SOLUTIONS-10

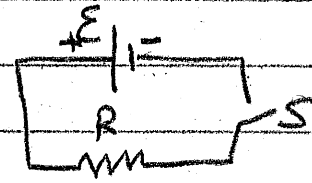
Devices Battery $\overset{+}{}\!\mid\!\mid\!\overset{-}{}$ ε

Capacitor $C = \frac{Q}{V}$ ∥-plate $C_0 = \varepsilon_0 \frac{A}{d}$

Resistor $R = \frac{V}{I}$ $R = \rho \frac{l}{A} = \frac{l}{\sigma A}$

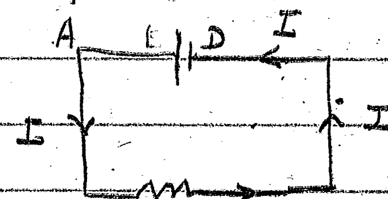
Ideal Battery and R

Close switch



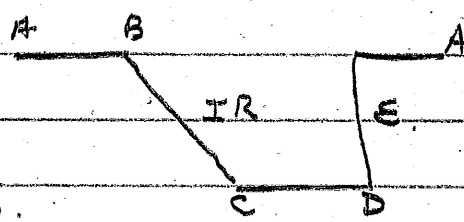
Current flows from + to − outside and from − to + inside battery − outside electrons flow

Inside ions flow



Potential varies as

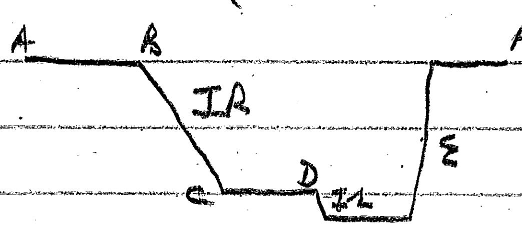
$I = \frac{\varepsilon}{R}$

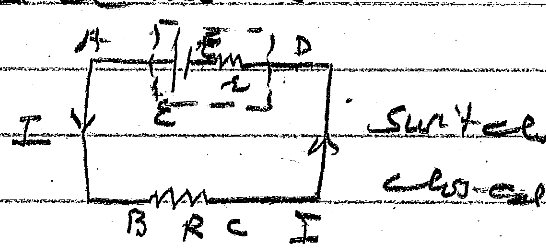


Power loss in R $P = \frac{\varepsilon^2}{R}$

Real Battery has resistance inside

Potential

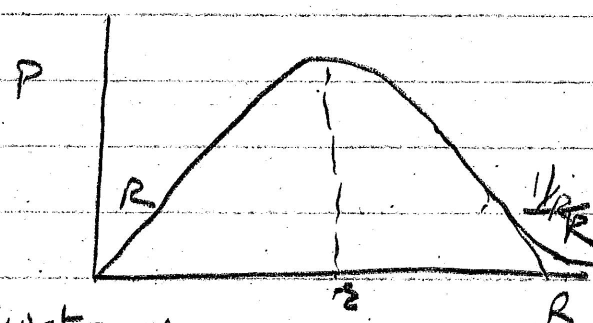




$$I = \frac{\varepsilon}{(R+r)}$$

Power in R $\quad P = \left(\frac{\varepsilon}{R+r}\right)^2 R.$

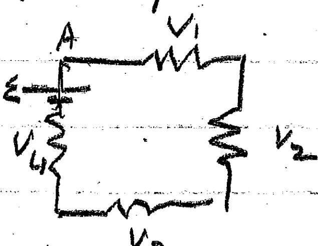
Vary R
Maximum
power when
$R = r$
Battery resistance



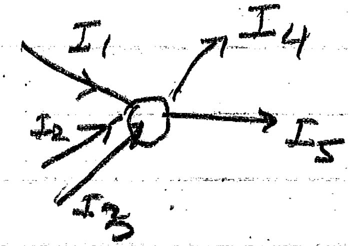
Kirchhoff's laws

__Loop Rule:__ Since potential at a point is unique, total change of potential around any closed loop must be zero

$$V_A - V_1 - V_2 - V_3 + \varepsilon = V_A.$$



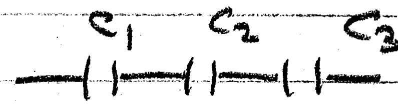
__JUNCTION RULE:__ Current measures flux of charge, charge is conserved so at a junction total outgoing current must be equal to total incoming current



$$I_4 + I_5 = I_1 + I_2 + I_3$$

More than One Capacitor

Series circuit



$$\frac{1}{C_s} = \sum \frac{1}{C_i}$$ charge is common

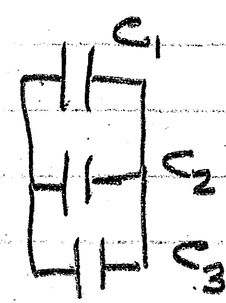
V's add.

Parallel Circuit

$C_p = \sum C_i$

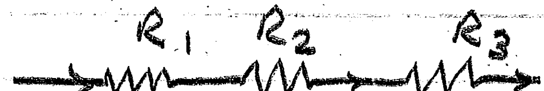
V's common

Q's add



More than one resistor

Series:

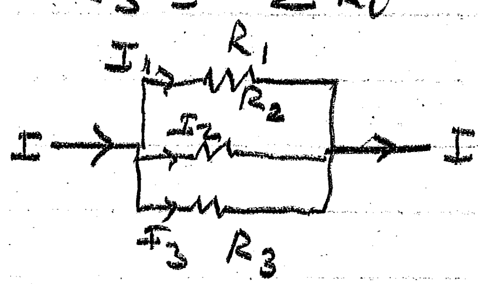


current is common

V's add $\quad R_s = \sum R_i$

Parallel

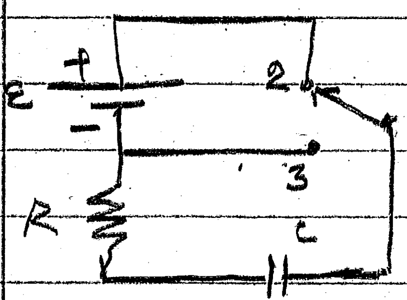
$I = I_1 + I_2 + I_3$



Potential drop is common, Currents add

$$\frac{1}{R_p} = \sum \frac{1}{R_i}$$

Charging / Discharging a capacitor.



At $t=0$, Connect 1 to 2 the battery will start moving charges through the wires so charges will begin to accumulate on the plates of C and current will flow through R. NO CURRENT FLOWS THROUGH C at ANY TIME.

At $t=0$

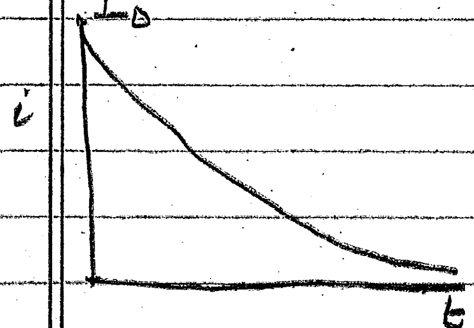
$$\mathcal{E} - I_0 R = 0 \qquad I_0 = \frac{\mathcal{E}}{R}.$$

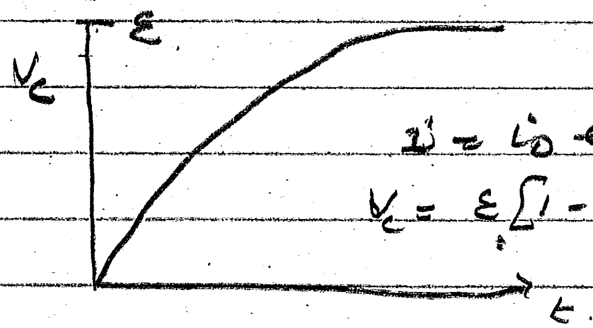
a short time later capacitor has charge on it

$$\mathcal{E} = iR + \frac{q}{C} \qquad i = \frac{1}{R}\left(\mathcal{E} - \frac{q}{C}\right)$$

i is less than I_0.

A long time later capacitor is fully charge $\mathcal{E} = \frac{Q}{C}$, $i = 0$.





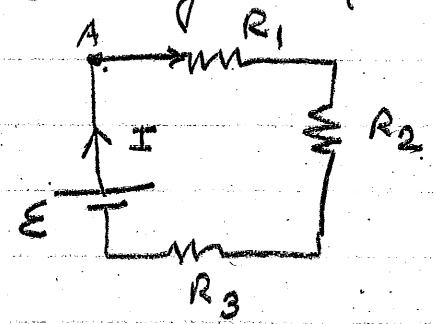
$$i = i_0 e^{-t/RC}$$

$$V_c = \mathcal{E}\left[1 - e^{-t/RC}\right]$$

If later you connect 1 to 3, capacitor discharges through R.

S-25 Kirchhoff's "Laws"

Loop Rule Since we are dealing with conservative forces potential at any point is unique, hence ~~total~~ change of potential around any ~~closed~~ loop must be zero.

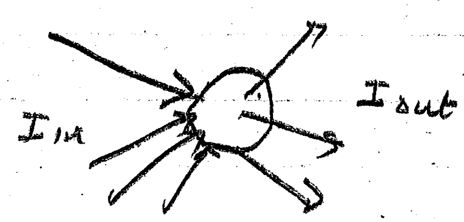


$$V_A - IR_1 - IR_2 - IR_3 + \mathcal{E} = V_A.$$

Note: V drops if you go along I; rises if you go against I.

JUNCTION RULE

Current measures flux of charge and charge is conserved so at a junction the total outgoing current must equal the total incoming current



$$\Sigma I_{out} = \Sigma I_{in}$$

5-26 RC

$$R = \frac{V}{I} \qquad C = \frac{Q}{V}$$

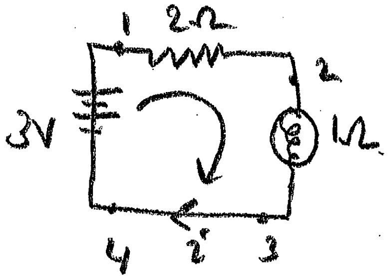
$$\text{so } RC = \frac{V}{I} \cdot \frac{Q}{V} = \frac{Q}{QT^{-1}} \rightarrow T^{1}$$

so RC has dimensions of time

5-27 T is controlled by both R and C. Charging a capacitor requires transfer of charge from the battery to the capacitor plates. The current must pass through R so R controls the rate of transfer. The quantity of charge needed to complete the process is controlled by C. Hence R and C both appear in the time constant.

23.5

Find the current



(a) $3V - 2\Omega\, i - 1\Omega\, i = 0$

$\Rightarrow \underline{\underline{i = 1A}}$

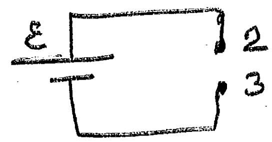
$\Delta V_{12} = V_2 - V_1 = -i\,(2\Omega) = -(1A)(2\Omega) = -2V$

$\Delta V_{22} = V_3 - V_2 = -i\,(1\Omega) = -(1A)(1\Omega) = -1V$

$\Delta_{34} = V_4 - V_3 = 0V$

(b) $\Delta V_{12} = V_2 - V_1 = 0V$ (no current flows)

$\Delta V_{23} = V_3 - V_2 = -3V$ $\longrightarrow$



$\Delta V_{34} = V_4 - V_3 = 0V$

$\boxed{23.12}$ Resistivities are in Table 22.1 [p. 732]

$$R_{copper} = \frac{(1.7\ 10^{-8}\ \Omega m)(0.2 m)}{\pi (0.5\ 10^{-3} m)^2}$$

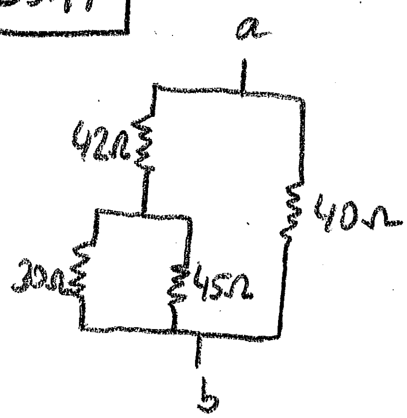
b/c $R = \frac{\rho l}{A}$

$$R_{Iron} = \frac{(9.7\ 10^{-8}\ \Omega m)(0.6 m)}{\pi (0.5\ 10^{-3} m)^2}$$

Since two wires are in series $R_{total} = R_{copper} + R_{iron}$

$$R_{total} = 4.33\ 10^{-3}\ \Omega + 74.1\ 10^{-3}\ \Omega = 78\ 10^{-3}\ \Omega = 78\ m\Omega$$

$\boxed{23.17}$



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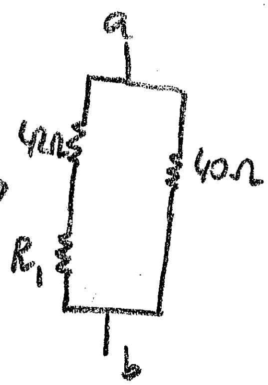
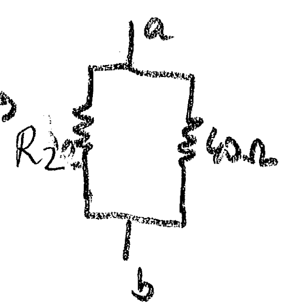


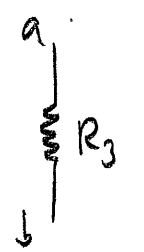
$$R_1 = \left(\frac{1}{30\Omega} + \frac{1}{45\Omega}\right)^{-1} \text{ (parallel)}$$
$$= 18\ \Omega$$

→



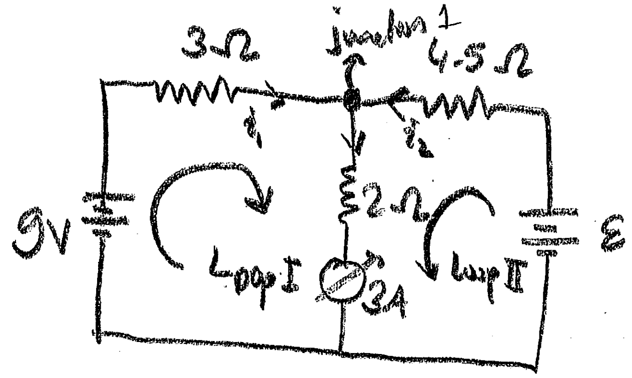
$R_2 = 42\Omega + 18\Omega = 60\Omega$ (series)

→



$$R_3 = \left(\frac{1}{60\Omega} + \frac{1}{40\Omega}\right)^{-1} = 24\Omega \text{ equivalent resistance.}$$

23.22



Loop I: $9V - i_1\, 3\Omega - 2\Omega \cdot 3A = 0$ (1)

Loop II: $\mathcal{E} - 4.5\Omega\, i_2 - 2\Omega \cdot 3A = 0$ (2)

and junction 1: $i_1 + i_2 = 3A$ (3)

3 unknowns $\mathcal{E}, i_1, i_2$, 3 equations (1), (2) and (3)

plug (3) in (1)

$9V - (3A - i_2)3\Omega - 6V = 0$

$3V - 9V + 3\Omega\, i_2 = 0$

$3\Omega\, i_2 = 6V \Rightarrow \boxed{i_2 = \frac{6V}{3\Omega} = 2A}$

find i_1 via (3)

$i_1 + i_2 = 3A$

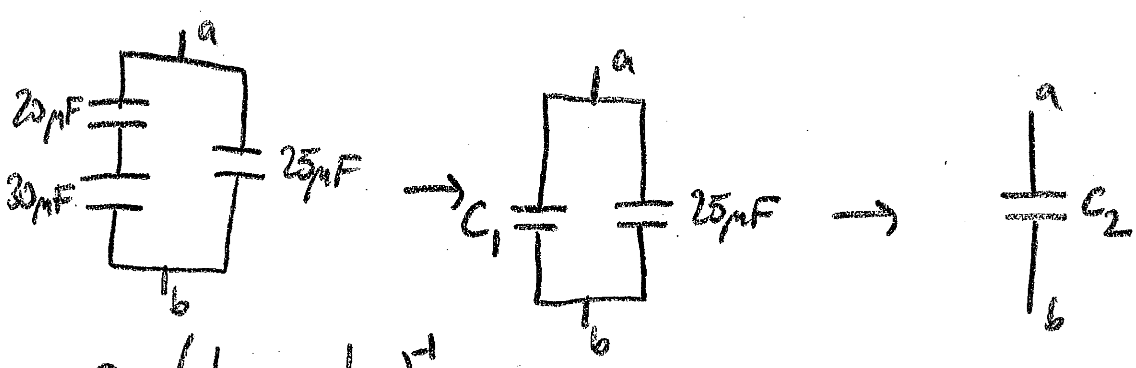
$\boxed{i_1 = 3A - i_2 = 3A - 2A = 1A}$

find $\mathcal{E}$ using (1) and (3)

$\mathcal{E} - (4.5\Omega)(2A) - (2\Omega)(3A) = 0$

$\mathcal{E} - 9V - 6V = 0 \Rightarrow \boxed{\mathcal{E} = 15V}$

23.34



$C_1 = \left(\frac{1}{25\mu F} + \frac{1}{30\mu F}\right)^{-1} = 12\mu F$

(series)

$C_2 = C_1 + 25\mu F = 12\mu F + 25\mu F$

$= 37\mu F$ (parallel)