PHYS 122 | Exam II | November 6, 2009
Prof. S.M. Bhagat

Name: SOLUTION

(Sign in ink, print in pencil)

Notes:

1) There are four (4) problems in this exam. Please make sure that your copy has all of them.
2) Please show your work, indicating clearly what formula you used and what the symbols mean. Just writing the answer will not get you full credit. In stating vectors give both magnitude and direction.
3) Write you answers on the sheet provided.
4) Do not forget to write the units.
5) Do not hesitate to ask for clarification at any time during the exam. You may buy a formula at the cost of one point.

Take care! God Bless You!

$$k_e = 9x10^9 \frac{N \cdot m^2}{C^2}, \mu_0 = 4\pi 10^{-7} \frac{T-m}{A}$$

$$\varepsilon_0 = 9x10^{-12} \frac{F}{m}$$

Mass of proton $m_p = 1.6x10^{-27}\,kg$

Mass of electron $m_e = 9x10^{-31}\,kg$

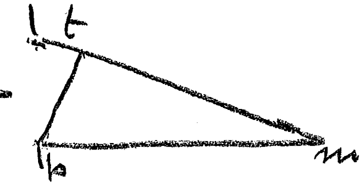
Elementary Charge $e = 1.6x10^{-19}\,C$

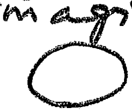
Problem 1a . In single slit diffraction of light of wavelength λ passing though a slit of width w the minima (dark spots) occur at angles

$$\sin\theta_m = \frac{m\lambda}{w}, m = 1,2,3...$$

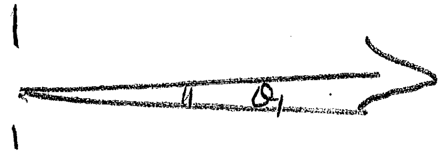
Why? (20)

In single slit diffraction N (very large) # of waves superpose to produce the diffraction pattern. To cause a dark spot all of them must add together to get a zero. If the path differences between the waves from the top point and the bottom point is $m\lambda$ [$m=1,2,3\ldots$] the slit can be split into an even # of parts in which a wave from each part is $\lambda/2$ away from the wave from its neighboring part, thereby leading to cancellation. One can imagine that the N, E-vectors form closed circles



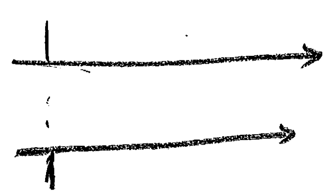
 $m=1$,  $m=2$,  $m=3$ . . . . .

Problem 1b Show that when openings or obstacles are large with respect to the wavelength of light, it is appropriate to use geometrical optics. (5)



When light goes through a slit of width, it spreads by the angle given by $\sin\theta_1 = \frac{\lambda}{w}$

If $w \gg \lambda$, $\theta_1 \to 0$, diffraction becomes negligible, light effectively travels in St. lines



Problem 2a Then lens maker's formular for a lens with two spherical surfaces is written as

$$\frac{1}{f} = (n-1)[\frac{1}{R_F} - \frac{1}{R_B}]$$

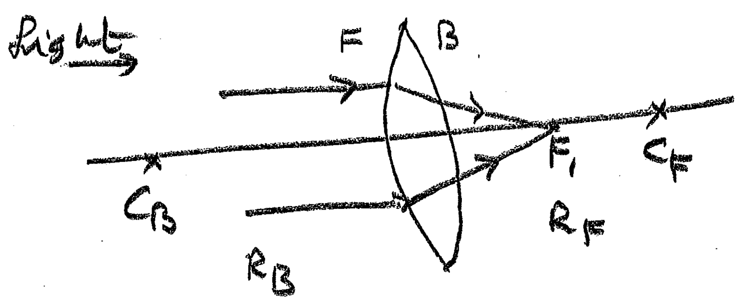
where f = focal length, n is refractive index of lens material and R_F, R_B are the radii of the two surfaces. How does this formula allow you to distinguish between a convergent lens and a divergent lens? Support your answers with appropriate diagrams. (18)

Lenses imply Refraction

Sign convention: distances along path of light +ive

against path of light −ive.

Convex lens



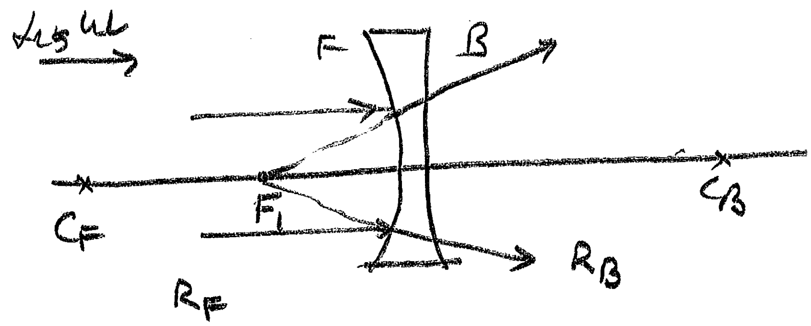
R_F +ive

R_B −ive

f +ive

Lens is convergent

Concave lens



R_F −ive

R_B +ive

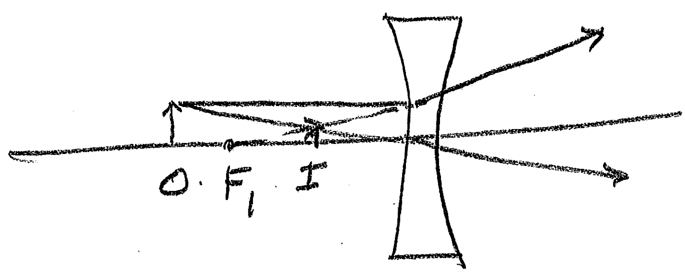
f −ive

Lens is Divergent

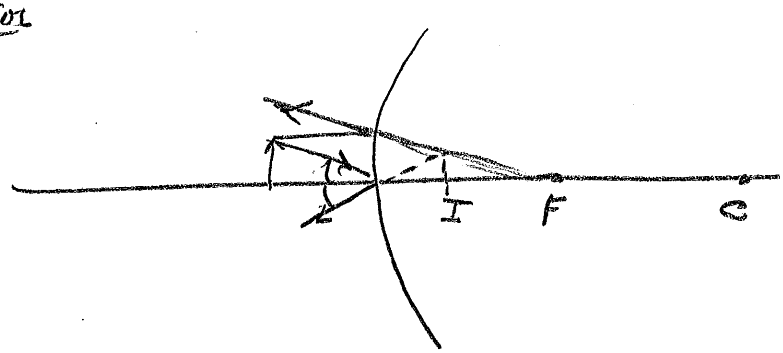
Problem 2b Can you use a divergent lens or mirror to produce a real image? Support your answer with a diagram. (7)

No, a divergent lens/mirror has f -ive so it can only produce virtual, upright, reduced images.

Lens



Mirror

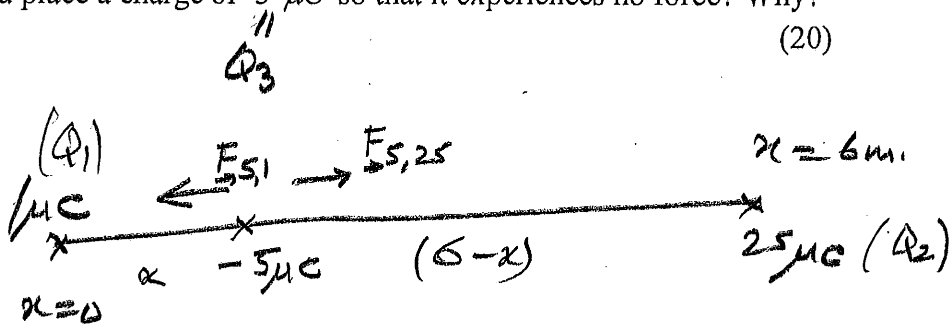


Problem 3a What is an $\underset{\rightarrow}{E}$ − *field* ? (5)

If a stationary charge (q) experiences a force when there is no visible agency applying the force, it must be located in an $\underset{\rightarrow}{E}$ - field.

Problem 3b A charge of 1 μC is located at x=0 and a charge of 25 μC is fixed at x=6m. Where would you place a charge of -5 μC so that it experiences no force? Why?

(20)



The Coulomb force acts along the line joining the charges

$$\vec{F} = \frac{k\, q_1 q_2}{r^2}\,\hat{r}$$

So to get cancellation Q_3 must be located on the x-axis and since Q_1, Q_2 both +ive Q_3 should be between them and close to Q_1. Put Q_3 at x.

The two forces on it are

$$\vec{F}_{5,1} = -\frac{k \times 5 \times 1 \times 10^{-12}}{x^2}\, N\hat{x}.$$

$$\vec{F}_{5,25} = +\frac{k \times 5 \times 25 \times 10^{-12}}{(6-x)^2}\, N\hat{x}$$

and we need

$$\vec{F}_{5,1} + \vec{F}_{5,25} = 0.$$

$$-\frac{\cancel{k \times 5} \times 1 \times \cancel{10^{-12}}}{x^2}\hat{x} + \frac{\cancel{k \times 5} \times 25 \times \cancel{10^{-12}}}{(6-x)^2} = 0.$$

or

$$\frac{1}{x^2} = \frac{25}{(6-x)^2}$$

$$\frac{1}{x} = \frac{5}{6-x}$$

$$5x = 6-x$$

$$\underline{x = 1m}$$

Problem 4a Write down Gauss' law in your own words. (5)

A stationary charge Q located at $r=0$, produces a Coulomb $\vec{E}$ field at r:

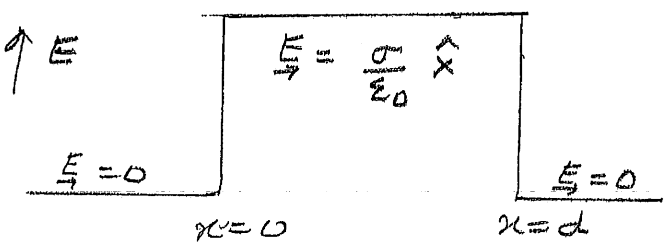
$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

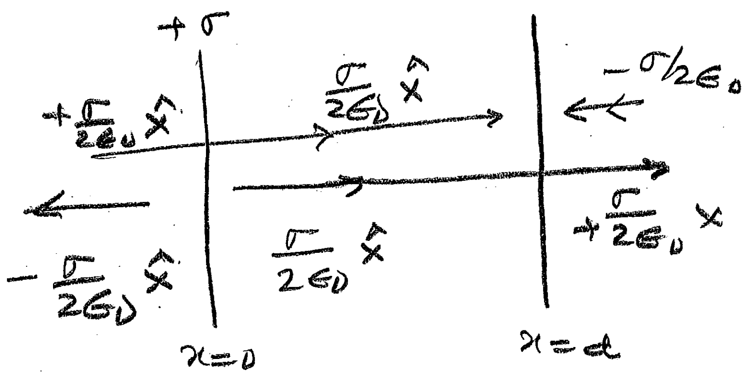
Hence, Gauss' law:

Total flux of $\vec{E}$ through a closed surface is determined solely by the enclosed charges:

$$\boxed{\sum_C \vec{E}\cdot\Delta\vec{A} = \frac{1}{\varepsilon_0}\sum q_i}$$

Problem 4b We are told that a thin sheet carrying a uniform charge density $\sigma C/m^2$ will produce an $\underset{\rightarrow}{E}-\textit{field}$ of $\pm\frac{\sigma}{2\varepsilon_0}\hat{n}$ where $\hat{n}$ is perpendicular to the sheet. How would you use two sheets to produce the field shown below? (12)





Put $+\sigma$ sheet at $x = 0$.

$-\sigma$ sheet at $x = d$

The two fields cancel when $x < 0$ or $x > d$.

But for $0 < x < d$ they add to produce $\underset{\rightarrow}{E} = \frac{\sigma}{\varepsilon_0}\hat{x}$.

Problem 4c Why is there a negative sign on the right side of the equation for change of potential energy in a Coulomb $\underrightarrow{E}$ $-$ *field* :

$$\Delta U_E = -\sum \underrightarrow{F_E} \bullet \underrightarrow{\Delta S}$$

(8)

Potential Energy is work stored in a System when it is assembled in the presence of a Conservative force. Here, $\underrightarrow{F_E}$ is the Conservative force. The force applied to assemble the system must be equal and opposite to the conservative force (to ensure no change in kinetic energy.)