PHYS 122 Exam II November 5, 2009

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Name: SOLUTION

(Sign in ink, print in pencil)

Notes:

1) There are six (6) problems in this exam. Please make sure that your copy has all of them.
2) Please show your work, indicating clearly what formula you used and what the symbols mean. Just writing the answer will not get you full credit. In stating vectors give both magnitude and direction.
3) Write you answers on the sheet provided.
4) Do not forget to write the units.
5) Do not hesitate to ask for clarification at any time during the exam. You may buy a formula at the cost of one point.

Take care! God Bless You!

$$k_e = 9x10^9 \frac{N \cdot m^2}{C^2}, \mu_0 = 4\pi 10^{-7} \frac{T-m}{A}$$

$$\varepsilon_0 = 9x10^{-12} \frac{F}{m}$$

Mass of proton	$m_p = 1.6x10^{-27} kg$
Mass of electron	$m_e = 9x10^{-31} kg$
Elementary Charge	$e = 1.6x10^{-19} C$

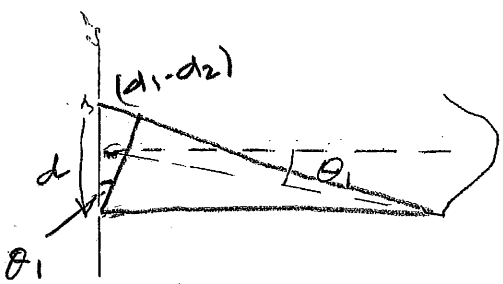
Problem 1 In 2-slit interference using light of wavelength λ, the first minimum occurs when

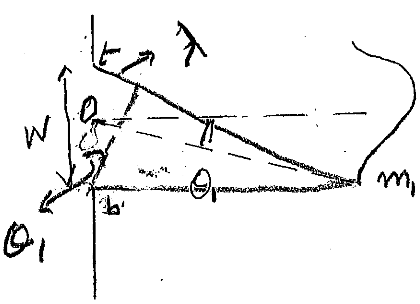
$$\sin\theta_1 = \frac{\lambda}{2d}$$

where d is the split separation. In single-slit diffraction the first minimum is at

$$\sin\theta_1 = \frac{\lambda}{w}$$

where w is the slit width. What accounts for the difference? (16)





In 2-slit interference, two coherent waves start in phase, one travels d_1 and the other d_2 until they meet at the screen. For a minimum (dark spot) $(d_1 - d_2) = (m + \frac{1}{2})\lambda$, $m = 0, \pm 1, \pm 2 \ldots$

For first minimum $(d_1 - d_2) = \frac{\lambda}{2}$ hence (see figure)

$$\sin\theta_1 = \frac{\lambda}{2d}$$

In single slit diffraction, N (large number) of coherent waves start in phase so to produce a minimum at the screen all of them add up to zero. This requires that the path difference between the wave from the top (t) and from the bottom (b) of the slit must be (see figure).

$$tm_1 - bm_1 = \lambda$$

If so, the wave coming from the center (o) is $\lambda/2$ behind that from b, the slit breaks into two halves, the waves from the lower half cancelling those from the upper half. Hence

$$\sin\theta_1 = \lambda/w$$

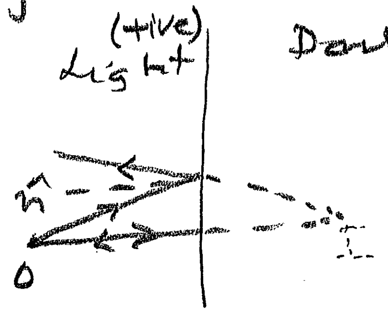
Problem 2 Plane and spherical mirrors form images given by the equations

$$\frac{1}{p}+\frac{1}{q}=\frac{2}{r}, m=-\frac{p}{q}$$

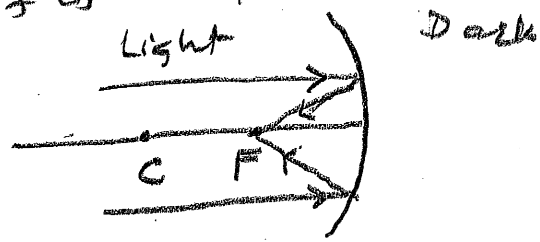
where p =object distance, q =image distance, r =radius of the mirror, m =magnification. How do you distinguish among a plane, a concave and a convex mirror? Support your answers with diagrams. (16)

<u>Plane Mirror</u> $r \to \infty$ (very large)

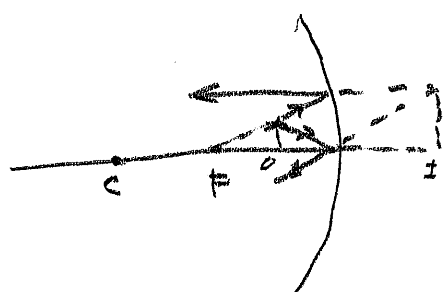
$q = -p$, $m = 1$. All images are virtual, upright and same size as object



<u>Concave mirror</u>: r is +ive, f is +ive, mirror is Convergent

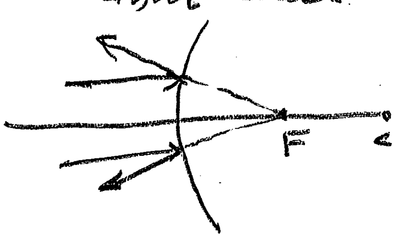


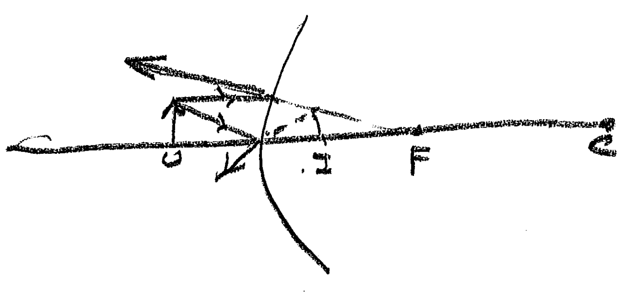
Variety of real images except when $p < f$, q becomes −ive, virtual upright, enlarged image



<u>Convex Mirror</u> r is −ive, f is −ive, Mirror is divergent. All images, virtual, upright, reduced

Light Dark





Problem 3 Show that with a convergent lens or mirror a real image can never get closer than the focal point. (16)

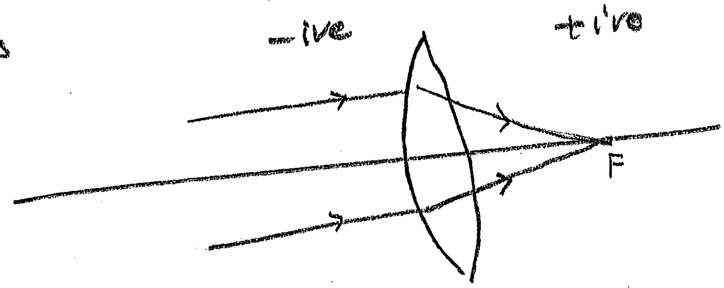
(Diagram Required)

The equation is

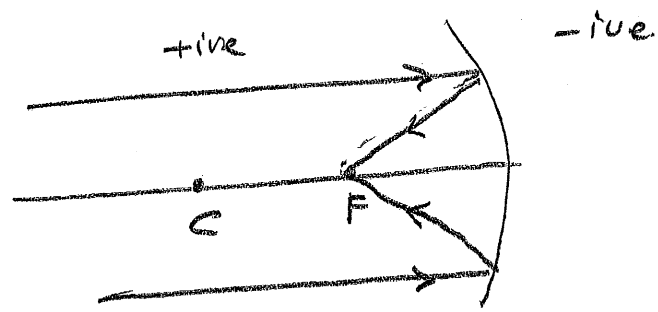
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}.$$

To make q smallest you must make $\frac{1}{p} = 0$, ie $p \to \infty$, $q \to f$, Parallel light is incident

Lens

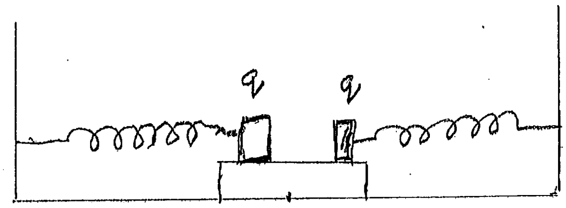


Mirror



Problem 4 The picture shows two equal charges (q) attached to two identical springs with spring constants $10^3\ N/m$. They are in equilibrium when the separation is $0.1\ m$. Calculate q. (neglect friction and each spring is squeezed by $0.05\ m$)

(16)



The charges are in equilibrium so total force on one charge is zero. Consider q on right

$\vec{F}_{sp} \leftarrow \square \rightarrow \vec{F}_E$

q

$\vec{F}_E = \dfrac{kq^2}{r^2}\hat{x}$ $\vec{F}_{sp} = -Cx\hat{x}$

$x = 0.05m,\quad C = 10^3 N/m$

$r = 0.1\ m$

$k = 9\times10^9 \dfrac{N\text{-}m^2}{c^2}$

$$\frac{9\times10^9\times q^2}{(0.1)^2} N\hat{x} - 10^3\times 0.05\ N\hat{x} = 0$$

$$q^2 = \frac{10^3\times 0.05\times (0.1)^2}{9\times10^9} c^2 = \frac{5}{9}\times10^{-10} c^2$$

$q = \underline{7.4\times10^{-6} C}$

Problem 5a State Gauss' law in your own words, defining the symbols precisely. (6)

A stationary charge Q at $r=0$ produces a Coulomb $\vec{E}$ field at r:

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

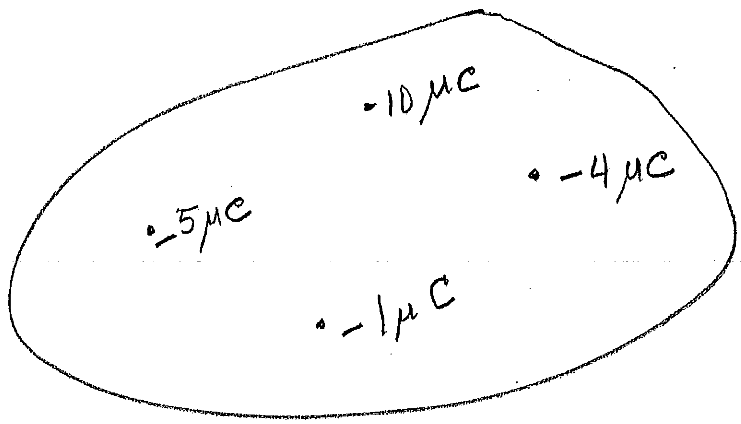
Hence, total flux of $\vec{E}$ through a closed surface is determined solely by the enclosed charges (sources, sinks)

$$\boxed{\sum_c \vec{E}\cdot\Delta\vec{A} = \frac{1}{\varepsilon_0}\sum Q_i}$$

Problem 5b The picture shows several stationary charges enclosed in a closed surface.

i) What is the total flux of the $\underset{\rightarrow}{E}$ – *field* through the surface? Why?

ii) What is the $\underset{\rightarrow}{E}$ – *field* on the surface? Why? (10)



By Gauss' Law

i) $$\sum_i \underset{\rightarrow}{E} \cdot \underset{\rightarrow}{\Delta A} = \frac{1}{\varepsilon_0} \sum Q_i$$

here $\sum Q_i = (10-5-4-1)\mu C = 0$

so $\sum_i \underset{\rightarrow}{E} \cdot \underset{\rightarrow}{\Delta A} = 0$

ii) One cannot say anything about the $\underset{\rightarrow}{E}$ field. To calculate $\underset{\rightarrow}{E}$ you would need to know the exact location of each charge as well as the coordinates of the point on the surface.

Problem 6a A parallel plate capacitor with plate area $A = 2m^2$ is filled with air and the plate separation is $1\,cm$. Put charges $\pm 10\mu C$ on the plates. What is the potential difference between the plates? Why? (8)

$A = 2m^2$

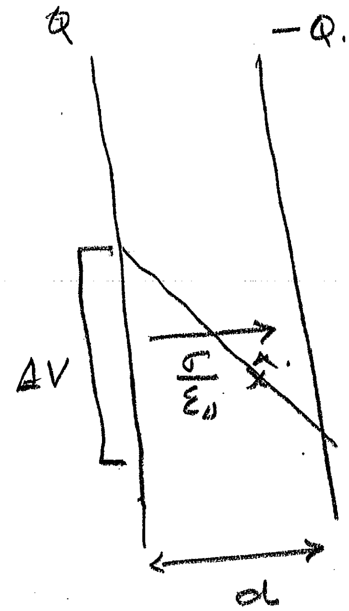
$d = 10^{-2}m$

Charge density on the plate

$\sigma = \frac{10^{-5}C}{2} = 5\times10^{-6}\ C/m^2$

It will produce an E-field

$\underset{\to}{E} = \frac{\sigma}{\varepsilon_0}\frac{V}{m}\hat{x} = \frac{5\times10^{-6}}{9\times10^{-12}}\ V/m\ \hat{x}$



$\Delta V = -\underset{\to}{E}\cdot\underset{\to}{\Delta S}$

$\underset{\to}{\Delta S} = d\hat{x}$

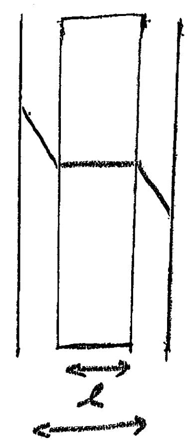
So $\Delta V = -\frac{5\times10^{-6}}{9\times10^{-12}}\times10^{-2}$ Volts

$= -0.55\times10^{4}$ Volts.

Problem 6b Place a conducting slab of area $2\,m^2$ and thickness $0.5\,cm$ between the plates. What is the potential difference between the plates? Why? (8)

Inside the Conductor charges are stationary

So $\underset{\to}{E} = 0$ & ∴ $\Delta V = 0$.



$\Delta V = -\underset{\to}{E}\cdot\underset{\to}{\Delta S}$

Now $\underset{\to}{\Delta S} = (d-l)\,\hat{x}$

$\Delta V = -\frac{0.55\times10^{4}\text{Volts}}{2}$

Problem 6c In order to place a charge Q on a capacitor C_0 you need to do $\frac{Q^2}{2C_0}$ joules of work. Where does the energy go? Why? (4)

As we saw in 6a when there is charge on the plates, there is an $\vec{E}$ - field between them, the work $\frac{Q^2}{2C_0}$ gets stored in the $\underline{E}$ - field.