FORMULAE – SOLUTIONS 2

If the amplitude is small, period of a pendulum is

$$t = 2\pi\sqrt{\frac{l}{g}}$$

l = length
g = acc. due to gravity

Energy $\frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}Mv^2$

Travelling wave:

$$D = A\sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T}\right) = A\sin(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f.$$

If $\underset{\to}{A} \parallel \hat{x}$ wave is longitudinal
If $\underset{\to}{A} \perp \hat{x}$ wave is transverse

$$v = \lambda f \rightarrow \text{speed}$$

Power transmitted by wave on stretched string

$$P_W = \frac{1}{2} A^2\omega^2 \frac{T}{V} \quad \text{where } V = \sqrt{\frac{T}{\mu}}$$

Intensity $I = \dfrac{\underline{\text{Power of Source}}}{4\pi R^2}$ ← SOUND

$\beta = (10\,dB)\log_{10}\frac{I}{I_0}$, $I_0 = 10^{-12}\,\text{Watt}/m^2$, $\beta = 10$, $\frac{I}{I_0} = 10$

$$I = \frac{1}{2} S_m^2 \omega^2 \frac{\gamma P_0}{v} \qquad S = S_m \sin(kx - \omega t)$$

$$P = P_0 - \gamma P_0 S_m k \cos(kx - \omega t)$$

vel. of sound $V = \sqrt{\frac{\gamma P_0}{\rho_0}} = \sqrt{\frac{\gamma k_B T}{m}}$

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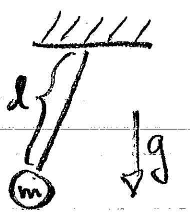
S-1

The period T of a pendulum, length ℓ, in a gravitational field with acceleration g is

$$T = 2\pi\sqrt{\ell/g}.$$



Thus, on Earth, we are given

$$1\,s = 2\pi\sqrt{\ell/9.8\tfrac{m}{s^2}}$$

$$\Longrightarrow \quad \ell = \frac{1}{(2\pi)^2}(1\,s)^2\left(9.8\tfrac{m}{s^2}\right)$$

$$\ell = 9.8/(4\pi^2)\,m$$

$$\boxed{\ell = 0.25\ m}$$

On the moon $g_M = g_E/6$, $T = 2\pi\sqrt{(0.25\,m)/(9.8\tfrac{m}{s^2}/6)}$

$$\boxed{T = 2.5\ s.}$$

We have $1s = 2\pi\sqrt{\ell/g_E}$, and want a new length, L, such that $1\,s = 2\pi\sqrt{L/g_M}$. Thus,

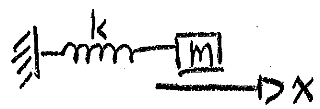
$$2\pi\sqrt{\frac{L}{g_M}} = 2\pi\sqrt{\ell/g_E}$$

so

$$\frac{L}{g_M} = \frac{\ell}{g_E}.$$ Since $g_M = g_E/6$,

we find $\dfrac{L}{g_E/6} = \dfrac{\ell}{g_E} \longrightarrow \boxed{L = \dfrac{\ell}{6}}.$

S-2



$$x(t) = (0.05\,m)\cos(\omega t)$$

frictionless $\Rightarrow$ Energy is conserved, which may be expressed as

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2.$$

At the extremes of the oscillation, the block is at a "turning point" so $v = 0$ and $E = \frac{1}{2}k(0.05\,m)^2$, thus all energy is potential.

Clearly, when $x=0$ $E = \frac{1}{2}mv^2$ and all energy is kinetic.

S-2 continued

So: (i) at $x=0$ kinetic energy is maximized,
and (ii) at $x=\pm 0.05m$ the potential energy is maximum.

If the kinetic and potential energies are equal

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

and

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = 2\left(\frac{1}{2}kx^2\right) = kx^2 .$$

From before, $E = \frac{1}{2}k(0.05\,m)^2$ at all times, so

$$kx^2 = \frac{1}{2}k(0.05\,m)^2$$

$$x = \sqrt{\frac{(0.05\,m)^2}{2}} = \boxed{0.035\,m} \text{ (iii)}$$

S-3

There is no change in frequency. In both cases $f = \frac{1}{2\pi}\sqrt{k/m}$.

When the spring/mass system is hung from the ceiling, it reaches an equilibrium position determined by k and g. However, g does not effect how the mass will oscillate about that equilibrium point. The frequency of oscillation in all cases is a result of the restoring force $\vec{F} = -kx\hat{x}$ from equilibrium, and is therefore independent of gravity. [In this case force is $\vec{F} = -ky\hat{y}$]

S-4

(i) While the wave propagates in the x direction (the argument of sin is $(6.28x - 12.56t)$), the Amplitude points in the $\hat{y}$ direction: the wave is transverse.

S-4

(ii) $\vec{y} = 0.01 \sin(6.28x - 12.56t)\,\hat{y}$

$= A \cdot \sin(kx - \omega t)\,\hat{y}$

So: $\left.\begin{array}{l} A = 0.01\ m \\ k = 6.28\ 1/m \\ \omega = 12.56\ Hz \end{array}\right\}$ $\boxed{\lambda = \frac{2\pi}{k} = 1.00\ m}$ $\boxed{f = \frac{\omega}{2\pi} = 2.00\ Hz}$

$\boxed{v = \lambda f = 2.00\ m/s}$ $\quad \vec{v} = 2\,m/s\ \hat{x}$

-OR- $\vec{y} = A \sin\left(2\pi \frac{x - vt}{\lambda}\right)\hat{y}$

So, put $6.28x - 12.56t$ into the form $2\pi \frac{x - vt}{\lambda}$:

$6.28x - 12.56t = 6.28(x - 2.00t)$

$= 2\pi \frac{x - 2.00t}{1.00}$

Thus $\boxed{\begin{array}{l} \lambda = 1.00\ m \\ v = 2.00\ m/s \\ f = v/\lambda = 2.00\ Hz \end{array}}$

8-5

(i) double A: $P_i = \frac{1}{2}(2A)^2\omega^2\frac{T}{v}$

$$= \frac{1}{2}\,4A^2\omega^2\frac{T}{v}$$

$$= 4\left(\frac{1}{2}A^2\omega^2\frac{T}{v}\right)$$

$$= 4P$$

doubling A ⟶ quadruple P.

(ii) halve ω: $P_{ii} = \frac{1}{2}A^2\left(\frac{\omega}{2}\right)^2\frac{T}{v}$

$$= \frac{1}{4}\left(\frac{1}{2}A^2\omega^2\frac{T}{v}\right)$$

$$= P/4$$

halving ω ⟶ quartering P.

(iii) increase T by a factor of 3:

Note $v = \sqrt{T/\mu}$ ⟵ involves T

So

$$P_{iii} = \frac{1}{2}A^2\omega^2\frac{3T}{\sqrt{3T/\mu}}$$

$$= \frac{3}{\sqrt{3}}\left(\frac{1}{2}A^2\omega^2\frac{T}{\sqrt{T/\mu}}\right)$$

$$= \sqrt{3}\left(\frac{1}{2}A^2\omega^2\frac{T}{v}\right)$$

$$= \sqrt{3}\,P$$

triple T ⟶ muliply P by $\sqrt{3}$.

CHAP 15

15-2 $V_1 = 150\ m/s$ when $T_1 = 75N$

$T_2 = ?$ when $V_2 = 180\ m/s$

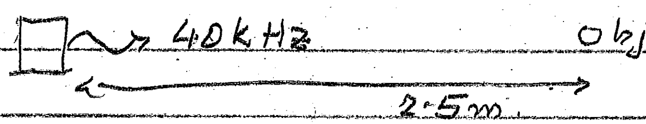
$$V_1 = \sqrt{\frac{T_1}{\mu}} \qquad V_2 = \sqrt{\frac{T_2}{\mu}}$$

$$\frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^2$$

$$T_2 = T_1\left(\frac{V_2}{V_1}\right)^2 = 75\left(\frac{180}{150}\right)^2$$

$$= 108N$$

15-3



$f = 4\times10^4\ Hz$

speed of sound in air at $20^\circ C$ (293 K)

$= 343\ m/s$ (p. 489, Table 15.1)

a) Wavelength $\lambda = \frac{343}{4\times10^4}\ m$

$= 8.6\times10^{-3}\ m. = 8.6\ mm$

b) Time of travel

$t = \frac{2\times2.5}{343}\ sec = 0.015\ sec. = 15\ msec.$

15-22 $\lambda = 450nm = 450\times10^{-9}m$ (Blue)

speed of light in air (vacuum) $= 3\times10^{8} m/s$.

$\lambda = 650nm = 650\times10^{-9}m$ (Red)

$$f_{Blue} = \frac{3\times10^{8}}{450\times10^{-9}} = 6.7\times10^{14} Hz$$

$$f_{Red} = \frac{3\times10^{8}}{650\times10^{-9}} = 4.6\times10^{14} Hz$$

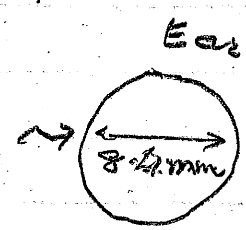
15-25 In dB we intensity is measured by decibels

$$\beta = (10dB)\log_{10}\frac{I}{I_0}$$

with $I_0 = 10^{-12}$ watt/m^2

Hence

$$20db = 10\log_{10}\frac{I}{I_0}$$



or $\log\frac{I}{I_0} = 2$, $\frac{I}{I_0} = 10^{2}$

So $I = 10^{-10}$ watt/m^2

Intensity is energy transport/m^2 so energy delivered to Ear will be $r = 4.2mm$

$$E = I\cdot\pi r^2 = 10^{-10}\times3.142\times(4.2\times10^{-3})^2$$

$$= 5.5\times10^{-15} \text{ Joule}$$

15-30 $\beta = (10\,dB)\log_{10}\frac{I}{I_0}$ $I_0 = 10^{-12}$ watt/m^2

so if $I = 3\times10^{-6}$ W/m^2

$$\beta = 10\log_{10}\frac{3\times10^{-6}}{10^{-12}} = 64.77\ dB$$