SOLUTIONS - 1

FORMULAE

$$\vec{F} = -kx\hat{x} \qquad M\vec{a} = \vec{F}$$

$$\vec{x} = A\cos(\omega t + \theta_0)\hat{x}$$

$$\vec{v} = -A\omega \sin(\omega t + \theta_0)\hat{x} \;;\; v_{max} = A\omega$$

$$\vec{a} = -A\omega^2 \cos(\omega t + \theta_0)\hat{x} \qquad a_{max} = A\omega^2$$

$$\omega = \sqrt{\frac{k}{M}} \;,\; \omega = \frac{2\pi}{T} \;,\; f = \frac{1}{T} \;,\; T = 2\pi\sqrt{\frac{M}{k}}$$

$$P_{sp} = \frac{1}{2}kx^2$$

$$K_f + P_f = K_i + P_i \qquad \frac{1}{2}MV^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

$$\vec{F} = -Mg\sin\theta\,\hat{\tau} \;,\; \theta \ll 1 \;\; \vec{F} = -Mg\theta\,\hat{\tau}$$

$$\vec{a} = -\frac{g}{l}s\,\hat{\tau} \qquad T = 2\pi\sqrt{\frac{l}{g}}$$

$$P_g = Mgl(1-\cos\theta) = Mgl\frac{\theta^2}{2} \quad \text{for} \quad \theta \ll 1$$

Chap 14

4. Heart beats 75 in 1min

75 in 60 secs

Period

$$T = \frac{60}{75} = 0.8 \text{ sec.}$$

Frequency $f = \frac{1}{T} = 1.25 \text{ Hz}$

7. Here $\theta_0 = 0$ at $t = 0$, object at A.

Max speed = 0.40 m/sec.

period $T = 2$ sec.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \text{ rad/sec.} = 3.14 \text{ rad/s.}$$

$$A\omega = 0.4 \text{ m/sec}$$

a) $$A = \frac{0.4}{\pi} = 0.13 \text{ m.}$$

b) x at $t = 0.25$ sec

$$x = (0.13 \cos 0.25\pi) \text{ m} \qquad \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$= 0.09 \text{ m}$$

17. $T = .2$ sec

$$T = 2\pi\sqrt{\frac{M}{k}}$$

a) Double mass T goes to $2 \times \sqrt{2} = 2.83$ sec.

b) $M \to \frac{M}{2}$ T " " $\frac{2}{\sqrt{2}} = 1.41$ secs

c) Amplitude is doubled : period is independent of A.

d) $k \to 2k$. $\quad T \to \frac{2}{\sqrt{2}} = 1.41$ sec.

20 $M = 0.2$ kg

$f = 2$ Hz

$x = 0.05$ m.

$V_x = -0.3$ m/s when $x = 0.05$ m.

a) Period $T = \frac{1}{f} = 0.5$ sec

c) Calculate Amplitude. $\quad \omega = 2\pi f = 4\pi$ rad/sec.

$$0.05 = A\cos(\omega t + \theta_0)$$

$$-0.3 = -A\omega \sin(\omega t + \theta_0)$$

$$\cos(\omega t + \theta_0) = \frac{0.05}{A}$$

$$\sin(\omega t + \theta_0) = \frac{0.3}{4\pi A}$$

square and add

$$1 = \left(\frac{0.05}{A}\right)^2 + \left(\frac{0.3}{4\pi A}\right)^2$$

$$= (25\times10^{-4} + 5.7\times10^{-4})\frac{1}{A^2}$$

$$A = 5.5\times10^{-2}\text{ m}$$

b) $V_{max} = A\omega = 5.5\times10^{-2}\times 4\pi = 0.69$ m/s

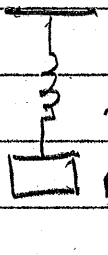
d) Energy $= \frac{1}{2}MV_{max}^2 = \frac{1}{2}\times0.2\times(0.69)^2 = 0.048$ Joules

26 $T = 5.5\,s$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$l = \frac{T^2}{4\pi^2} g = \frac{(5.5)^2}{4 \times \pi^2} \times 9.8 = \underline{7.5m}$$

44 Let acc. due to gravity be g_p.



$-k\Delta y\,\hat{y} - Mg_p\hat{y} = 0$

$\Delta y = 31.2\text{ cm}.$

≡m $\Delta y = -\dfrac{Mg_p}{k} = 0.312\text{ m}.$

$g_p = 0.312 \times \dfrac{k}{M}$

Next, period is $T = \dfrac{14.5}{10} = 1.45\text{ sec.} = 2\pi\sqrt{\dfrac{M}{k}}$

$\sqrt{\dfrac{k}{M}} = \dfrac{2\pi}{1.45}\ (\text{sec})^{-1}$

$g_p = 0.312\left(\dfrac{2\pi}{1.45}\right)^2 = 15.9\ \text{m/s}^2$

49 $M = 0.1\text{ kg}$ $\qquad \omega = \sqrt{\dfrac{2.5}{0.1}} = 5\text{ rad/s}$

$k = 2.5\text{ N/m}$

$v = 0.2\text{ m/s}$ when $x = -0.05\text{m}.$

$-0.05 = A\cos(\omega t + \theta_0) = A\cos(5t + \theta_0)$

$0.2 = 5A\cos(5t + \theta_0)$

$\left(\dfrac{0.05}{A}\right)^2 + \left(\dfrac{0.2}{5A}\right)^2 = 1$

$\left(\dfrac{41\times10^{-4}}{A^2}\right) = 1 \qquad A = \sqrt{41}\times10^{-2} = 6.4\times10^{-2}\text{ m}$

Energy $\dfrac{1}{2}kA^2 = \dfrac{1}{2}kx^2 + \dfrac{1}{2}MV^2$

$x = 0.03\text{m}$

$\frac{1}{2}\times2.5\times41\times10^{-4} = \frac{1}{2}\times2.5\times9\times10^{-4} + \frac{1}{2}\times0.1\times V^2$

$\frac{1}{2}\times0.1\,V^2 = \frac{1}{2}\times2.5\times32\times10^{-4}$

49 (cont'd) $V^2 = \frac{2.5 \times 32 \times 10^{-4}}{0.1} (m/sec)^2$

$V = 0.283 \text{ m/sec}$

55 $M = 2.5 \text{ kg}$

$k = 20 \text{ N/m}$

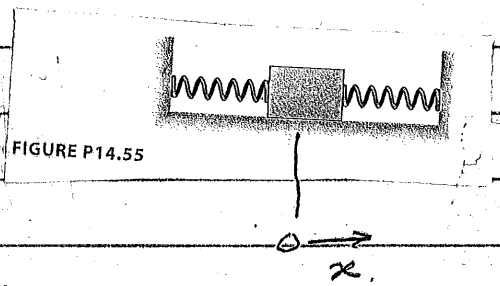


FIGURE P14.55

If you move block to the right, left spring pulls by force

$-kx\hat{x}$

and right spring pushes by force

$-kx\hat{x}$

so net force on block is

$\vec{F} = -2kx\hat{x}$

hence

$\omega = \sqrt{\frac{2k}{M}} = \sqrt{\frac{2 \times 20}{2.5}} = 4 \text{ rad/s}$

$f = \frac{\omega}{2\pi} = \frac{4}{2 \times 3.142} = 0.64 \text{ Hz}$