Wave Optics: INTERFERENCE and DIFFRACTION

RADIATION: Electromagnetic wave

LIGHT: Transverse E.M. wave

λ_0: $400\,nm < \lambda_0 < 800\,nm$ [In Vacuum]

f: $4\times10^{14} < f < 8\times10^{14}$ Hz

Speed $c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3\times10^8$ m/sec in vacuum

$v = \frac{c}{n}$ in medium, $n \geq 1$

$\lambda_n = \frac{\lambda_0}{n}$ [FREQUENCY DOES NOT CHANGE, SO λ MUST!].

We can represent a light wave travelling along x as an $\vec{E}$-wave

$$E = E_m \sin(kx - \omega t + \phi), \quad \vec{E}_m \perp \hat{x}$$

$$k = \frac{2\pi}{\lambda}$$

$$\omega = 2\pi f$$

E_m = amplitude, ϕ = phase.

SUPERPOSITION:

Recall that when more than one ~~wave~~ wave is present at the same point at the same time, the net effect is obtained by making an algebraic sum.

Let us consider two light waves

$$E_1 = E_m \sin(kx - \omega t + \phi_1)$$

$$E_2 = E_m \sin(kx - \omega t + \phi_2)$$

That is, they have the same wavelength and the same frequency but the phases are different.

As discussed in class, emission of light involves an electron jumping from one energy level to another in its parent atom and each jump lets out a wave train of about 3m long. Since there are "zillions" of atoms we have enormous number of wave trains with arbitrary phases so a light wave from a source has a phase which varies randomly in time.

If you superpose E_1 and E_2 you will get

$$E = E_1 + E_2$$
$$= 2 E_m \cos\left(\frac{\phi_1 - \phi_2}{2}\right) \sin\left(kx - \omega t + \frac{\phi_1 + \phi_2}{2}\right)$$

That is, a wave whose amplitude is

$$Amp = 2 E_m \cos\left(\frac{\phi_1 - \phi_2}{2}\right)$$

Intensity $I \propto (Amp)^2$

So $I \propto 4 E_m^2 \cos^2 \left(\frac{\phi_1 - \phi_2}{2}\right)$

[The factors $\frac{1}{2} \epsilon_0 c$ are left out].

Two totally different situations arise.

<u>Case I</u> The sources of E_1 and E_2 are

INCOHERENT

That is, $(\phi_1 - \phi_2)$ is a random function of time. If so, I is also a random function of time. The observed value will be a time average:

$$\langle I \rangle \propto 4E_m^2 \langle \cos^2\left(\frac{\phi_1-\phi_2}{2}\right) \rangle.$$

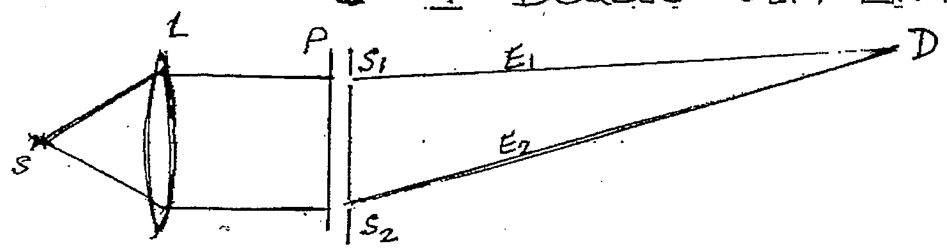
But $$\langle \cos^2\left(\frac{\phi_1-\phi_2}{2}\right) \rangle = \frac{1}{2}$$

So $$\langle I \rangle \propto 2E_m^2$$

Hence, for two incoherent sources the total intensity is just the sum of the two intensities. Two light bulbs just increase brightness.

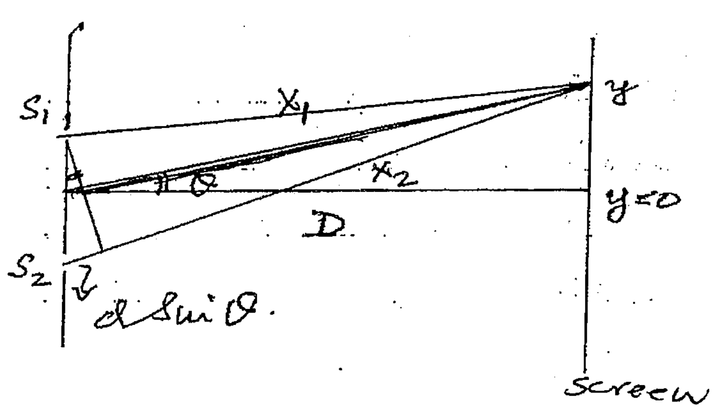
Case II The sources of E_1 and E_2 are COHERENT ~~Coherent~~. That is, the waves E_1 and E_2 are specially prepared in such a way that $(\phi_1 - \phi_2)$ is fixed (independent of time for our discussion) at any given location. [This is the case we discussed for Sound waves ~~a few~~ last weeks ~~ago~~].
So how do we get two coherent light sources. We discuss two examples.

I DOUBLE SLIT INTERFERENCE



S is a point source of light located at the focal ~~point~~ point of a convergent lens. After passing through the lens the light becomes a parallel beam. The corresponding wave front is a plane P travelling toward the right. Now place a ~~screen~~ plate with two small holes each of width ~~W~~ separated by d. Assume $W \ll d$. The waves which emerge from S_1, S_2 are both derived from the SAME wave front so at S_1 & S_2 they have same phase (say zero). By the time they arrive at the detector D their phases would have changed (see details below) but $(\phi_1 - \phi_2)$ does NOT vary with time. We have two coherent sources producing E_1, E_2 at D.

[In your expt. the Source is a laser which produces a parallel



$x_1 \to$ path length of wave from S_1.

θ: locates y on screen

beam. S_1, S_2 are slits in a plate and you used a screen to view the interference pattern).

Separation ~~bet~~ between S_1 and $S_2 = d$

distance to screen = D

Position of detector = y [$y = 0$ at mid-pt of sources].

x_1 = distance travelled by E_1

x_2 = distance travelled by E_2

At y: Phase of E_1, $\phi_1 = \frac{2\pi}{\lambda} x_1$.

Phase of E_2; $\phi_2 = \frac{2\pi}{\lambda} x_2$.

$$\left(\frac{\phi_1 - \phi_2}{2}\right) = \frac{2\pi}{\lambda}\left(\frac{x_1 - x_2}{2}\right)$$

If $(x_1 - x_2) = M\lambda$, $M = 0, \pm 1, \pm 2, \ldots$

$$\frac{\phi_1 - \phi_2}{2} = M\pi$$

$$\cos^2\left(\frac{\phi_1 - \phi_2}{2}\right) = 1.$$

So at such points I will be maximum

CONDITION FOR MAXIMA

$$(x_1 - x_2) = M\lambda, \quad M = 0, \pm 1, \pm 2, \ldots$$

However, if $(x_1 - x_2) = (m + \frac{1}{2})\lambda$, $m = 0, \pm 1, \pm 2, \ldots$

$$\frac{\phi_1 - \phi_2}{2} = (m + \tfrac{1}{2})\pi$$

$$\cos^2 (m + \tfrac{1}{2})\pi = 0. \qquad \underline{I = 0}$$

CONDITION FOR MINIMA

$$(x_2 - x_1) = \left(m + \tfrac{1}{2}\right)\lambda.$$

From the picture you can see that

$$(x_2 - x_1) = d \sin\theta.$$

So

$$d \sin\theta_M = M\lambda \quad [\text{Maxima}]$$

$$d \sin\theta_m = \left(m + \tfrac{1}{2}\right)\lambda \quad [\text{Minima}].$$

and, of course, all angles are small, ~~i.e.~~ $\frac{\lambda}{d} \ll 1$.

Consider the y-coordinate of the M^{th} maximum.

$$\frac{y_M}{D} = \tan\theta_M = \sin\theta_M$$

$$= \frac{M\lambda}{d}.$$

Similarly, its next neighbor has

$$\frac{y_{M+1}}{D} = \frac{(M+1)\lambda}{d}.$$

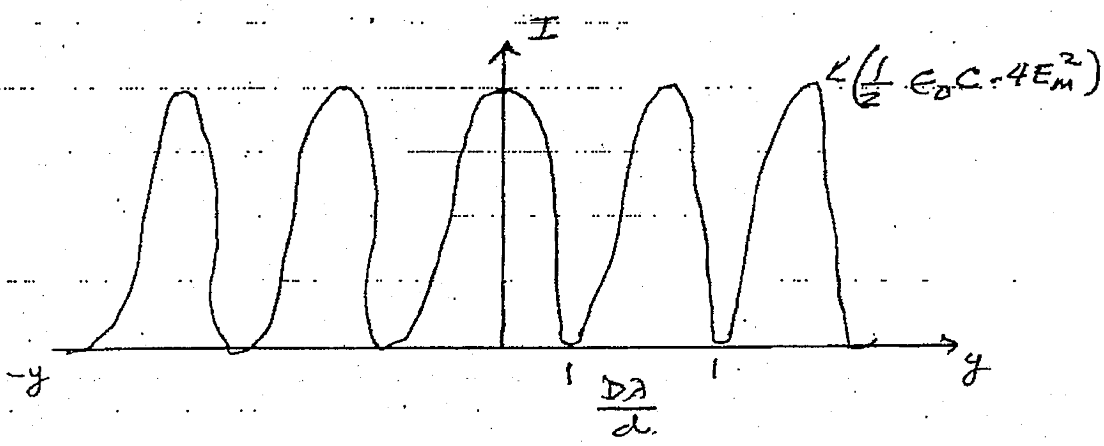
So

$$y_{M+1} - y_M = \frac{D\lambda}{d}.$$

So for two slit Interference

$$I = \frac{1}{2}\epsilon_0 C . 4E_m^2 \cos^2\left(\frac{\phi_1 - \phi_2}{2}\right)$$

and consists of equally spaced $\left(\frac{D\lambda}{d}\right)$ equal intensity $\left(\frac{1}{2}\epsilon_0 C\, 4E_m^2\right)$ fringes.



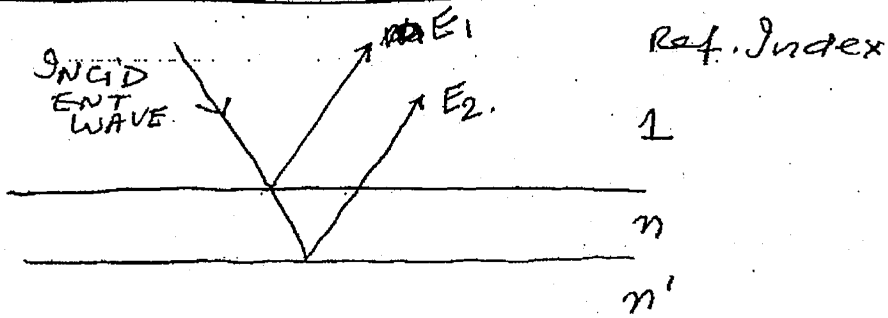
Average of Intensity on Screen space

$$\langle I \rangle = \frac{1}{2}\epsilon_0 c \cdot 2E_M^2 \;!! \quad \left[\langle \cos^2\theta \rangle = \frac{1}{2}\right]$$

JUST TWICE THE INTENSITY DUE TO ONE WAVE

II THIN FILM INTERFERENCE

A thin film of thickness t and refractive index n is deposited on



a block of refractive index n'. A light wave is incident on the top surface at an angle of incidence of a fraction of a degree ($i \simeq 0$, $r \simeq 0$, $R \simeq 0$). On reflection from the top surface we get one wave (designated E_1), part of the light enters the film and gets reflected at the bottom surface producing the second wave (E_2). Both E_1 and E_2 are derived from the same incident wave so their phase difference is a fixed quantity depending on the thickness t. E_1 & E_2 are Coherent.

The conditions for maxima and minima are of course,

$$x_2 - x_1 = m\lambda \qquad m = 0, \pm 1, \pm 2, \ldots$$

or

$$x_2 - x_1 = \left(m + \tfrac{1}{2}\right)\lambda \qquad m = 0, \pm 1, \pm 2$$

However, now we ~~also~~ must also consider what happens to the phase when a wave ~~goes reflected~~ ~~undergoes~~ undergoes reflection.

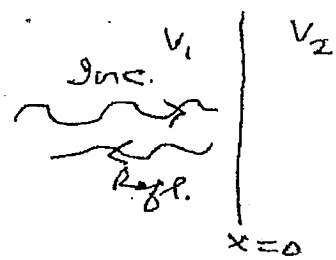
Recall, what we learnt while studying reflection of waves on stretched strings except now we cast it in terms of E-wave.

Incident wave

$$E_i = E_{mi} \sin(kx - \omega t)$$

Reflected wave

$$E_r = E_{mr} \sin(kx + \omega t)$$



Boundary at x=0

and $$\frac{E_{mr}}{E_{mi}} = \frac{V_1 - V_2}{V_1 + V_2}.$$

Let us compare waves at $x=0$ where the reflection occurs.

$$E_i = E_{mi} \sin(-\omega t) = E_{mi} \sin(\omega t + \pi)$$

$$E_r = E_{mr} \sin \omega t$$

Two cases arise

i) $V_1 < V_2$ $[n_1 > n_2]$

$\frac{E_{mr}}{E_{mi}}$ is $-$ive.

$$E_i = E_{mi} \sin(\omega t + \pi)$$

$$E_r = + E_{mr} \sin(\omega t + \pi)$$

No Phase change.

ii) $V_1 > V_2$ $[n_1 < n_2]$

$\frac{E_{mr}}{E_{mi}}$ is $+$ive.

$$E_i = E_{mi} \sin(\omega t + \pi)$$

$$E_r = E_m \sin \omega t$$

phase change of π on reflection.

Now let us consider interference between E_1 and E_2.

First, extra distance travelled by E_2 is $2t$ but refractive index is n so wavelength in medium is $\frac{\lambda_0}{n}$ where λ_0 is wavelength in air.

Next, if $n' > n$ $[v' < v]$. there is π phase change for both E_1 and E_2 so condition for maximum is

$$2nt = M\lambda_0 \qquad M = 0, \pm 1, \pm 2, \ldots$$

However, if $n' < n$ [$v' > v$]. only E_1 has a phase change, while E_2 has none so Condition for maximum becomes

$$2nt \; [redacted] \; \frac{\lambda_0}{2} = m\lambda_0$$

Notice a phase change of π is like a path difference of $\frac{\lambda_0}{2}$.

Colors of thin films of oil on water, or the surface of soap bubbles, arise because of thin film interference.

Non-reflecting glass is produced by depositing a thin layer of transparent material and ensuring destructive interference for $\lambda_0 \sim 600$ nm [Green light].

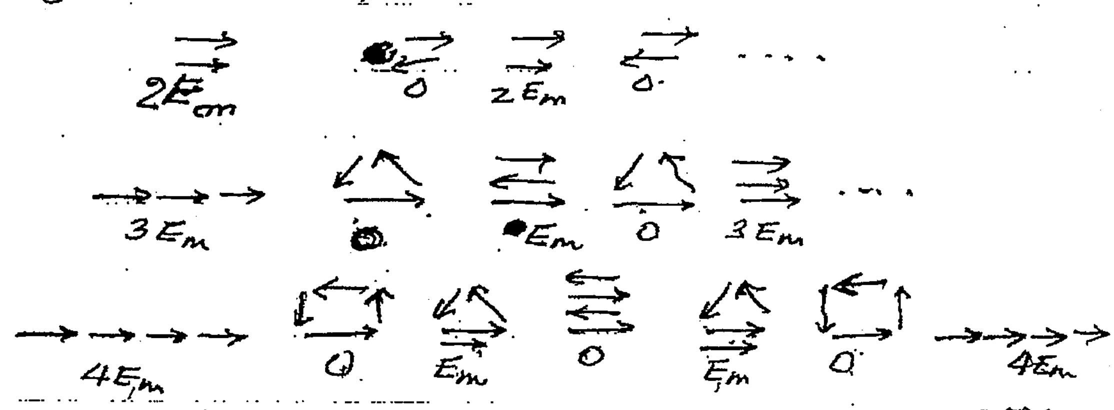
MULTIPLE SOURCE INTERFERENCE – ALL SOURCES COHERENT.

\# of Sources Amplitudes

2.

3.

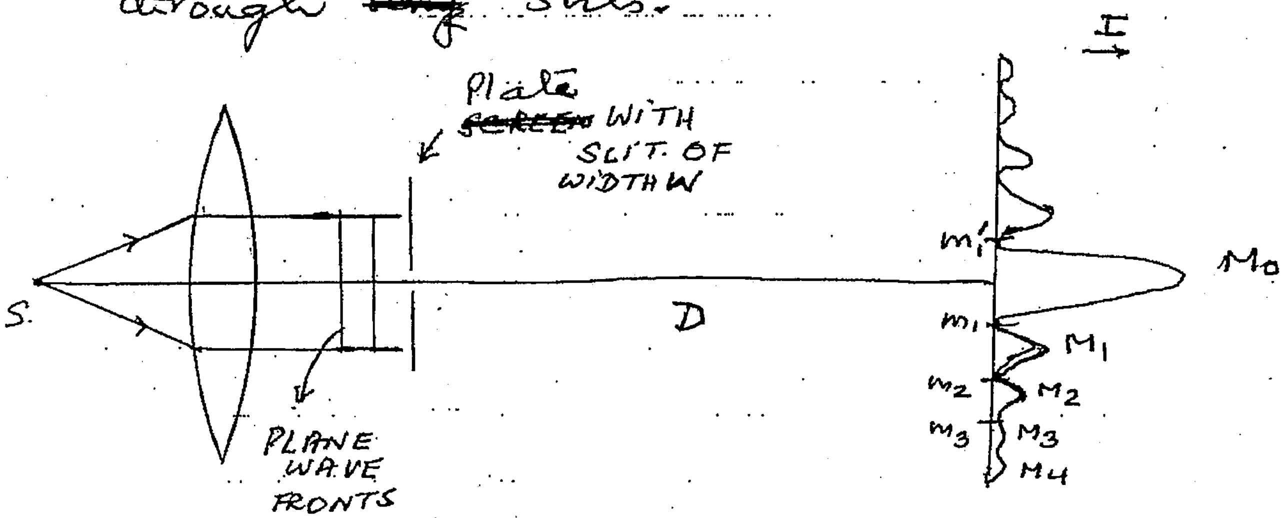
4



11/18

DIFFRACTION - SINGLE SLIT

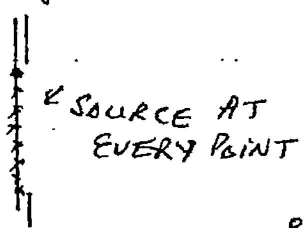
Diffraction arises because of superposition of a very large number of waves. Experimentally, it manifests itself by the spreading of a wave when it passes through an opening whose size is comparable to the wavelength that is why sound exhibits diffraction when it goes through doors and windows while ~~light~~ diffraction of light is observable only when light goes through ~~tiny~~ narrow slits.



EXPT: S is a point source of light of wavelength λ. It is placed at focal point of lens so after passing through lens we get a parallel beam which is pictured as a plane wave front travelling to the right. We place a ~~screen~~ plate with a narrow slit of width W and let the light fall on a screen a distance D away. What you observe is a series of maxima, $M_0, M_1, M_2 \ldots$ where the central one (M_0) is brightest and the intensity reduces rapidly as you go from $M_0 \to M_1 \to M_2 \to M_3 \to \ldots$ $m_1, m_2, m_3 \ldots$ locate the "dark" spots in between.

In the laboratory upstairs you use a laser as a light source as that produces a parallel beam and hence plane wave fronts.

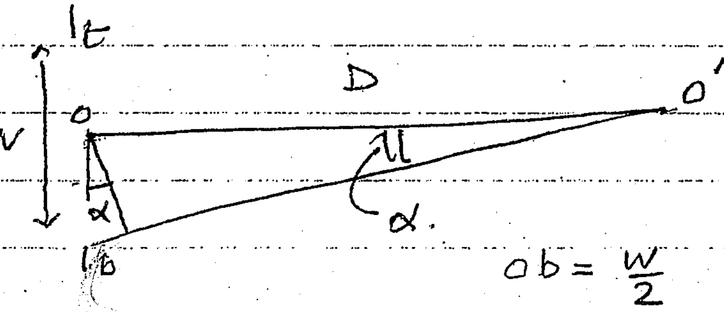
Our challenge is to construct a simple model which will allow us to understand the observations. We begin by recalling Huygen's construct that every point on a wave front is a source. Thus, it is quite reasonable to claim that the part of the wave front exposed by the slit



gives rise to a large number (say $N \gg 1$) of waves all of which start in step (in phase) from the wave front. So now we must try to ~~understand~~ explain how N ~~#~~ waves arriving at the screen conspire to produce the intensity pattern observed by us.

CENTRAL MAXIMUM (M_0): All of the waves arrive at the screen in phase. Why?

The max. path difference is between a wave coming from ctr of slit O and one from the edge (b) and therefore



$ob = \frac{W}{2}$

The angle $\alpha = \frac{W}{2D}$

$$\Delta_{max} = (bO' - OO') = \frac{W}{2} \cdot \frac{W}{2D} = \frac{W^3}{4D}$$

For a typical case $W = 10^{-4}$ m, $D \simeq 1$ m so

$$\Delta_{max} = \frac{10^{-8}}{4} \text{ m} \simeq 2.5 \text{ nm} \approx \frac{\lambda_{light}}{200}$$

Which is a very small fraction of λ.

Hence, if each wave contributes an amplitude E_m the total amplitude at M_0 would be given by the vector addition

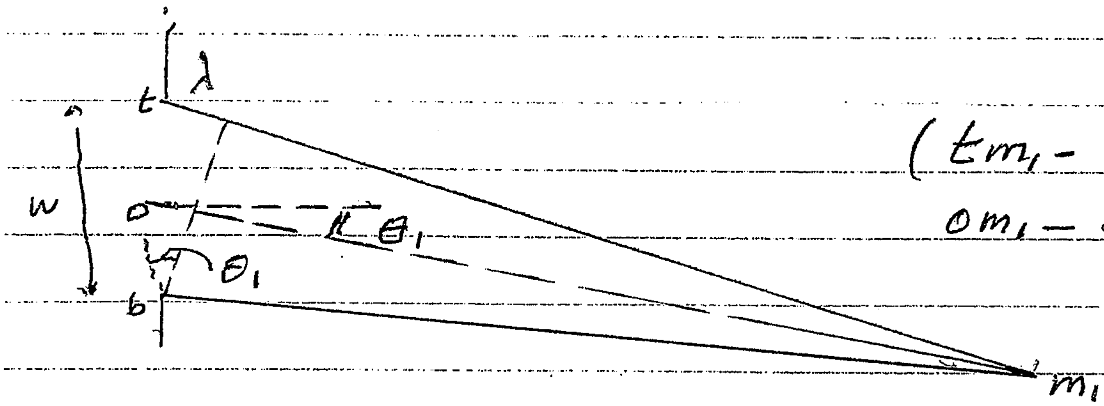
$$\underset{E_m}{\longrightarrow} + \underset{E_m}{\longrightarrow} + \underset{E_m}{\longrightarrow} + \cdots\cdots = NE_m$$

and Intensity at M_0 would be proportional to $N^2E_m^2$

$I_0 \propto N^2E_m^2$ [The consts. $\frac{1}{2}\epsilon_0 c$ are omitted].

FIRST MINIMUM (m_1).

Here the total Intensity is zero and therefore our N vectors must add together to produce a NULL result.



$$(tm_1 - bm_1) = \lambda$$

$$om_1 - bm_1 = \frac{\lambda}{2}.$$

This will happen if the path difference between the wave coming from the top edge (t) and that coming from the bottom edge (b) is exactly λ. Why?

Note that if $(tm_1 - bm_1) = \lambda$, $(om_1 - bm_1) = \frac{\lambda}{2}$ and we can claim that the slit can be split into two parts such that for every wave coming from the lower half there will be one coming from the upper half which is $\frac{\lambda}{2}$ behind and so they will cancel one another. Amazing, isn't it? Only waves can do this. Particles never

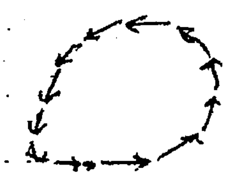
The angle θ_1 which locates m_1 is therefore given by

$$\sin\theta_1 = \frac{\lambda}{W}$$

Thus the central maximum, which is bounded by m_1 and m_1' will have a width of $2\theta_1$. The smaller the ~~b~~ W the larger the spread due to diffraction.

In terms of the E_m vectors m_1 happens because:

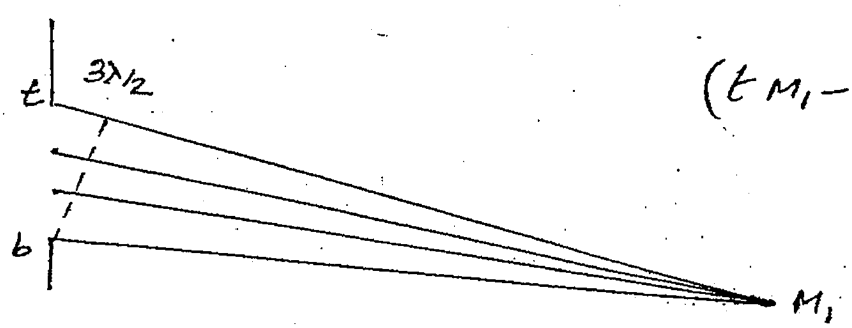
The string of length NE_m has



$\sum E_m \equiv 0$.

been wound around so it closes on itself.

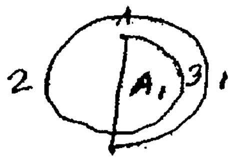
First Maximum M_1



$$(tM_1 - bM_1) = \frac{3\lambda}{2}$$

Now the waves arrange themselves so that the path difference between the wave from t and that from b is $\frac{3\lambda}{2}$. Effectively, the slit splits into 3 equal parts, two of which cancel one another so that only $\frac{1}{3}$rd of the sources contribute to the amplitude at M_1.

To calculate the amplitude at M_1 let us wind our string of length NE_m some more until it looks like.



The sum of all the vectors is A_1 and.

$$A_1 \cdot \frac{3\pi}{2} = NE_m.$$

amplitude at M_1

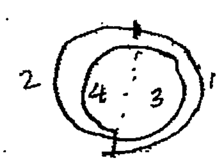
$$A_1 = \frac{2}{3\pi} NE_m.$$

$$I_1 \propto \frac{4}{9\pi^2} N^2 E_m^2$$

$$\frac{I_1}{I_0} = \frac{4}{9\pi^2}$$

M_1 is barely $\frac{1}{20}$ th as intense as M_0.

<u>Second Minimum</u>: (M_2). This requires us to wind the string even more so it looks like

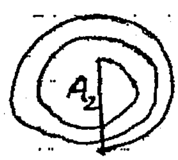


$\sum E_m \equiv 0.$

We need $t_{M_2} - b_{M_2} = 2\lambda$
The slit splits into 4 equal parts
Each quarter cancels its neighbor.

Second Maximum M_2

Continue winding further



$$A_2 \cdot \frac{5\pi}{2} = N E_m$$

$$A_2 = \frac{2}{5\pi} N E_m .$$

$$\frac{I_2}{I_0} = \frac{4}{25\pi^2}$$

I_2 is nearly 62 times ~~weaker~~ smaller than I_0.
Subsequent minima/maxima follow from the above discussion.

TWO-SLIT EXPT: INTERFERENCE + DIFFRACTION

In discussing the two slit case above we assumed that $w \ll d$ so that the central maximum for diffraction became much broader than the width of the interference fringes and that allowed us to discuss the interference effect alone. In ~~po~~ practice w and d can be quite comparable and what you observe is a ~~diffraction pattern~~

diffraction-cum-interference pattern: diffraction maxima with interference fringes in them.

Shown below are intensity patterns for the case $d = 3w$.

INTERFERENCE

DIFFRACTION

BOTH