

THE E-FIELD (COULOMB) / GRAVITATIONAL FIELD

FORCE BETWEEN TWO POINT CHARGES

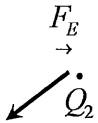
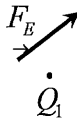
$$\vec{F}_E = k_e \frac{Q_1 Q_2}{r^2} \hat{r}$$

$$k_e = 9 \times 10^9 \frac{N \cdot m^2}{C^2}$$

Q_1, Q_2 same sign

$$\vec{F}_E \parallel +\hat{r}$$

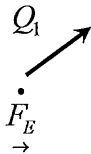
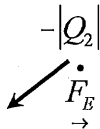
→ REPULSIVE (BOTH FORCES OUTWARD)



Q_1, Q_2 have opposite signs.

$$\vec{F}_E \parallel -\hat{r}$$

→ ATTRACTIVE



FORCE BETWEEN TWO POINT MASSES

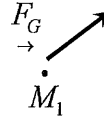
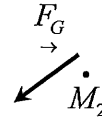
$$\vec{F}_G = -\frac{GM_1 M_2}{r^2} \hat{r}$$

$$G = 6.7 \times 10^{-11} \frac{N \cdot m^2}{(kg)^2}$$

F_G ALWAYS ATTRACTIVE

→

$F_G \parallel -\hat{r}$ (BOTH FORCES INWARD)



NOTICE THAT THE FORCES OCCUR AS ACTION-REACTION PAIRS IN EVERY CASE.

$$\left[\begin{array}{c} F_{12} \\ \rightarrow \end{array} = - \begin{array}{c} F_{21} \\ \rightarrow \end{array} \right]$$

THE EQUATIONS REPRESENT TWO FORCES.

Many Point Charges Force on Q_i

$$\vec{F}_i = k_e \sum_{j \neq i} \frac{Q_i Q_j}{r_{ij}^2} \hat{r}_{ij}$$

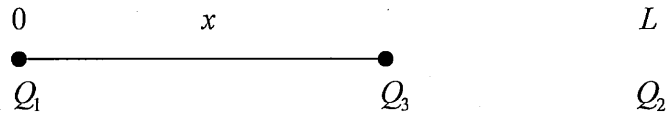
Note: Right side involves addition of vectors.

SPECIAL CASES

1. Q_1 at $x=0$, Q_2 at $x=L$. Where to locate Q_3 so \vec{F}_3 force on Q_3 is zero.

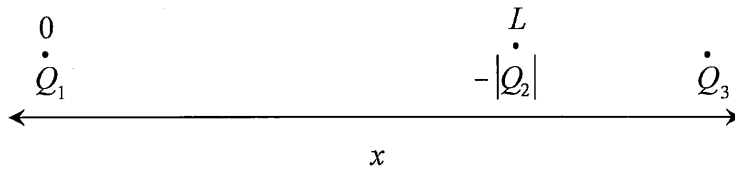
Q_3 must be on the line joining Q_1 and Q_2 .

$$x = \frac{L}{1 + \sqrt{\frac{Q_2}{Q_1}}}$$

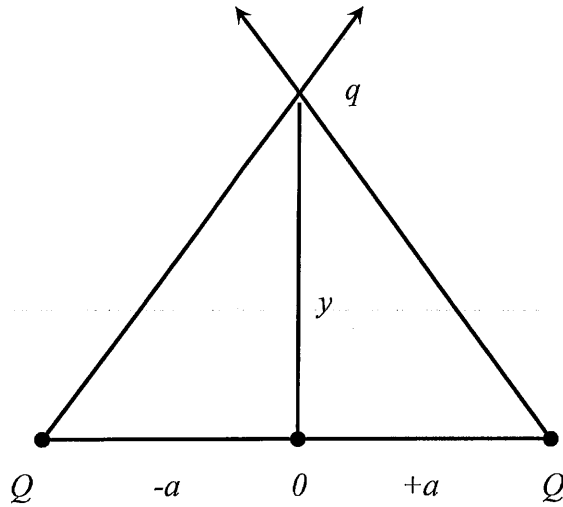


2. Q_1 at $x=0$, $-|Q_2|$ at $x=L$. \vec{F}_3 will be zero if

$$x = \frac{L}{\sqrt{\frac{Q_1}{Q_2}} - 1} \text{ when } Q_1 > |Q_2|$$



3. Q at $x=-a$, Q at $x=+a$. What is force on q at $(0, y)$



$$\vec{F}_E(y) = \frac{2k_e Qy}{(y^2 + a^2)^{3/2}} \hat{y}$$

What if we have $-|q|$ at $(0, y)$

$$\frac{-2k_e |q| Qy \hat{y}}{(y^2 + a^2)^{3/2}}$$

Let us make $y \ll a$.



In this case, force is proportional to displacement y and opposite to it so $-|q|$ will show Linear Harmonic oscillations.

QUESTION: Why is there a force between two charges(masses) when they are far apart from one another.

To answer this we develop the concept of a FIELD

COULOMB E - FIELD

If there is a charge Q sitting at $x=0$, the space around it is not empty. Q creates a coulomb E field which permeates all of space. If you place a test charge q in this E field, it experiences a force $\vec{F}_E = q \vec{E}$

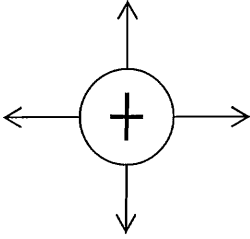
GRAVITATIONAL (G_F) FIELD

If there is a mass M sitting at $x=0$, the space around it is not empty. M creates a gravitational (G_F) field which permeates all of space. If you place a test mass m in the G field it experiences a force

$$\vec{F}_G = m \vec{G}_F$$

1. Single +ive charge Q at $r=0$.

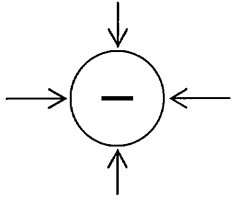
$$\vec{E} = \frac{k_e Q}{r^2} \hat{r}$$



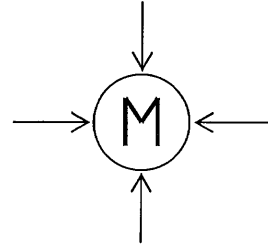
Acts like a "source" of an \vec{E} field which points radially outward

2. Single -ive charge at $r=0$.

$$\vec{E} = -k_e \frac{|Q|}{r^2} \hat{r}$$



Acts like a "sink" of \vec{E} field which pts. Radially inward.

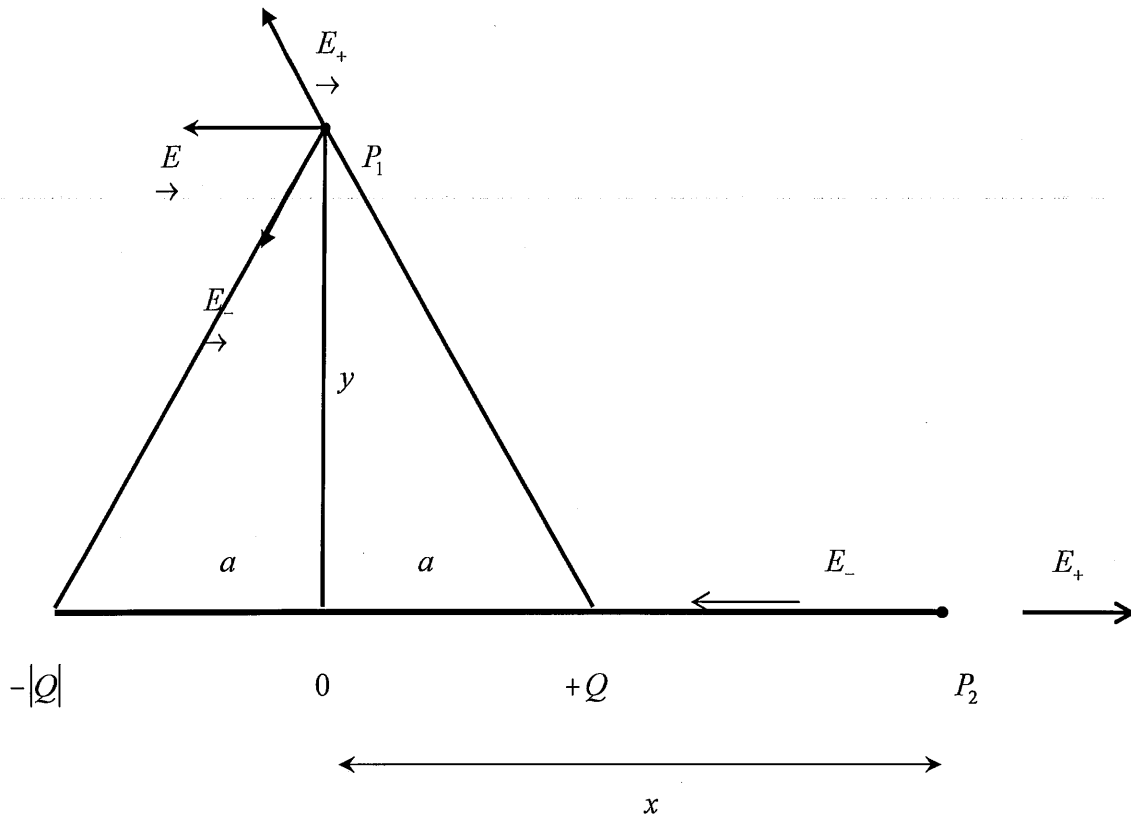


$$\vec{G}_r = -\frac{GM}{r^2} \hat{r}$$

ALWAYS INWARD RADIALLY.

3. \vec{E} field of Dipole: $-|Q|$ at $x=-a$

$+Q$ at $x = a$



at $P_1 = (0, y)$ $\vec{E}(y) = -\frac{k_e 2aQ}{(y^2 + a^2)^{3/2}} \hat{x}$

at $P_2 = (x, 0)$ $\vec{E}(x) = -\frac{k_e 4aQx}{(x^2 - a^2)^2}$

Next, define dipole moment $\vec{p} = 2aQ\hat{x}$

$\vec{E}(y) = \frac{-k_e \vec{p}}{y^3}$

$\vec{E}(x) = \frac{2k_e \vec{p}}{x^3}$

When $x, y \gg a$ that is far away from dipole