Week 12 – SOLUTIONS

A stationary charge q experiences a force in an $\underset{\rightarrow}{E}$-field

$$\underset{\rightarrow}{F_E} = q\underset{\rightarrow}{E}$$

A stationary $\underset{\rightarrow}{Q}$ generates a Coulomb $\underset{\rightarrow}{E}$

$$\underset{\rightarrow}{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} \qquad \boxed{\sum_c \underset{\rightarrow}{E}\cdot\underset{\rightarrow}{\Delta A} = \frac{1}{\varepsilon_0}\sum Q_i} \quad \text{Gauss}$$

A moving charge experiences a force which is perpendicular to its velocity at all times when it is located in a $\underset{\rightarrow}{B}$ field

$$\underset{\rightarrow}{F_B} = q\left[\underset{\rightarrow}{v}\times\underset{\rightarrow}{B}\right]$$

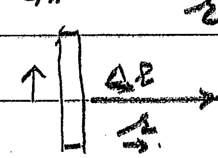
$q\underset{\rightarrow}{v} \parallel$ Thumb
$\underset{\rightarrow}{B} \parallel$ Fingers
$\underset{\rightarrow}{F_B} \perp$ Palm

A current is a moving charge. Hence, current I in a conductor Δl feels a force

$$\underset{\rightarrow}{F_I} = I\left[\underset{\rightarrow}{\Delta l}\times\underset{\rightarrow}{B}\right]$$

A current generates a $\underset{\rightarrow}{B}$ field $\underset{\rightarrow}{\Delta B} = \frac{\mu_0}{4\pi}\frac{I\,\Delta l\times\underset{\rightarrow}{r}}{r^3}$

Hence the $\underset{\rightarrow}{B}$-field circulates around the current.

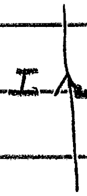


This leads to Ampere's law

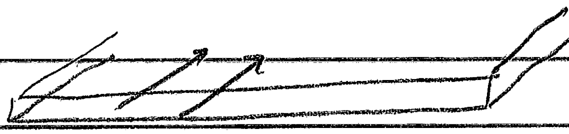
Circulation of $\underset{\rightarrow}{B}$ field around a closed loop is determined ~~only~~ by currents threading the surface on which the loop is drawn, only currents within the loop count

$$\sum_c \underset{\rightarrow}{B}\cdot\underset{\rightarrow}{\Delta l} = \mu_0\sum I_i$$

Ampere's law Applications



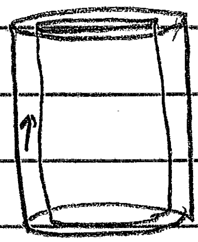
Single wire $\underset{\sim}{B} = \frac{\mu_0 I}{2\pi r}\,\hat{\phi}$



sheet of thickness t carrying current density $\underset{\rightarrow}{J} = -J\hat{z}$

Below sheet $\underset{\sim}{B} = -\frac{\mu_0 J t}{2}\,\hat{x}$

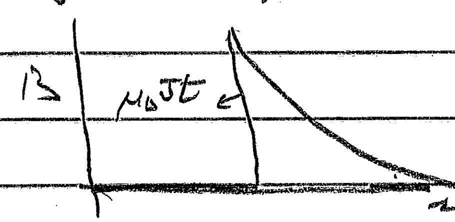
Above " $\underset{\sim}{B} = +\frac{\mu_0 J t}{2}\,\hat{x}$



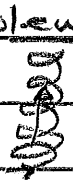
cylindrical sheet of radius R and thickness t ($\ll R$).

$r < R$ $\quad \underset{\sim}{B} = 0$

$r > R$ $\quad \underset{\sim}{B} = \frac{\mu_0 I}{2\pi r}\,\hat{\phi}$. on surface $\underset{\sim}{B} = \mu_0 J t\,\hat{\phi}$



Solenoid $\underset{\sim}{B} = \mu_0 n I\,\hat{y}$



cannot use amperes law but it can be shown that for a single ring of radius a centered at zero and carrying current I, B field at y is

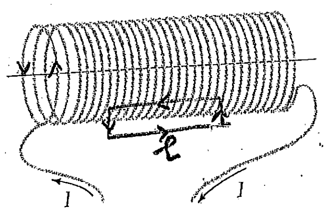
$$\underset{\sim}{B} = \frac{\mu_0}{4\pi}\,\frac{2 I \pi a^2}{(a^2+y^2)^{3/2}}\,\hat{y} = \frac{\mu_0}{4\pi}\,\frac{2\mu}{(a^2+y^2)^{3/2}}$$

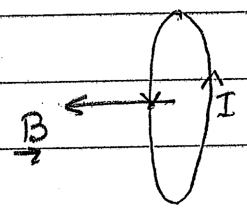
where Magnetic moment is $\underset{\sim}{\mu} = I\pi a^2\,\hat{y}$

S-32 A tightly wound long solenoid consists of a large number of closely spaced rings with a common axis (see figure). It produces a uniform field inside it. Use Ampere's law to show that for the case shown (ccw current in solenoid)

$$\underset{\rightarrow}{B} = -\mu_0 n I \hat{x}$$

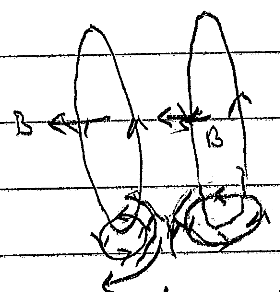
where n= No. of turns/meter of the solenoid. (n= N/L)





a single ring, with current as shown will produce a field along $-\hat{x}$ on its axis.

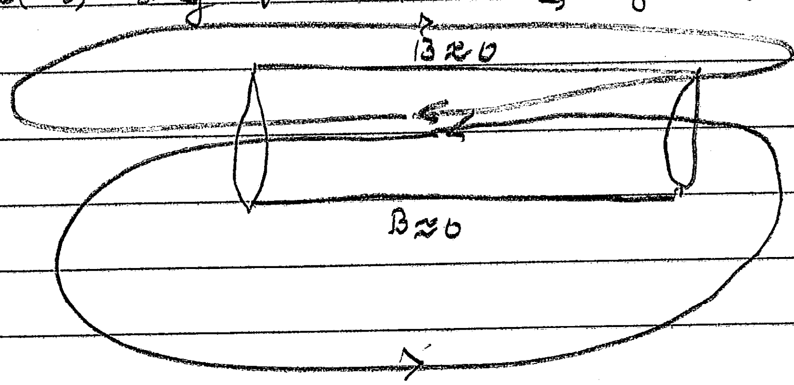
Consider two neighboring rings, close



along the radius the fields cancel.

So $B_r = 0$.

If solenoid is long & narrow $\underset{\rightarrow}{B}$-field must look like



So just outside $B \approx 0$ as $\underset{\rightarrow}{B}$ field lines must circulate so one should say

$$\underset{\rightarrow}{B} = -B\hat{x} \quad \text{inside.}$$

$$B_{?} = 0$$

$$B_{outside} = 0$$

Appropriate Ampere loop is as shown on figure above

$$\sum_c \underset{\rightarrow}{B} \cdot \underset{\rightarrow}{\Delta l} = Bl + 0 + 0 + 0. \quad \text{Amp-meter.}$$

The total current threading the loop is

$$\sum I_i = \mu_0 n l I$$

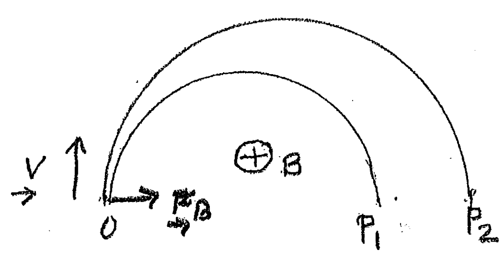
hence

$$Bl = \mu_0 n I l$$

so for solenoid

$$\underset{\rightarrow}{B} = -\mu_0 n I \hat{x}.$$

S-33 In a mass spectrometer the beam at 0 consists of two kinds of particles with same mass (M) but different charges q_1, q_2 entering with a velocity $\underset{\rightarrow}{v} = v\hat{y}$. For $\underset{\rightarrow}{B} = -B\hat{z}$ and the paths shown, what is the sign of the charge (+ive or –ive)? Where will the larger charge land, P_1 or P_2? Justify your answer.



In order to follow the path shown since $\underset{\rightarrow}{V} = V\hat{y}$ and $\underset{\rightarrow}{B} = -B\hat{z}$ and $\underset{\rightarrow}{F_B} = q[\underset{\rightarrow}{v} \times \underset{\rightarrow}{B}]$ which have to be along $\hat{x}$ so q must be negative.

The radius of the cyclotron orbit

$$R = \frac{MV}{qB}$$

Both particles have same mass.

$$R_1 = \frac{MV}{q_1 B} \qquad R_2 = \frac{MV}{q_2 B}$$

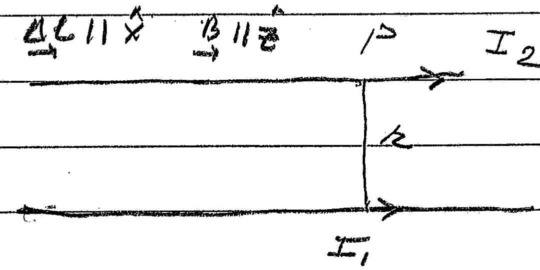
But larger q will have smaller R
so larger q lands at P_1.

<u>5-34</u> The current-current force arises because a current generates a $\underset{\to}{B}$-field $\underset{\to}{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$ and a current carrying conductor feels a force in a $\underline{B}$-field. $\underset{\to}{F}_I = I\,\underset{\to}{\Delta l} \times \underset{\to}{B}$

$\underset{\to}{\Delta l} \parallel \hat{x}$ $\quad \underset{\to}{B} \parallel \hat{z}$

Let us begin with I_1 at a point it will produce a field

$$\underset{\to}{B}_1 = \frac{\mu_0 I_1}{2\pi r}\hat{z}$$

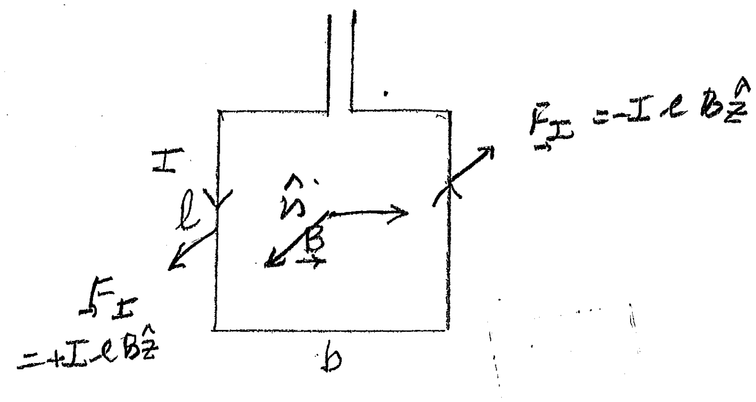


$\underset{\to}{\Delta l} \times \underset{\to}{B}$ along $-\hat{y}$

Now put I_2 of length l_2, force on I_2 due to I_1

$$\underset{\to}{F}_{I_2, I_1} = I_2\left[\underset{\to}{\Delta l_2} \times \underset{\to}{B}_1\right] = -\mu_0 \frac{I_1 I_2 l_2}{2\pi r}\hat{y}$$

S-35 Shown is a coil of width b and length ℓ suspended vertically in a $\underset{\rightarrow}{B}$-field. How would you make it work like a motor? The coil is free to rotate about its vertical axis.



Begin by establishing current I as shown.

The $\vec{B}$ field establishes forces

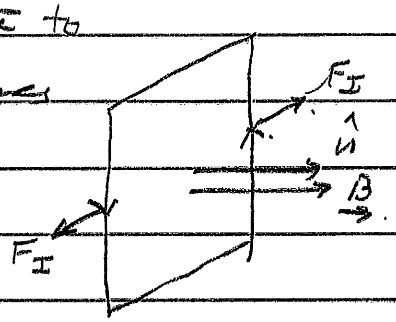
$\vec{F}_I = -I\ell B\hat{z}$ right wire

$\vec{F}_I = +I\ell B\hat{z}$ left wire

They will cause a torque

$$\vec{\tau} = I\ell B\frac{b}{2}\hat{y} + I\ell B\frac{b}{2}\hat{y}$$

and the coil will rotate to the position when $\hat{n}$ becomes parallel to $\vec{B}$ and torque becomes zero.



As soon as $\hat{n}$ overshoots further, reverse I

coil will continue to experience a torque parallel to $\hat{y}$ and you have a dc motor

So establish a current & switch it every half cycle to keep the motor going.

24-4

Earth's $\vec{B}$-field: 5×10^{-5} T

Field at center of one loop: $B = \frac{\mu_0 I}{2R}$

" " " " N loops: $B = N\frac{\mu_0 I}{2R}$.

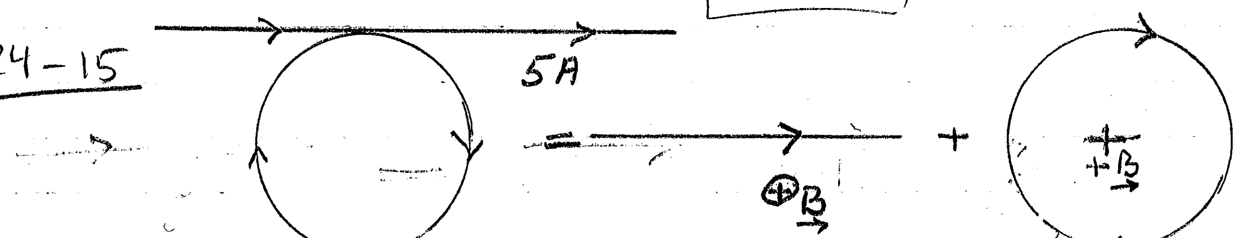
Note: The direction is assumed to be taken care of already, so we don't need to concern ourselves with vectors. Only magnitude is of interest.

$$B = \frac{\mu_0 N I}{2R} \Longrightarrow I = \frac{2RB}{\mu_0 N}$$

$$= \frac{2\,(0.50\text{ m})(5\times10^{-5}\text{ T})}{(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}})(200)}$$

$$= \boxed{0.2\text{ A.}}$$

24-15



Straight wire with loop = Superposition of straight wire and one loop, that is

$$\vec{B}_{total} = \vec{B}_{wire} + \vec{B}_{loop}$$

At center of loop $\vec{B}_{loop} = -\frac{\mu_0 I}{2R}\hat{z}$ where R is the radius of the loop.

A distance r from the straight wire $\vec{B}_{wire} = -\frac{\mu_0 I}{2\pi r}\hat{z}$

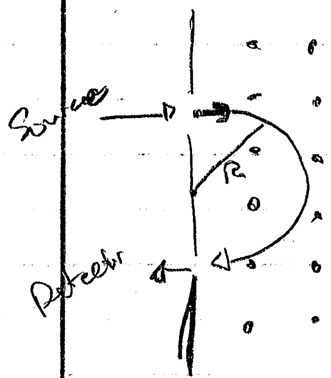
By right hand rule, Both B_{loop} and B_{wire} point into the page. Thus

$$\vec{B}_{tot} = -\frac{\mu_0 I}{2\pi r}\hat{z} + -\frac{\mu_0 I}{2R}\hat{z} = -\frac{\mu_0 I}{2}\left(\frac{1}{\pi r} + \frac{1}{R}\right)\hat{z}$$

$$= -\frac{(4\pi\times10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}})(5.0\text{ A})}{2}\left(\frac{1}{0.01\text{ m}} + \frac{1}{\pi(0.01\text{ m})}\right)\hat{z}$$

$$= \boxed{-4.1\times10^{-4}\text{ T}}\,\hat{z}$$

24.28



We need to calculate the Diameter

q +ive

$\vec{B} = B\hat{z}$

$\vec{B}$ - variable, uniform mag. field.

Net charge is 1+

i.e, $q = +1.6 \times 10^{-19}$ C

Mass is $85 m_p$, m_p is the proton mass

$$m = 85 m_p = 1.42 \times 10^{-25} \text{ kg}$$

The radius a charged particle traces out is given by

$$R = \frac{mv}{qB} = \frac{(1.42 \times 10^{-25} \text{ kg})(2.3 \times 10^5 \text{ m/s})}{(1.6 \times 10^{-19} \text{ C})\ (0.80 \text{ T})}$$

$$\boxed{R = 2.6 \times 10^{-1} \text{ m}}$$

The distance is $2R = \boxed{5.2 \times 10^{-1} \text{ m.}}$

24.31

$\vec{B}$ points in the $\hat{y}$ direction. The force due to $\vec{B}$ is given by $\vec{F} = q\vec{v} \times \vec{B}$. Since $\vec{B}$ is in the $\hat{y}$ direction, $\vec{F}$ cannot be in the $\hat{y}$ direction. Thus, the $\hat{y}$-component of velocity is constant.

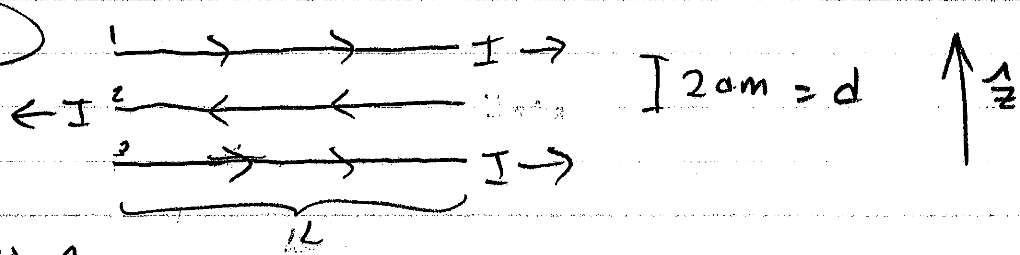
$$v_y = v \sin\theta = (5.5 \times 10^5 \tfrac{m}{s})(\sin 30°)$$

It travels this fast for 10 μs, starting at $y = 0$.

So $$y = v_y t = (5.5 \times 10^5 \tfrac{m}{s})(\sin 30°)(10 \times 10^{-6} \text{ s})$$

$$\boxed{y = 2.8 \text{ m.}}$$

24-33



$I = 10\ A$
$L = 50\,cm$

THE FORCE BETWEEN 2 WIRES w/ OPPOSITE CURRENTS IS REPULSIVE AND GIVEN BY

(1) $F = \dfrac{\mu_0 I_1 I_2}{2\pi d} L$ WHERE L IS LENGTH, d = SEPARATION.

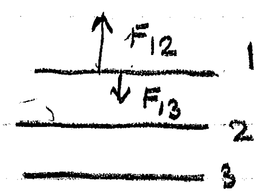
THEN, FOR OUR SYSTEM,

$$|F_{12}| = |F_{23}| = \frac{\mu_0 I^2}{2\pi d} L$$

FOR THE CURRENTS IN THE SAME DIRECTION, THERE IS AN ATTRACTIVE FORCE GIVEN BY (1) SO FOR F_{13},

$$|F_{13}| = \frac{\mu_0 I^2 L}{2\pi (2d)}$$

ADDING FORCES,



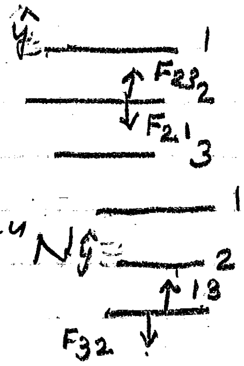
FORCE ON 1 $= (|F_{12}| - |F_{13}|)\hat{z}$

$$= \frac{\mu_0 I^2 L}{2\pi}\left(\frac{1}{d} - \frac{1}{2d}\right)\hat{z}$$

$$= \frac{\mu_0 I^2 L}{2\pi d}\left(\frac{1}{2}\right)\hat{z} = 2.5\times10^{-4}\ N\ \hat{y}$$

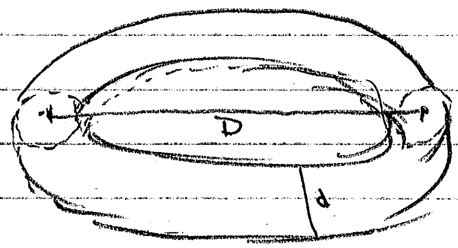
FORCE ON 2 $= (|F_{12}| - |F_{23}|) = 0\ N\ \hat{y}$

FORCE ON 3 $=$ $-$ FORCE ON 1 $= -2.5\times10^{-4}\ N\hat{y}$



29-41

CONSIDER: $\vec{\mu} = I A \hat{n}$

* 

$D = 3 \times 10^3$ km

$d = 1 \times 10^3$ km

THE AREA OF THE "LOOP": $A = \pi r^2 = \pi \left(\frac{D}{2}\right)^2$

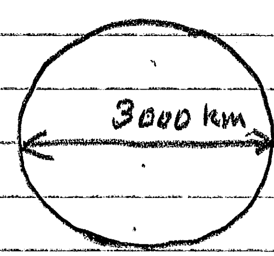
THEN, WE KNOW THE DIPOLE MOMENT μ IS GIVEN BY:

$$AI = \mu \Rightarrow I = \frac{\mu}{A}, \quad \mu = 8 \times 10^{22}\ Am^2$$

SO THAT NOW,

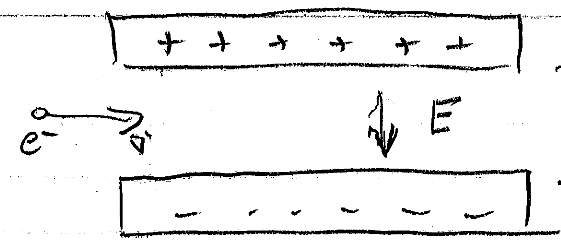
$$I = \frac{\mu}{\pi \left(\frac{D}{2}\right)^2} = \frac{4\mu}{\pi D^2} = 1.13 \times 10^{10}\ A$$

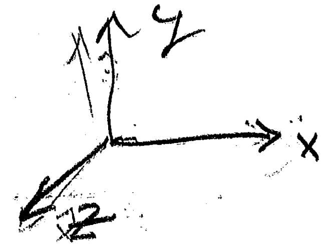
* We are pretending that this can be thought of as a "wire" of diameter



and we dont worry about the wire thickness.

(51)





$V_0 = 1.0\times10^{7}$ m/s $\quad d = 1$ cm

THE POTENTIAL DIFFERENCE BETWEEN PLATES $\Delta V = 200V$

WE NEED THE MAGNETIC FIELD TO BE STRONG ENOUGH TO EXERT A FORCE THAT CANCELS THE ELECTRIC FIELD'S FORCE. NOW:

$\vec{F}_E = qE\hat{y}$ electron has -ive charge

We ~~need to~~ cancel this with $\vec{F}_B$ along $-\hat{y}$,
so $\vec{B}$ will have to be along $-\hat{z}$

$\vec{F}_B = -qvB\hat{y}$ and $\vec{F}_E + \vec{F}_B = \vec{F}$

WE NEED THIS TO BE 0:

$$0 = B = \frac{E}{V} = \frac{200}{10^{-2}\times10^{7}} = 2\times10^{-3}\,T$$

$$\Rightarrow \vec{B} = -2\times10^{-3}\,T\hat{z}$$

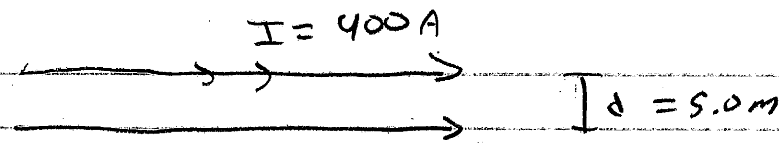
... motion, $\vec{v} = v_0\hat{y}$

So TAKE $\vec{B} = B_0\hat{n}$ ... $(\hat{y}\times\hat{n})$

Now, ... $= 2.0\times10^{-3}$ T

Thus, ... $\vec{B} = -B_0\hat{x}$

55



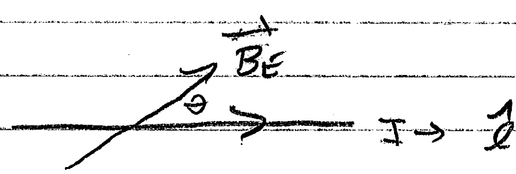
WE KNOW THE FORCE DUE TO EARTH'S $\vec{B}_E$ GIVEN BY:

$$\vec{F}_E = I\vec{l} \times \vec{B}_E$$

AND FORCE DUE TO OTHER WIRE:

$$|\vec{F}_W| = \mu_0 L \frac{I^2}{2\pi d}$$

FOR THIS PROBLEM, WE MUST ASSUME THAT EARTH'S FIELD MAKES SOME ANGLE θ WITH THE WIRES:



THEN,

$$|\vec{F}_E| = ILB_E \sin\theta \Rightarrow \frac{|\vec{F}_E|}{IL} = B_E \sin\theta$$

ALSO,

$$|\vec{F}_W| = \mu_0 L \frac{I^2}{2\pi d} \Rightarrow \frac{|\vec{F}_W|}{IL} = \mu_0 \frac{I}{2\pi d}$$

COMPARING,

$$B_E \sin\theta = (5\times10^{-5}\,T)\sin\theta$$

$$\frac{\mu_0 I}{2\pi d} = \frac{(4\pi\times10^{-7})(400)}{2\pi(5)}\,T = 1.6\times10^{-5}\,T$$

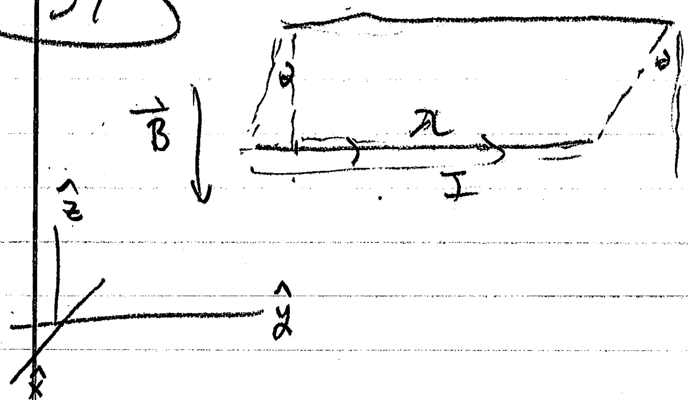
SO THAT SETTING THESE EQUAL,

$$5\sin\theta_c = 1.6 \Rightarrow \theta_c = \sin^{-1}\left(\frac{1.6}{5}\right)$$

THUS, FOR $\theta > \theta_c$, $|\vec{F}_E| > |\vec{F}_W|$ $\quad$ $\theta = \theta_c$, $|\vec{F}_E| = |\vec{F}_W|$

FOR $\theta < \theta_c$, $|\vec{F}_E| < |\vec{F}_W|$

59



$\theta = 10^\circ = \frac{10}{180}\pi$

$\lambda = 50\,g/m$

$I = 10\,A$

$\vec{B} = -|B|\,\hat{z}$

$L =$

WE KNOW THE FORCE ON THE WIRE BY GRAVITY

$$\vec{F}_g = -mg\hat{z} = -\lambda L g\,\hat{z}$$

AND THE FORCE DUE TO $\vec{B}$

$$\vec{F}_B = I\vec{l} \times \vec{B} = IL|B|(\hat{y} \times \hat{z}) = IL|B|\,\hat{x}$$

THUS THE TENSION IN THE STRING, $\vec{T} = T_z\hat{z} + T_x\hat{x}$

$$T_z = -|\vec{F}_g| = T\cos\theta$$

FOR $\theta = 10^\circ$, $\cos\theta \approx 1$, $\sin\theta \approx \theta$

$$T_x = -T\sin\theta \approx -T\theta \quad , \quad T_z \approx T = \lambda L g$$

SO THAT, $-T\theta + IL|B| = 0$

$$\text{OR} \quad |B| \approx \frac{T\theta}{IL} = \frac{\lambda L g}{IL}\theta = \frac{\lambda g}{I}\theta$$

$$= \frac{\lambda g}{I}\theta \approx .0087\,T$$