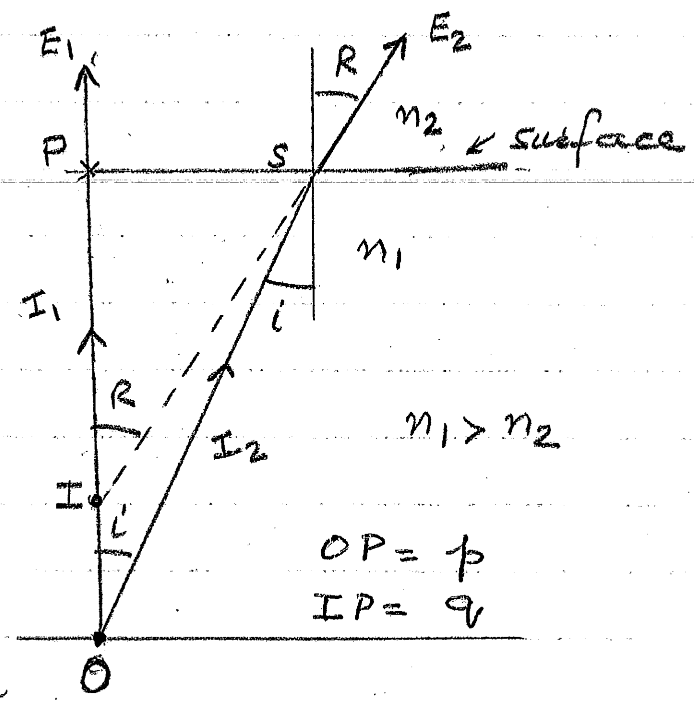
FORMATION OF IMAGES – REFRACTION AT A SINGLE SURFACE [SIGN CONVENTION: ALONG LIGHT +ive, AGAINST LIGHT –ive]

I: Apparent depth of Water in a pool.

Supposing you are standing at the edge of a Swimming and look straight down. If the actual depth of water is d meters what value do you perceive?



We can Solve this problem by putting a point object O at the bottom and locate its image formed by the water as the light refracts through its surface. Look at the picture

OPTICAL SYSTEM SO all distances are measured from P.

Take two rays starting from O:

I_1 makes angle of incidence zero and gives rise to E_1

I_2 makes angle of incidence i and Causes E_2 satisfying

$$n_2 \sin R = n_1 \sin i$$

Since you are looking straight down all angles are small.

The Virtual IMAGE at

I [q is –ive].

is located by intersection of E_1 and E_2

(extended backwards).

Next, from the picture we see

$$\tan R = \frac{SP}{IP} \qquad (1)$$

$$\tan i = \frac{SP}{OP} \qquad (2).$$

Divide ② by ①

$$\frac{IP}{OP} = \frac{\tan i}{\tan R}$$

$$\simeq \frac{\sin i}{\sin R} \qquad \left[\begin{matrix} i \ll 1 \\ R \ll 1 \end{matrix}\right].$$

$$= \frac{n_2}{n_1}$$

Clearly IP = apparent depth
OP = Rl. depth

$$\frac{d_{app}}{d} = \frac{n_2}{n_1}$$

for water $n = 1.33$
for air $n = 1$.

$$\text{So} \quad \frac{d_{app}}{d} = \frac{3}{4}.$$

So if water is 80 cm deep, to a person at the edge it will appear to be only 60 cm [small children should be warned before they jump in and suddenly find that they are too short].

FORMATION OF IMAGES — THIN LENSES

This case also involves refraction so we have the same sign convention:

distances measured along path of light: +ive
distances measured against path of light: −ive.

Lens consists of a transparent material which has two curved surfaces. We will deal with surfaces which are spherical so only two radii are required.

Lens maker's formula Focal length f is given by

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_F} - \frac{1}{R_B}\right]$$

here n = refractive index of material which is placed in air ($n=1$)

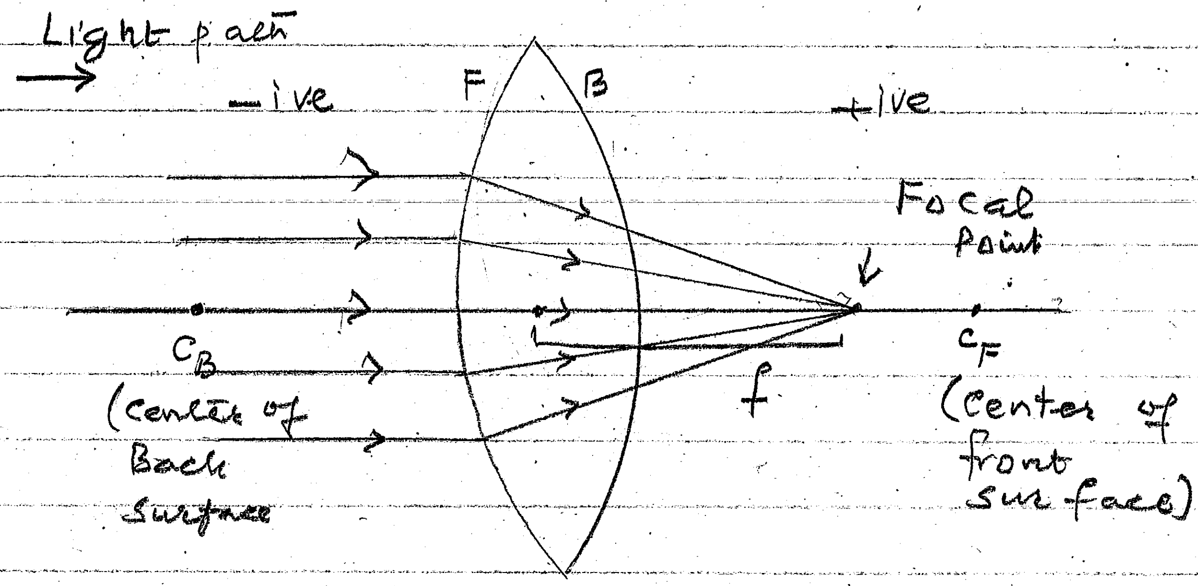
R_F = radius of front surface (facing the incident light)

R_B = radius of back surface.

Thin lenses the thickness of the lens is much smaller than R_F and R_B.

Two Cases arise

I Convergent lens



In this case R_F is +ive

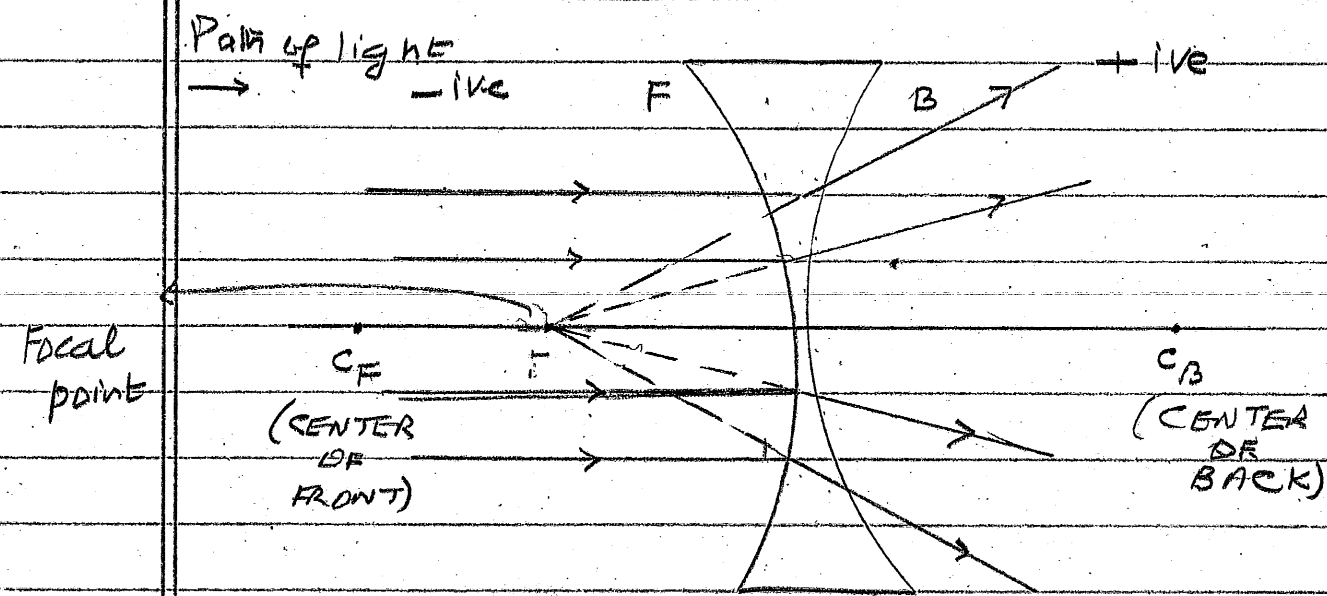
R_B is -ive.

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_F} - \frac{1}{R_B}\right]$$ is positive

Because f is positive focal point is to the right of the lens hence parallel light falling on the lens will be made to converge to a point, as shown above.

Light actually goes through the focal point, you can project it on a screen.

II DIVERGENT LENS



Here R_F is -ive

R_B is +ive

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_F} - \frac{1}{R_B}\right] \text{ is negative}$$

Because f is negative focal point is to the left of the lens hence parallel light falling on the lens will appear to diverge from a point as shown above.

Notice, no light actually goes through the focal point, it is a VIRTUAL point. (negative distance). You CANNOT project it on a screen.

But $CC' = OO'$

so

$$\frac{II'}{OO'} = \frac{q-f}{f} = \frac{q}{p}$$

$$\frac{q}{f} - 1 = \frac{q}{p}$$

$$\frac{q}{f} = 1 + \frac{q}{p}$$

write

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$m = -\frac{q}{p}$$

and these two equations describe all possible images formed by the lens.

We can recall all the cases which arose in convergent mirror case —

i) $p \to \infty$ $q \to f$ $m = 0$

ii) $p > 2f$ $q < 2f$ m -ive and less than 1.

(iii) $p = 2f$, $q = 2f$ m -ive and equal to 1.

(iv) $p < 2f$ $q > 2f$ m -ive and greater than 1

(v) $p < f$ q -ive, virtual image, m +ive.

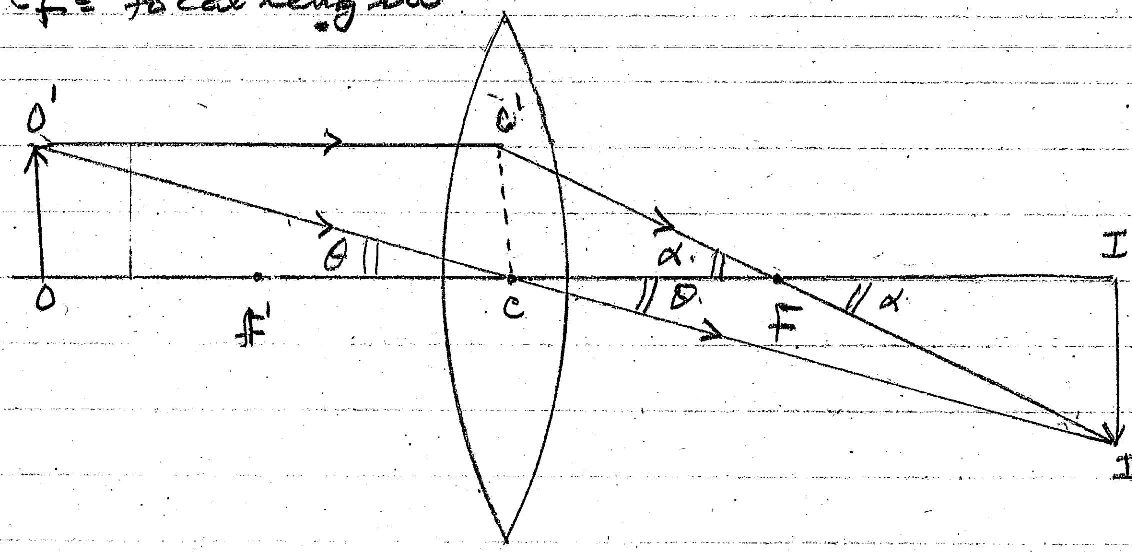
IMAGE FORMATION

CONVERGENT LENS.

$OC = p$ object distance

$IC = q$ image distance

$CF =$ focal length



Note: All angles are supposed to be small and thickness is small that is why lateral shift is negligible and the ray through C goes straight

First, look at magnification $m = -\frac{q}{p}$.

$$\frac{II'}{OO'} = \left|\frac{q}{p}\right|.$$

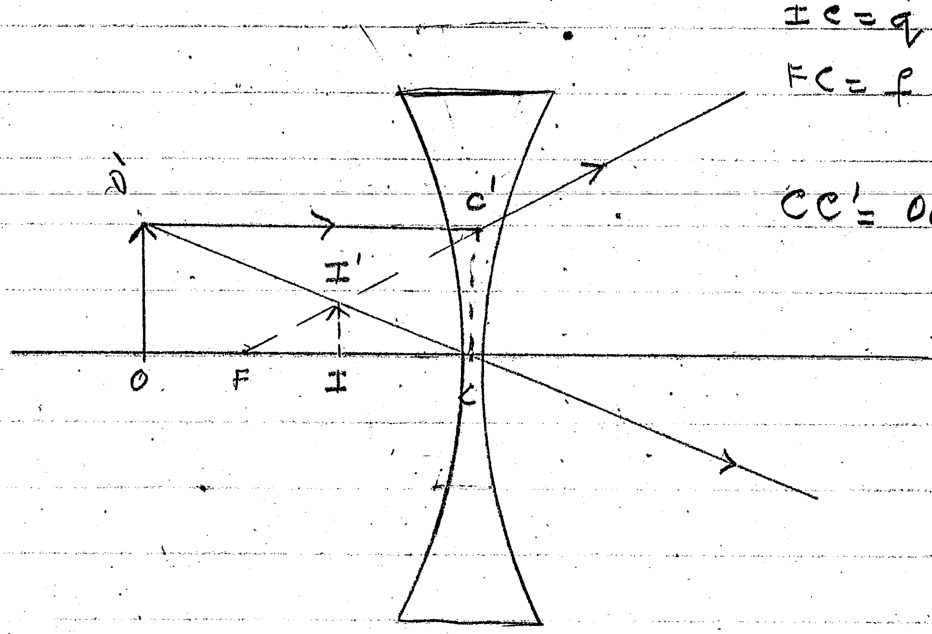
and indeed II' is inverted, m is -ive!

Next from angles α.

$$\frac{II'}{IF} = \frac{CC'}{CF}$$

$$\frac{II'}{CC'} = \frac{IF}{CF} = \frac{q-f}{f}$$

DIVERGENT LENS



$OC = p$

$IC = q$ [-ive]

$FC = f$ [-ive]

$CC' = OO'$

In this case ALL IMAGES ARE VIRTUAL, UPRIGHT and REDUCED ($m < 1$). [like in CONVEX MIRROR].

Now

$$\frac{CC'}{II'} = \frac{f}{f-q} = \frac{p}{q}$$

so

$$\frac{f-q}{f} = \frac{q}{p}$$

$$1 - \frac{q}{f} = \frac{q}{p}$$

$$\frac{q}{p} - 1 = -\frac{q}{f}$$

$$\frac{1}{p} - \frac{1}{q} = -\frac{1}{f}$$

But both f and q are -ive, Hence again

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \text{with } m = -\frac{q}{p}.$$