PHYS 122 | Exam I | October2, 2009
Prof. S. M. Bhagat

Name: SOLUTION

(Sign in ink, print in pencil)

Notes

1) There are four (4) problems in this exam. Please make sure that your copy has all of them.
2) Please show your work indicating clearly what formula you used and what the symbols mean. Just writing the answer will not get you full credit. In stating vectors give both magnitude and direction.
3) Write your answers on the sheet provided.
4) Do not forget to write the units
5) Do not hesitate to ask for clarification at any time during the exam. You may buy a formula at the cost of one point.

Take Care! God Bless You!

$$\mu_0 = 4\pi 10^{-7}\frac{T-m}{A}$$

$$\varepsilon_0 = 9x10^{-12}\frac{F}{m}$$

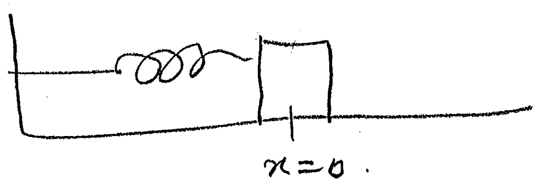
Mass of proton $m_p = 1.6x10^{-27}\,kg$

Mass of electron $m_e = 9x10^{-31}\,kg$

Elementary Charge $e = 1.6x10^{-19}\,C$

Problem 1

A spring-mass system consists of a mass M attached to a spring of spring constant k and placed as shown on a horizontal frictionless table. The spring is unstretched at x = 0.

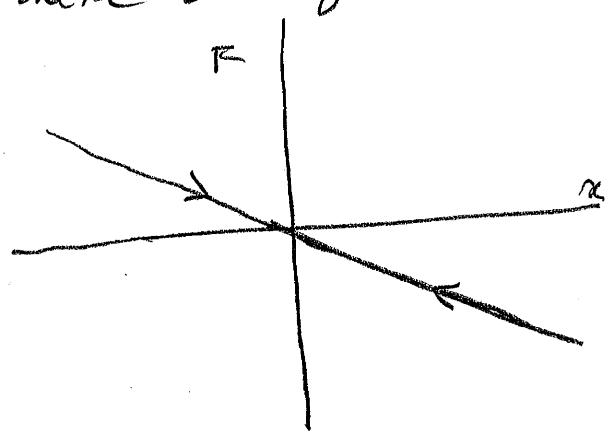


a) If you pull the mass to x = A and let go, why does it oscillate? (10)

The spring force is

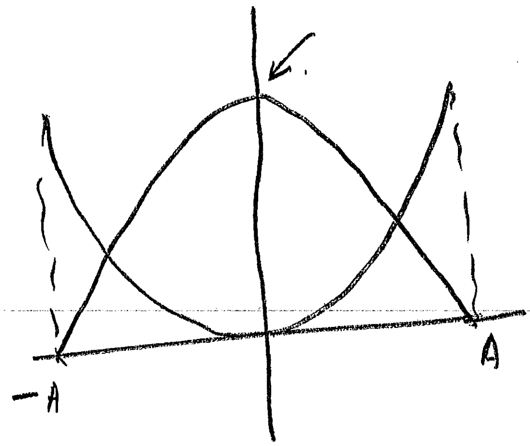
$$\vec{F}_{SP} = -kx\,\hat{x}$$

So as long as $x \neq 0$, this force being a restoring force works to bring mass back to Em $(x=0)$. However, when it gets to $x=0$, it cannot stop and keeps going and when it eventually stops at $\pm A$ the force is there to bring it back.



b) At what values of x will the velocity be maximum? Why? (5)

Velocity is maximum when $x=0$ b/c at $x=0$, Potential energy is zero.



c) At what values of x will the kinetic energy be equal to the potential energy? Why? (10)

Conservation of energy tells us that

$$\frac{1}{2}kx^2+\frac{1}{2}Mv^2=\frac{1}{2}kA^2$$

if $\frac{1}{2}kx^2=\frac{1}{2}Mv^2$ each must be

$\frac{1}{2}$ of $\frac{1}{2}kA^2$ so

$$\frac{1}{2}kx^2=\frac{1}{2}\cdot\frac{1}{2}kA^2$$

$$x^2=\frac{A^2}{2}$$

$$x=\frac{A}{\sqrt{2}}$$

Problem 2a

As written below the expression for the deviation from equilibrium D has several quantities missing

$$D = \ldots \sin(x - vt)$$

here

D is a physical quantity
x is a length
V a speed
and t a time

Justify the quantities you need to introduce and explain their physical significance. (10)

D is a physical quantity so it must have dimensions. Sine fn. is dimensionless so we must introduce A and write

$$D = A \sin(x - vt)$$

where A will have the dimensions of D

Sine fn. has no dimensions so its argument cannot have dimensions so we must introduce a length λ such that

$$D = A \sin \frac{(x - vt)}{\lambda}$$

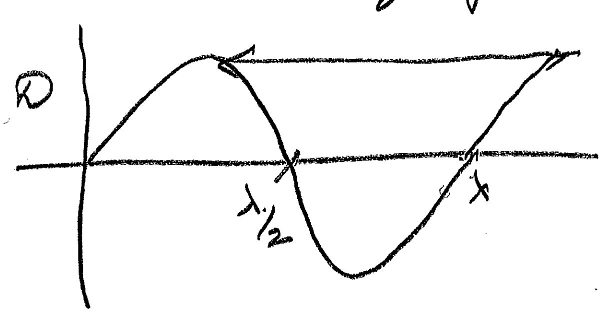
Sine fn. repeats every 2π so for convenience put in 2π & write

$$D = A \sin \frac{2\pi(x - vt)}{\lambda}$$ O.K. Now!

A has the physical significance of Amplitude — largest excursion D away from zero.

Plot D as a fn. of x for t=0

λ is the repeat distance — wavelength!



Problem 2b

A wave is written as

$$\underset{\rightarrow}{y} = 0.01(\cos 12.56x \sin 31.4t)\hat{y}$$

Where x is in meters and t in secs: $= A \cos(kx \sin \omega t)\hat{y}$

(i) Is this wave longitudinal or transverse? (3)
(ii) What is the wavelength? (4)
(iii) What is the frequency? (4)
(iv) Is this a travelling wave? Justify your answer. (4)

(i) Transverse wave b/c $\underset{\rightarrow}{A} \perp \hat{x}$

(ii) $k = \frac{2\pi}{\lambda} = 12.56\ m^{-1}$

$\lambda = \frac{2\pi}{12.56} = 0.5\ m.$

(iii) $\omega = 2\pi f = 31.4\ rad/s$

$f = \frac{\omega}{2\pi} = \frac{31.4}{2\pi} = 5\ Hz$

(iv) This is not a travelling wave. b/c it has nodes at $\lambda/4$ and every $\frac{\lambda}{2}$ thereafter.

Problem 3a

What is sound? (5)

Any mechanical wave whose frequency is between 20Hz and 20kHz because our Ears detect it!

Problem 3b

The speed of sound in a gas is written as

$$v = \sqrt{\frac{\gamma k_B T}{m}}, \text{ where } k_B = 1.38\times10^{-3} J/K$$

Why is there a γ in this equation? (5)

Sound is a displacement wave
If displacement varies with position volume (V) changes.
If volume changes Pressure (P) must change.
Question: What is the relationship between P & V?
Answer Freq. of sound greater than 20 Hz. Variations are so rapid that em with surroundings is not possible. No heat exchange with surroundings. $\boxed{V \uparrow P \downarrow}$. If $\Delta Q = 0$, process is

$\Delta Q = 0$

adiabatic the equation is $PV^{\gamma} = \text{const}$ since

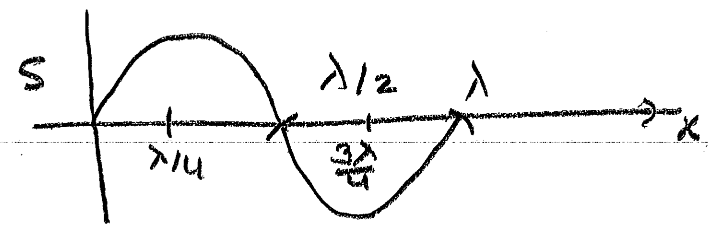
$\gamma = \frac{C_p}{C_v}$

Problem 3c

Draw two diagrams showing sound as a periodic displacement wave and as its corresponding pressure wave. (10)

Displ. $S = S_m \sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right)$

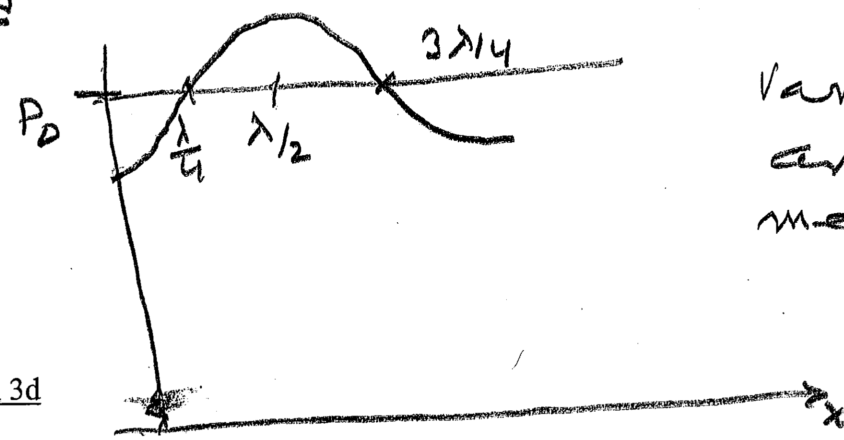
$t=0$



Variation around zero

Press $P = P_0 - \gamma P_0 S_m k \cos(kx - \omega t)$

$t=0$



Variation around mean pressure P_0!

Problem 3d

What is the seed of sound in a vacuum chamber? (5)

There is NO sound in vacuum.

Problem 4a

What is light? (5)

Light is a transverse electromagnetic wave, speed in vacuum is 3×10^8 m/s and wave lengths in vacuum are $400\,nm < \lambda < 800\,nm$.

Problem 4b

Show that $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ has the dimensions of a velocity.

Because μ_0 is MLQ^{-2} and ε_o is $Q^2M^{-1}L^{-3}T^2$ (5)

$$\mu_0 \varepsilon_0 \rightarrow \cancel{M}\,L\,\cancel{Q^{-2}}\,\cancel{Q^{2}}\,\cancel{M^{-1}}\,L^{-3}\,T^{2}$$

$$\rightarrow L^{-2}T^{2}$$

$$\text{so } \frac{1}{\mu_0 \varepsilon_0} \rightarrow \frac{L^2}{T^2}$$

$$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} \rightarrow \frac{L}{T} \leftarrow \text{dimensions of velocity}$$

Problem 4c

Light and sound are both waves. List five notable differences between them. (10)

SOUND	LIGHT
There is no SOUND IN VACUUM	LIGHT WAVES EXIST IN VACUUM
IN GASES SOUND IS LONGITUDINAL ($\vec{s}_m \parallel \hat{x}$)	LIGHT IS TRANSVERSE ($\vec{E}, \vec{B} \perp \hat{x}$)
Speed in Air 340 m/s	Speed in Air 3×10^8 m/s
Frequencies $20\,Hz < f < 20{,}000\,Hz$	$\sim 10^{14}$ Hz
Mechanical wave Intensity $\propto s_m^2 \omega^2$	Electromagnetic wave Intensity $\propto E_m^2$ or B_m^2

Problem 4d

In E and B fields, the energy densities are $\eta_E = \frac{1}{2}\varepsilon_0 E^2$ and $\eta_B = \frac{B^2}{2\mu_0}$. If $E = cB$ and $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$, which is larger, η_E or η_B? Why? (5)

$$\eta_E = \frac{1}{2} \epsilon_0 E^2$$

$$= \frac{1}{2} \epsilon_0 c^2 B^2$$

$$= \frac{1}{2} \epsilon_0 \cdot \frac{1}{\mu_0 \epsilon_0} B^2$$

$$= \frac{B^2}{2\mu_0} = \eta_B.$$

So $\eta_E = \eta_B$ when $E = cB$