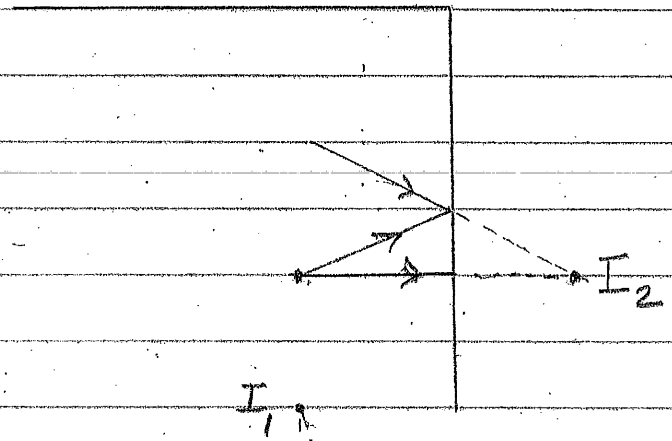
$\theta_f = 90° - \theta_a = 90° - 58.7°$

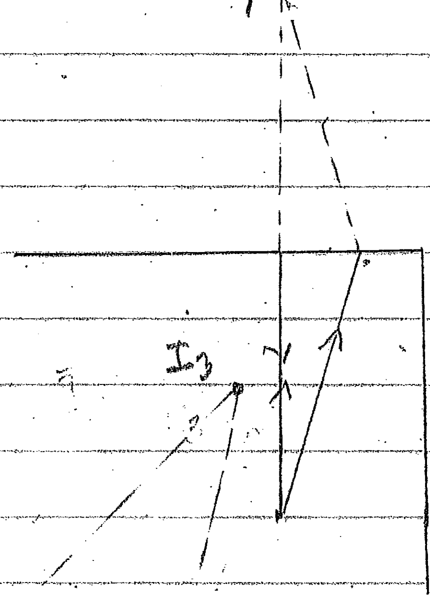
★ $\theta_f = 31.3°$

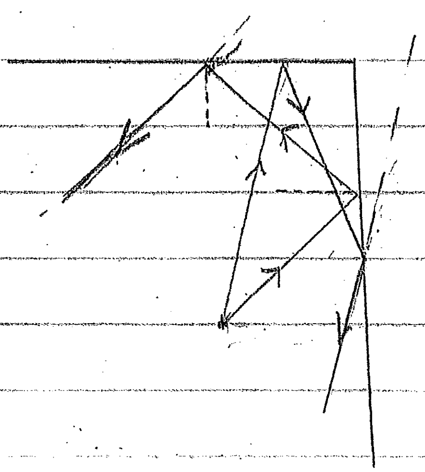
Ray Diagrams

Prob 18-11

You need two rays to locate an image.

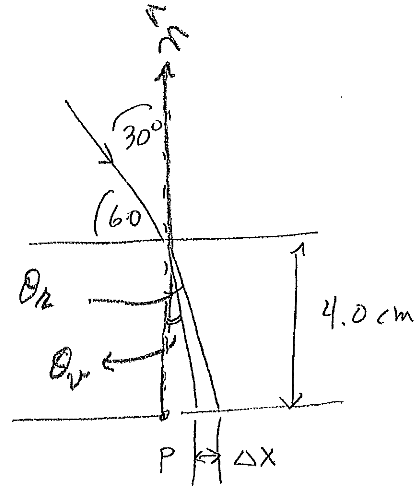






ON REFLECTION FROM BOTH MIRRORS LIGHT REVERSES DIRECTION

18.21 ANGLES WITH RESPECT TO NORMAL



using $\lambda_{red} = 700$ nm

and $\lambda_{vio} = 400$ nm.

we need to use figure 18.27 in the textbook to approximate the index of refraction values n_r, n_v for the 2 wavelengths:

looks like $n_r \simeq 1.57$ and $n_V \simeq 1.60$ for flint glass;

now we can use Snell's law to get the 2 refraction angles:

(don't forget to use 30° and not 60° for θ_i)

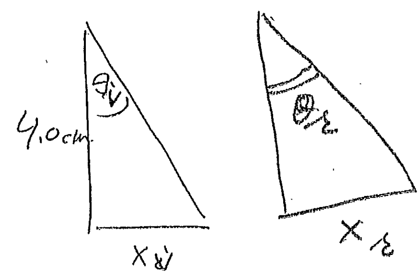
$(\sin 30° = \frac{1}{2})$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_r = \frac{n_{air}}{n_r}\sin\theta_{air} = \frac{1}{1.57}\sin 30° \Rightarrow \theta_r = 18.57°$$

$$\sin\theta_v = \frac{n_{air}}{n_v}\sin\theta_{air} = \frac{1}{1.60}\sin 30° \Rightarrow \theta_v = 18.21°$$

to get their separation distance upon exiting use the known depth of material and $\tan\theta_R = \frac{X_R}{depth}$, $\tan\theta_V = \frac{X_V}{depth}$



$$\Rightarrow X_{red} = d\tan\theta_r$$
$$= 4.0\text{ cm} \cdot \tan 18.57° = 1.344 \text{ cm}$$

$$X_{vio} = d\tan\theta_V$$
$$= 4.0\text{ cm} \cdot \tan 18.21° = 1.316 \text{ cm}$$

(notice how small of a difference)

so the separation is

$$\Delta X = X_{red} - X_{vio} = 1.344 - 1.316 = 0.028 \text{ cm} = \boxed{0.28 \text{ mm}}$$

the violet light is refracted more ($n_V > n_R$) so it lands closer to point P