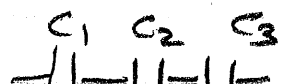
SOLUTIONS – 11

Devices: Battery $\overset{+}{|}\!\!\vdash^{-}$ $\mathcal{E}$

Capacitor $C = \frac{Q}{V}$, $\parallel$-plate $C_0 = \frac{\epsilon_0 A}{d}$, $U_E = \frac{Q^2}{2C_0}$

$$\boxed{\eta_E = \tfrac{1}{2}\epsilon_0 E^2}$$

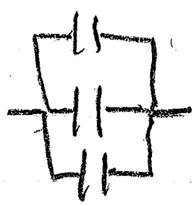
Many Capacitors



Series.

Q – common
V's add

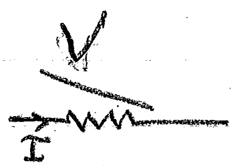
$\frac{1}{C_s} = \sum \frac{1}{C_i}$



V common, Q's add

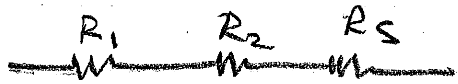
$C_p = \sum C_i$

Resistor

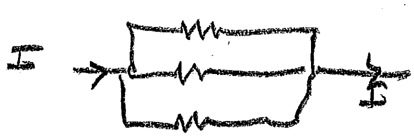


$R = \frac{V}{I}$, $R = \frac{\ell}{\sigma A}$, $I = \vec{J}\cdot\vec{A}$ $\boxed{\vec{J} = \sigma \vec{E}}$

Many Resistors



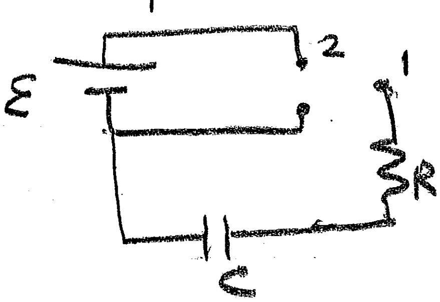
Series
I common
V's add.
$R_s = \sum R_i$



V – common
I's add.

$\frac{1}{R_p} = \sum \frac{1}{R_i}$

Capacitor – Resistor – Battery.



$\underline{t=0}$ $1 \to 2$ charging

$i = \frac{\mathcal{E}}{R} e^{-t/RC}$, $V_C = \mathcal{E}\left[1 - e^{-t/RC}\right]$

$t \to 0$ $1 - 3$ Discharging

$i = \frac{\mathcal{E}}{R} e^{-t/RC}$ $V_C = \mathcal{E} e^{-t/RC}$

<u>Note</u> NO CURRENT EVER FLOWS BETWEEN CAPACITOR PLATES

$$q = EC\left[1 - e^{t/RC}\right] \qquad \tau = RC.$$

POWER LOSS IN RESISTOR

$$P = I^2 R = \frac{V^2}{R}.$$

FORCE ON MOVING CHARGE due to $\underset{\rightarrow}{B}$.

$$\underset{\rightarrow}{F_B} = q\left[\underset{\rightarrow}{V} \times \underset{\rightarrow}{B}\right]$$

Charge moves in a <u>PLANE</u> perpendicular to B. Motion is circular

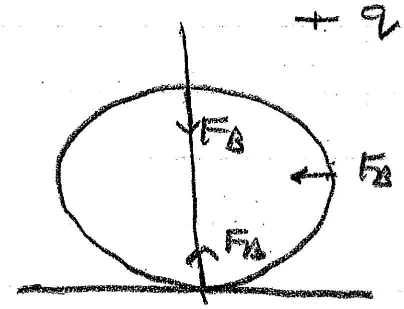
$\underset{\rightarrow}{V} = V\hat{x}$ at $t=0$ when

$\underset{\rightarrow}{B} = -B\hat{z}$ turned on

$$\underset{\rightarrow}{F_B} = -qVB\hat{r}$$

provides

$$\underset{\rightarrow}{F_c} = -\frac{Mv^2}{R}\hat{r}$$



So $R = \dfrac{Mv}{qB}$, angular velocity $\dfrac{v}{R} = \dfrac{qB}{M} = \omega$

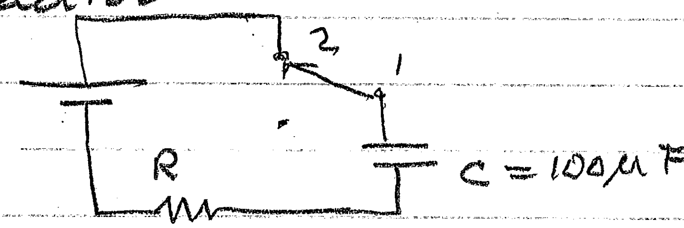
Direction is Parallel or antiparallel to $\underset{\rightarrow}{B}$.

Ampere's law

$$\sum_c \underset{\rightarrow}{B} \cdot \Delta \underset{\rightarrow}{l} = \mu_0 \sum I_i$$

Chap 23

37. Want to charge capacitor to 87% of $\mathcal{E}$ in 8 sec.



$V_C = \mathcal{E}\left[1 - e^{-t/RC}\right]$

$0.87\,\mathcal{E} = \mathcal{E}\left[1 - e^{-t/RC}\right]$

$\mathcal{E}\, e^{-t/RC} = 0.13\,\mathcal{E}$

$\ln e^{-t/RC} = \ln 0.13 = -2.04$

$\frac{t}{RC} = 2.04$

$R = \frac{t}{2.04\,C} = \frac{8}{2.04 \times 10^{-4}}$

$= 4 \times 10^{4}\,\Omega$

39 Discharge

$V_C = \mathcal{E} e^{-t/RC}$

$q = \mathcal{E} C e^{-t/RC} = Q e^{-t/RC}$

$C = 10^{-5}\,F$

$R = 10^{3}\,\Omega$

$Q = 20\,\mu C$

$q = 10\,\mu C$

$10 = 20\, e^{-t/RC}$

$0.5 = e^{-t/RC}$

$-0.69 = -t/RC$

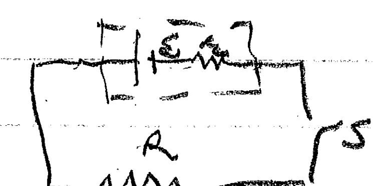
$t = 0.69 \times R \times C$

$= 0.69 \times 10^{3} \times 10^{-5} = \underline{6.9\,ms}$

48

$\mathcal{E} = 1.5\,V$

$r = 1\,\Omega$



close switch $I = \frac{\mathcal{E}}{R+r}$, $P = \left(\frac{\mathcal{E}}{R+r}\right)^{2} R$.

48 cont'd $R = 0.25\,\Omega$ $\quad P = \left(\frac{1.5}{1.25}\right)^2 0.25 = 0.36\,W$

$R = 0.5\,\Omega$ $\quad P = \left(\frac{1.5}{1.5}\right)^2 0.5 = 0.5\,W$

$R = 1.0\,\Omega$ $\quad P = \left(\frac{1.5}{2}\right)^2 1. = 0.56\,W$

$R = 2.0\,\Omega$ $\quad P = \left(\frac{1.5}{3}\right)^2 2. = 0.5\,W$

$R = 4.0\,\Omega$ $\quad P = \left(\frac{1.5}{5}\right)^2 4 = 0.36\,W$

<u>65</u> Discharging $\quad q = Q e^{-t/RC}$
$RC = 10\,ms$

$$u_E = \frac{q^2}{2C} = \frac{Q^2}{2C} e^{-\frac{2t}{RC}}$$

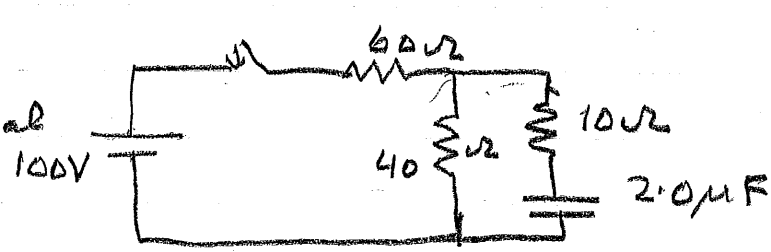
To get $q = \frac{Q}{2}$ $\quad e^{-t/RC} = 0.5$ $\quad \frac{t}{RC} = 0.69$

$t = 6.9\,ms$

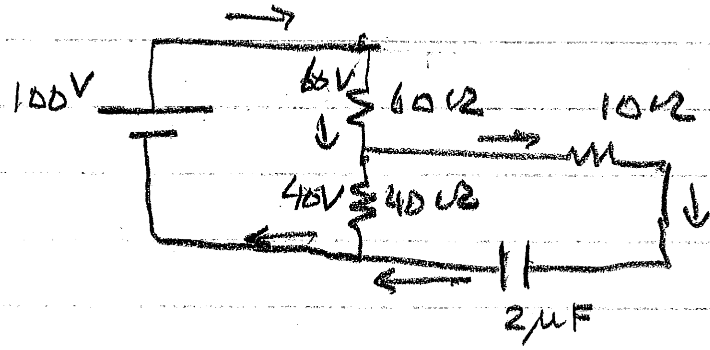
$q^2 = \frac{Q^2}{2}$ $\quad e^{-2t/RC} = 0.5$ $\quad \frac{2t}{RC} = 0.69$

$t = 3.45\,ms$

<u>70</u> a When the switch was closed potential across $R = 10\,\Omega$, $C = 2.0\,\mu F$ is 40V



No Current inside C



So $Q(2) = 40 \times 2 \times \mu F = 80\,\mu C$.

b) When switch opens capacitor discharges through $40\,\Omega + 10\,\Omega = 50\,\Omega$

$R = 50\,\Omega$

$C = 2 \times 10^{-6}\,F$

$$q = Q e^{-t/RC}$$

$$0.1 = e^{-t/RC}$$

$$-2.3 = -t/RC$$

$$t = 2.3RC = 2.3 \times 50 \times 2 \times 10^{-6}\,\text{sec}$$

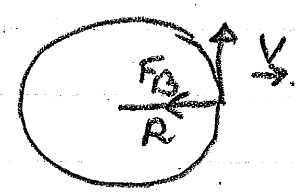
$$= 230\,\mu\text{sec}.$$

5-28 In an $\underset{\to}{E}$ field a stationary charge experiences a force. In a $\underset{\to}{B}$ field a moving charge experiences a force perpendicular to its velocity at all times.

$$\underset{\to}{F_E} = q\,\underset{\to}{E} \qquad \underset{\to}{F_B} = q\,[\underset{\to}{v} \times \underset{\to}{B}]$$

S-29 (i) Since the force $\vec{F}_B = q[\vec{v} \times \vec{B}]$, it is always perpendicular to the displacement and therefore does NO work.

(ii) The force $\vec{F}_B = -qvB\hat{r}$ provides the centripetal force $\vec{F}_c = -\frac{Mv^2}{R}\hat{r}$



hence $\frac{Mv^2}{R} = qvB$

angular velocity

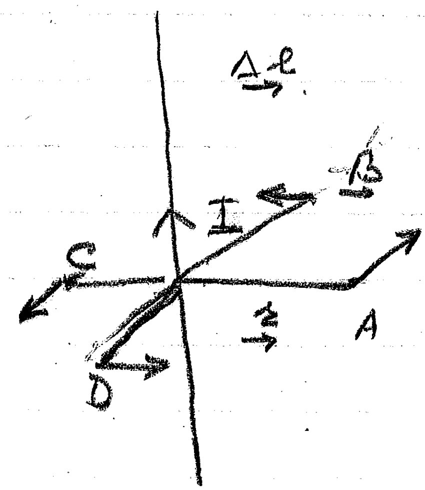
$$\omega = \frac{v}{R} = \frac{qB}{M}$$

is independent of v.

S-30 Take $\Delta \vec{l} \parallel$ y-axis

at pt A $\vec{r} = r\hat{x}$

$\Delta\vec{l} \times \vec{r}$ is along $-\hat{z}$



at pt B $\vec{r} = -r\hat{z}$

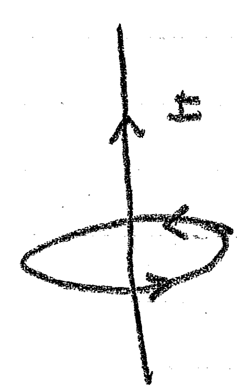
$\Delta\vec{l} \times \vec{r}$ is along $-\hat{x}$

at pt C $\vec{r} = -r\hat{x}$

$\Delta\vec{l} \times \vec{r}$ is along $+\hat{z}$

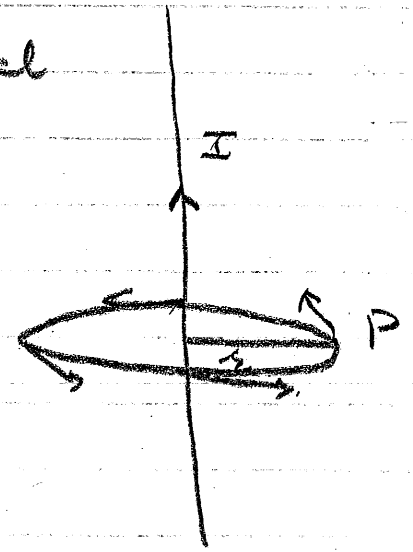
at pt D $\vec{r} = +r\hat{z}$

$\Delta\vec{l} \times \vec{r}$ is along $+\hat{x}$

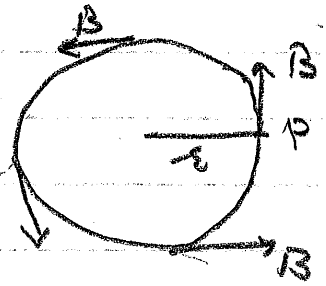


$\therefore$ Vector $\vec{B}$ circulates around I.

S-31 We have cylindrical symmetry about the y-axis. Hence $\vec{B}$ can be function of r only & must circulate around I.



Let us choose for the closed loop a circle of radius r centered at I. Magnitude of B is same on all points of circle. $\vec{B}$ is tangent to circle so



$$\sum_c \vec{B}\cdot\Delta\vec{l} = (B\,2\pi r)$$

By ampere's law

$$\sum_c \vec{B}\cdot\Delta\vec{l} = \mu_0 \sum I_i = \mu_0 I$$ (I is the only current).

So $$B\,2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

azimuthal angle