SOLUTIONS – 4

FORMULAE

Standing waves on string both ends fixed $\frac{n\lambda_n}{2} = L$, $f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

Both ends open, Both ends closed

Sound $\frac{n\lambda_n}{2} = L \quad f_n = \frac{n}{2L}\sqrt{\frac{\gamma k_B T}{m}}$

$n = 1, 2, 3.$

One end open – one closed

$$\frac{(2n-1)\lambda_n}{4} = L \quad f_n = \frac{(2n-1)}{4L}\sqrt{\frac{\gamma k_B T}{m}}$$

Interference – Sound from both sources starts in step

Maxima $(d_1 - d_2) = m\lambda$, $m = 0, \pm 1, \pm 2, \ldots$

Minima $(d_1 - d_2) = (m + \frac{1}{2})\lambda$, $m = 0, \pm 1, \pm 2 \ldots$

Beats $f_B = |f_1 - f_2|$

Light speed in vacuum $c = 3 \times 10^8 m/s$

Speed in medium $v = \frac{c}{n}$, n = refractive Index (Table on p. 554)

frequency does not change so wavelength in medium

$$\lambda_n = \frac{v}{f} = \frac{c}{nf} = \frac{\lambda_0}{n}$$

Two Slit Interference:

Maxima

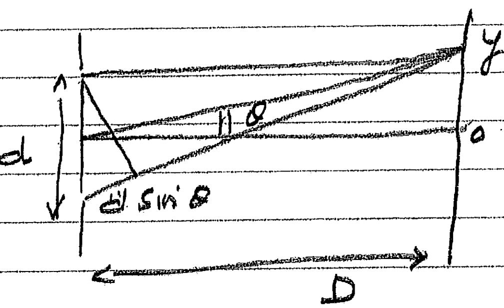
$d\sin\theta_M = M\lambda$

$M = 0, \pm 1, \pm 2, \pm 3, \ldots$

Minima

$d\sin\theta_m = \left(m + \frac{1}{2}\right)\lambda$

$m = 0, \pm 1, \pm 2, \pm 3, \ldots$



$\frac{y_M}{D} = \tan\theta_M \qquad \sin\theta_M \simeq \tan\theta_M$

$y_M = \frac{MD\lambda}{d}, \quad y_{M+1} - y_M = \frac{D\lambda}{d}$

Diffraction Grating

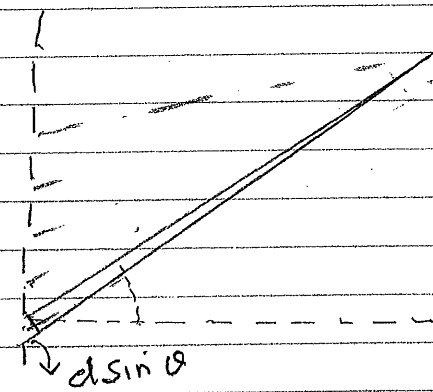
d is separation between adjacent slits so for

Maxima

$d\sin\theta_M = M\lambda$

$M = 0, \pm 1, \pm 2, \pm 3, \ldots$

Called Diffraction orders.

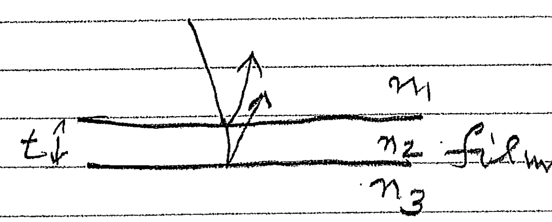


Thin Film Interference

Two Reflections

If $n_2 > n_1$ $(V_2 < V_1)$ Phase change π

$n_3 > n_2$ $(V_3 < V_2)$ " " "



Maximum

$2t = \lambda_{n_2} = \frac{\lambda_0}{n_2}$

Minimum

$2t = \frac{\lambda_n}{2} = \frac{\lambda_0}{2n_2}$

If $n_2 > n_1$ Phase change π

$n_3 < n_2$ No phase change

Max $2t = \frac{\lambda_{n_2}}{2} = \frac{\lambda_0}{2n_2}$, Min $2t = \lambda_{n_2} = \frac{\lambda_0}{n_2}$

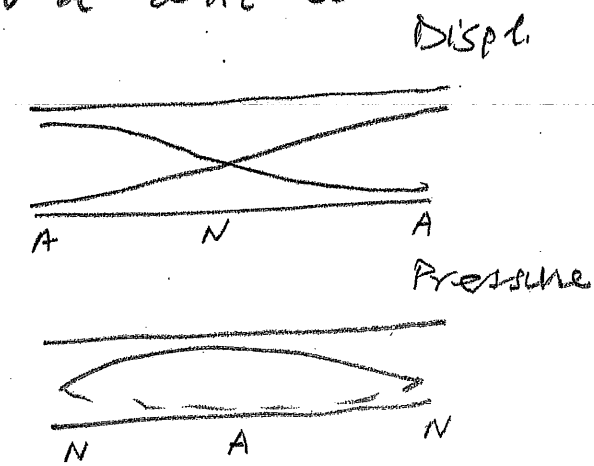
16.20

(a) for open-open tube

$$f_n = n\left(\frac{v}{2L}\right) \Rightarrow L = \frac{nv}{2f_n}$$

for shortest tube we should look for the fundamental frequency that is $n=1$. We want f to be 20 Hz so

$$L_1 = \frac{340\ m/s}{2 \times 20\ 1/s} = 8.5\ m$$

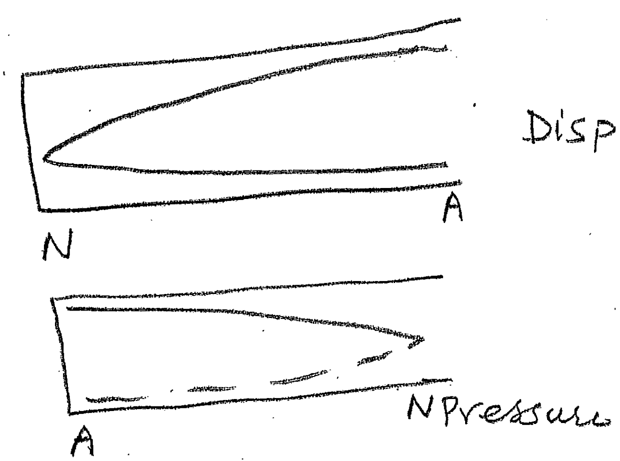


(b) open-closed tube

$$f_n = (2n-1)\frac{v}{4L} \Rightarrow L = (2n-1)\frac{v}{4f_n}$$

$n=1$ for minimum L

$$L_{min} = \frac{340\ m/s}{4 \times 20\ Hz} = 4.25\ m$$

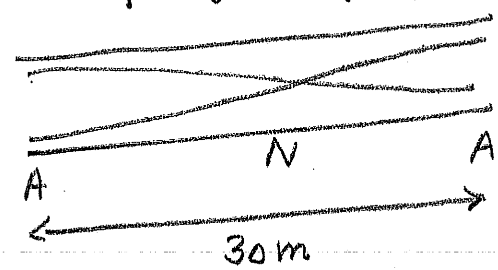


16.26

(a) $f_m = m\left(\frac{v}{2L}\right) \Rightarrow$ fundamental frequency is f_1

$L = \lambda/2$

$f_1 = \frac{340 m/s}{2 \times 30 m} = 5.67\ Hz$



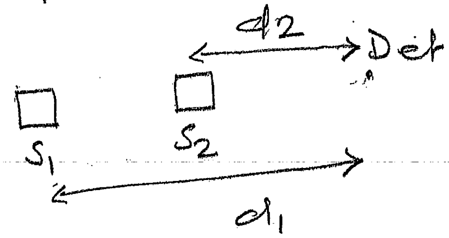
(b) Human ear lowest frequency is greater than 20 Hz. So the frequency of the lowest harmonic to be heard is $f_4 = 4 f_1 = 22.68\ Hz$

(c) When the temperature drops the speed of sound decreases* therefore the frequency will also decrease since it is proportional to the speed of sound.

$$V = \sqrt{\frac{\gamma k_B T}{m}}$$

16.30 Minima $(d_1 - d_2) = (m + \frac{1}{2})\lambda$, $m = 0, \pm 1, \pm 2, \ldots$

for destructive interference the distance $(d_1 - d_2)$ between the speakers is $(m + \frac{1}{2})$ times the wavelength. The shortest distance is $\frac{\lambda}{2}$. Using $\lambda = \frac{v}{f}$ we can find the separation.



$$L = \frac{\lambda}{2} = \frac{v}{2f} = \frac{340\,m/s}{2 \times 686\,Hz} = 24.78\,cm$$

16.36 FLUTE, one end open, one closed $f_n = \frac{2n-1}{4L}\sqrt{\frac{\gamma k_B T}{m}}$

$f_{beat} = |f_{flute} - f_{fork}| = 4\,Hz$ so her initial frequency is 523 ± 4 Hz.

Since f is proportional to $\frac{1}{L}$ so if she lengthens her flute then it means her initial frequency was higher than the tuning fork because she is in fact decreasing the flute's frequency in order to match the fork.

So initial frequency was $(523 + 4)\,Hz = 527\,Hz$

16.45 $f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$ for string $n = 1, 2, 3,$

$$f_{k_1} = k_1 f_1 = k_1\left(\frac{1}{2L}\sqrt{\frac{T}{\mu}}\right) = k_1\left(\frac{1}{2L}\sqrt{\frac{mg}{\mu}}\right)$$

$$f_{k_2} = k_2 f_1 \Rightarrow \frac{f_{k_1}}{k_1} = \frac{f_{k_2}}{k_2} \Rightarrow \frac{f_{k_1}}{f_{k_2}} = \frac{k_1}{k_2}$$

comparing the frequencies one gets

$\frac{80\,Hz}{64\,Hz} = 1.25$ this ratio can be $5/4$ or $10/8$ or any "respectable" fraction. However astronauts claim that there are no standing waves between these frequencies, therefore we find that $k_1 = 5$ and $k_2 = 4$. (Suppose $k_1 = 10$ and $k_2 = 8$ then they should be observing a standing wave with frequency $\frac{9}{8} \cdot 64\,Hz = 72\,Hz$) $(k_3 = 9)$

So we find

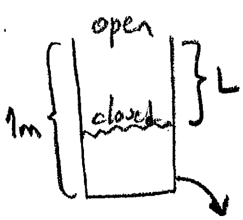
$$g = \left(\frac{f_k}{k} \cdot 2L\right)^2 \frac{\mu}{m} = \left(\frac{80\,Hz}{5} \cdot (2 \times 2m)\right)^2 \frac{(5g/2.5m)}{1\,kg}$$

remember $\mu = \frac{M_{rope}}{L_{rope}}$

linear density of the rope

$= 8.192\,m/s^2$

16.56 $\quad f_n = \frac{(2n-1)v}{4L} \quad n = 1, 2, \ldots$

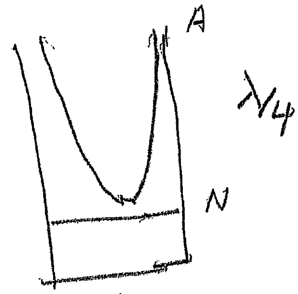


for open-closed tube boundary conditions

shortest length $L = \lambda/4$.

$$\lambda = \frac{v}{f} = \frac{340}{580}\ m$$

$$\text{so} \quad L = \frac{m \times 340\ m/s}{4 \times 580\ Hz}$$



$L_1 = 0.146m$ height of the water is $h_1 = 1m - L_1$

$= 0.854m = \underline{85.4cm}$

Next length $\frac{3\lambda}{4} = L_2$

$L_2 = 3 \times 0.146\ m = 0.438m$

so height of water

$h_2 = 56.2\ m$

Next is $\frac{5\lambda}{4} = L_3$

$L_3 = 5 \times 0.146m = 0.73m$

$h_3 = 0.27m$.

That is it! because $\frac{7\lambda}{4} > 1m$!

17.2

(a) $t = \frac{x}{v} = \frac{1m}{3 \cdot 10^8 m/s} = \frac{1}{3} \cdot 10^{-8} s = \frac{10}{3}$ ns

(b) $v_n = \frac{c}{n}$, $n_{diamond} > n_{glass} > n_{water} > n_{air} > n_{vacuum}$

$$\Rightarrow v_{diamond} < v_{glass} < v_{water} < v_{air} < v_{vacuum} \quad (1)$$

since $x = vt$ we have

$$x_{diamond} = v_{diamond} \cdot \frac{10}{3} ns = \left(\frac{\overbrace{v_{vacuum} \cdot \frac{10}{3} ns}^{1 meter}}{n_{diamond}} \right) = \frac{1 meter}{n_{diamond}}$$

$$x_{diamond} = \frac{1 meter}{2.419} = 0.413 \text{ meters}$$

$$x_{glass} = \frac{1m}{1.52} = 0.658 m$$

$$x_{water} = \frac{1m}{1.332} = 0.751 m$$

$$x_{air} = \frac{1m}{1.00029} = 0.9997 m$$

$$x_{vacuum} = \frac{1m}{1} = 1m$$

The higher the n, the smaller the speed of light in the medium so in $\frac{10}{3} \times 10^{-9}$ sec it travels least in diamond!

multiplying everything in relation (1) we see that

$$x_{diamond} < x_{glass} < x_{water} < x_{air} < x_{vacuum}$$

17.4

(a) the wavelength of the light in a medium is given with the equation

$$\lambda_n = \frac{\lambda_{vacuum}}{n} \quad \text{therefore} \quad \lambda_{air} = \frac{\lambda_{vac}}{n_{air}} \Rightarrow \lambda_{vac} = \lambda_{air} n_{air}$$

the frequency is $f = \frac{c_{vac}}{\lambda_{vac}} = \frac{3.10^8 m/s}{\lambda_{air}\, n_{air}}$

and $c_{solid} = \lambda_{solid} \cdot f = \lambda_{solid} \cdot \frac{3.10^8 m/s}{\lambda_{air}\, n_{air}} = \frac{420nm \times 3.10^8 m/s}{670nm \times 1.00029}$

$$c_{solid} = 1.88 \times 10^8 m/s$$

(b) $$f = \frac{c_{vac}}{\lambda_{vac}} = \frac{c_{vac}}{\lambda_{vac}} \cdot \frac{n_{solid}}{n_{solid}} = \frac{c_{vac}}{n_{solid}} \cdot \left(\frac{\lambda_{vac}}{n_{solid}}\right)^{-1} = c_{solid} \cdot (\lambda_{solid})^{-1}$$

$f = \frac{c_{solid}}{\lambda_{solid}} = \frac{c_{vac}}{\lambda_{vac}}$ frequency does not change!

$$f = \frac{3.10^8 m/s}{\lambda_{air} \cdot n_{air}} = \frac{3.10^8 m/s}{670nm \times 1.00029} = \frac{3.10^8 \cancel{m}/s}{670 \times 10^{-9} \cancel{m} \times 1.00029} = 4.476 \times \frac{10^{14}}{s}$$

A Hertz is $\frac{1}{s}$ so $f = 4.476 \times 10^{14} Hz$

17.8

$$y_{M+1} - y_M = \frac{D\lambda}{d}$$

the fringe spacing $\Delta y = \frac{\lambda D}{d}$ we have been asked to find out what the slit spacing d is.

$$d = \frac{\lambda D}{\Delta y} = \frac{(589\,nm)(150\,cm)}{4\,mm} = \frac{(589\times10^{-9}m)(150\times10^{-2}m)}{4\times10^{-3}m}$$

$$= 22087.5\times10^{-8}m \simeq 0.22\times10^{-3}m = 0.22mm$$

17.9

the location of the minima is given with

$$y_m = \left(m+\frac{1}{2}\right)\frac{\lambda D}{d}$$

[For minima also " $y_{m+1} - y_m = \frac{\lambda D}{d}$ "].

we know that the distance between 5th and 1st minima, is 6.0 mm. that is,

$y_5 - y_1 = 6.0mm$ $\left[y_5 - y_1 = \frac{4\lambda D}{d} \right]$

5th

$\frac{4\lambda D}{d}$

1st

so,

$$y_5 - y_1 = \frac{11\lambda D}{2d} - \frac{3\lambda D}{2d} = \frac{4\lambda D}{d} = 6.0\,mm$$

the wavelength is

$$\lambda = \frac{d\times 6.0\,mm}{4\times L} = \frac{0.2mm\times6.0mm}{4\times60\,cm}$$

$$= \frac{0.2\times10^{-3}m \times 6.0\times10^{-3}m}{4\times60\times10^{-2}m} = 5\times10^{-7}m = 500\times10^{-9}m$$

$$= 500\,nm$$

17.13 For Mth max, path difference between neighboring waves must be $M\lambda$ so $d\sin\theta_M = M\lambda$, d = separation of "neighbors"

$d = \frac{1\,cm}{1000}$ (i.e. 1000 slits per 1cm grating)

$d = \frac{10\,mm}{1000} = 0.01\,mm$

$\sin\theta_1 = \frac{1\times\lambda}{d} = \frac{550\times10^{-9}\,m}{0.01\times10^{-3}\,m} = 0.055 \Rightarrow \theta_1 = \arcsin 0.055$
$= 3.153$ degrees

$\sin\theta_2 = \frac{2\times\lambda}{d} = 2\times\sin\theta_1 \Rightarrow \theta_2 = \arcsin\theta_2 = 6.315$ degrees

17.15

$d = \frac{m\lambda}{\sin\theta_m} = \frac{2\times 600\,nm}{\sin 39.5} = 1886.56\,nm$

# of lines per millimeter is $N = \frac{1mm}{d} = \frac{1\times10^{-3}\,m}{1886.56\times10^{-9}\,m}$

$= 530.06$

N should be integer, so we round to the closest integer

$N = 530$

17-12 For Maxima $(d_1 - d_2) = M\lambda$
$M = 0, \pm1, \pm2$

here $M = 1$ so
$(d_1 - d_2) = \lambda = 550\,nm.$

17.21 Both reflections have phase change of π therefore

For constructive interference we have

$$2t = m\frac{\lambda}{n}$$

$m = 1, 2, \ldots$

the thickness of the film is proportional to m. Therefore for thinnest film we have to pick $m = 1$

And the thickness of the film "t" becomes

$$t = \frac{\lambda}{2n} = \frac{600\,\text{nm}}{2 \times 1.39} = 215.83\,\text{nm}$$