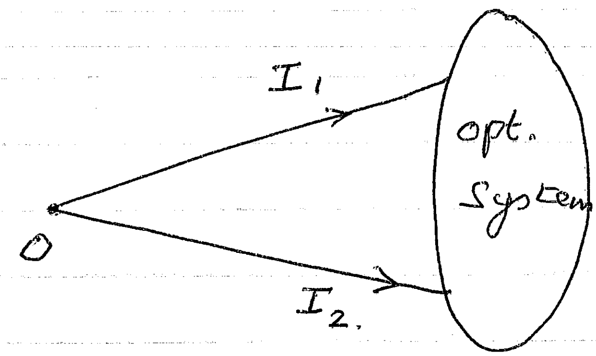
FORMATION OF IMAGES – MIRRORS.

<u>General Construct</u> to locate image of a point Object O using the laws of reflection and refraction to locate the path of light

<u>Step 1</u> Take two incident rays I_1 and I_2 starting from O.



<u>Step 2</u> Follow them through the optical system using

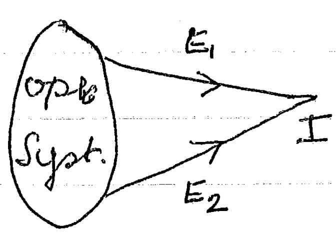
$$r = i \quad \text{for Reflection}$$

and

$$n_2 \sin R = n_1 \sin i \quad \text{for Refraction.}$$

<u>Step 3</u>: Locate the two rays E_1 and E_2 that emerge from the optical system. Two Cases arise:

<u>Case I</u>

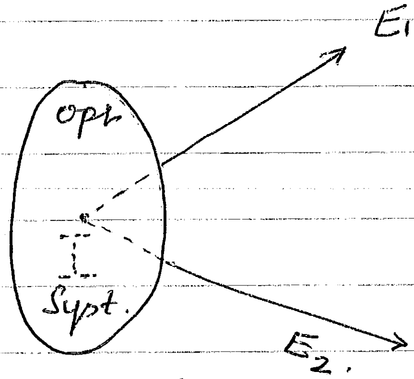


Point of intersection of E_1 and E_2 defines position of image I. Here, light actually goes through the point I so it is called a

REAL IMAGE

Note: A real Image Can be projected on a Screen.

Case II



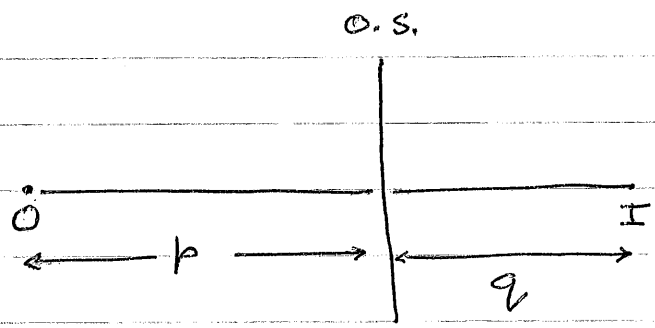
E_1, E_2 are diverging so they will not intersect. You will have to extrapolate the E_1, E_2 lines to locate the image I as the point from which E_1, E_2 "appear" to be coming. No light actually goes through I so it is called

VIRTUAL IMAGE

Note A virtual image cannot be projected on a Screen (your eye can see it)!

SOME DEFINITIONS

p = Distance of object from Opt. Syst. (O.S.)



q = Distance of image from opt. system.

Magnification $m = \frac{\text{Size of image}}{\text{Size of object}} = -\frac{q}{p}$.

The minus sign on the right side ensures that for a single element O.S. all real images (q +ive) are Inverted (m -ive) while all virtual images (q -ive) are upright (m +ive).

MIRROR

A mirror is an optical system in which only reflections occur, so there is a light side and a dark side. We use the "Sign" convention that distances are "+ive" on light ~~led~~ side and "-ive" on dark side.

1. PLANE MIRROR: PLANE, SILVERED Sheet of Glass

Two rays:

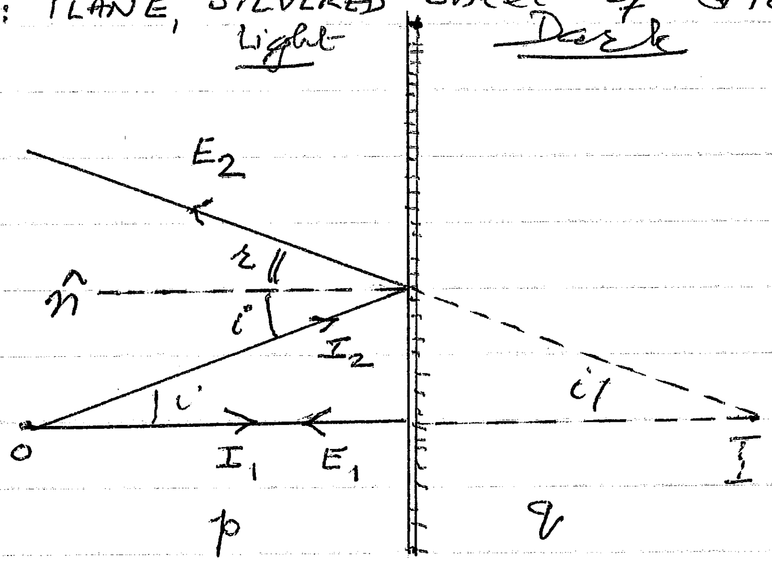
$I_1 \parallel \hat{n}$

$i=0, \; r=0$

$E_1 \parallel \hat{n}$

I_2: $r = i$

locates E_2.



E_1 and E_2 diverge.

Locate image by extrapolation at I.

q is -ive.

Image is virtual, clearly $|q| = p$.

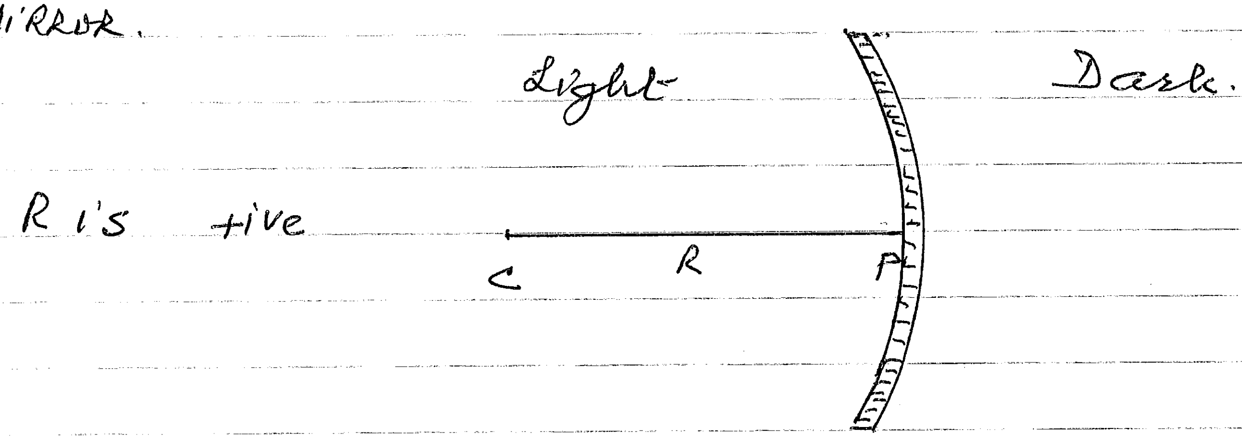
$$m = -\frac{q}{p} = +1.$$

upright, virtual Image is as far behind the mirror as the object is in front of it

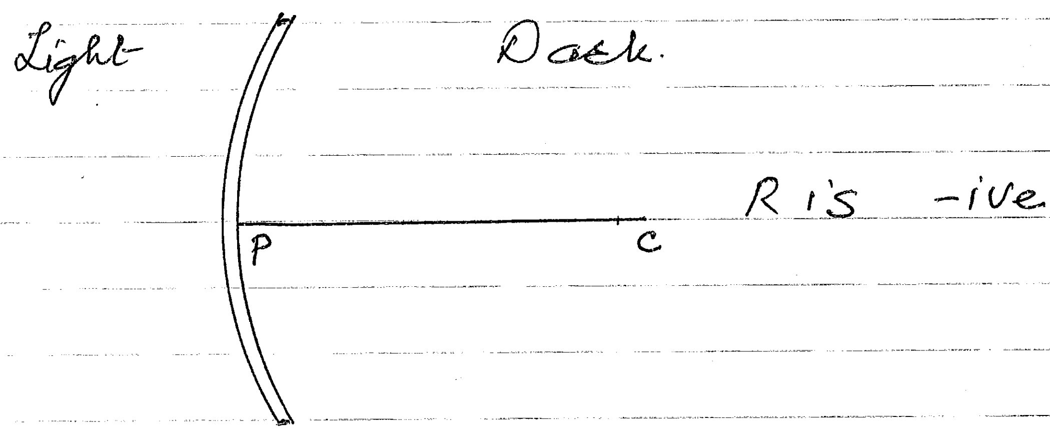
2 <u>SPHERICAL MIRRORS</u>: Mirror cut from a spherical shell of radius R.

<u>Two Cases arise</u>

A <u>CONCAVE</u>. CENTER OF SPHERE IN FRONT OF MIRROR.

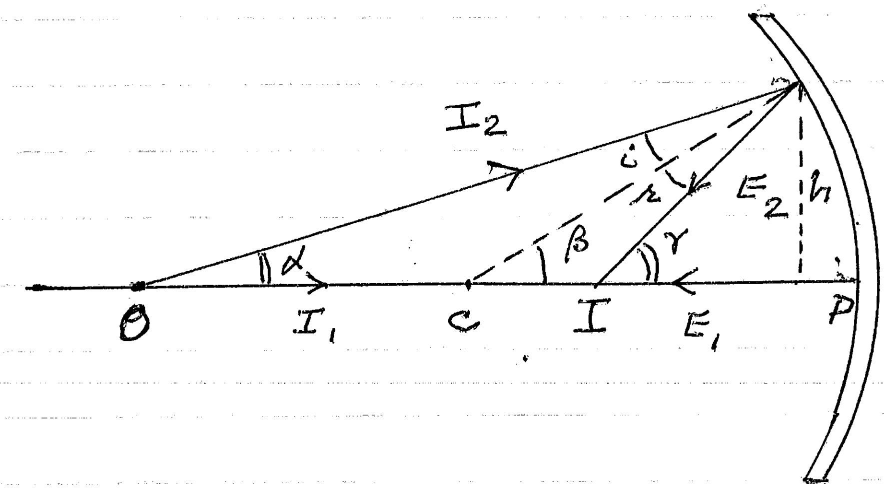


B <u>CONVEX</u> CENTER OF SPHERE BEHIND MIRROR.



<u>A</u> Images in Concave Mirror

We will consider only paraxial rays, that is all angles are taken to be very small so $\sin\theta \simeq \tan\theta \simeq \theta$.



Same construct. I_1, I_2 start from O, after reflection we get E_1, E_2, intersection locates I.

<u>Equations</u>

$$\beta = \alpha + i$$
$$\gamma = \beta + r$$
$$i = r$$

Hence $\quad \gamma = \beta + i = 2\beta - \alpha.$

or $\quad \gamma + \alpha = 2\beta.$

Angles are small, hence

$$\tan\gamma + \tan\alpha = 2\tan\beta.$$

$$\frac{h}{IP} + \frac{h}{OP} = \frac{2h}{CP}$$

or
$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R} \qquad (1)$$

and combined with

$$m = -\frac{q}{p} \qquad (2)$$

these two equations describe all possible images, and their sizes, formed by a concave mirror.

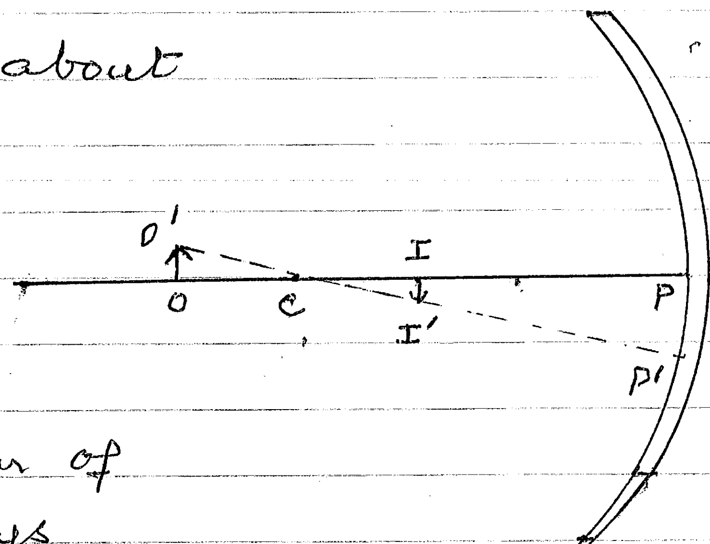
~~SPECIAL CASES~~
OBJECT SIZE.

a) Small object

Rotate picture about the center (of curvature) C

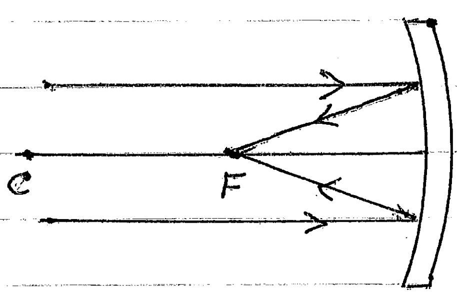
The above calculations apply.

Our assumption of paraxial rays requires that all objects are small.



SPECIAL CASES [Eqs (1) and (2)].

I $\quad p \to \infty, \quad q = \frac{R}{2}$



$p \to \infty$ Implies that incident line is a parallel beam,

and we learn that

when a parallel beam falls on the mirror, after reflection it converges to a point, This defines the focus and the focal length

$$f = R/2.$$

So $p \to \infty$, $q \to f$ ~~∞~~ $m \to 0$ Real Image

Concave Mirror makes parallel light converge to a point hence
Concave Mirror = CONVERGENT MIRROR.

NOTE NO REAL IMAGE CAN COME CLOSER THAN f!

II $p > R$ object lies beyond C.

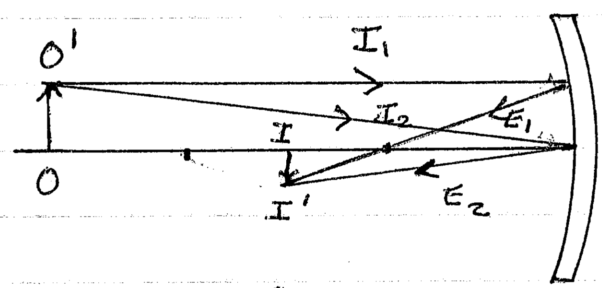
$$\frac{1}{q} = \frac{2}{R} - \frac{1}{p} = \frac{2p - R}{pR}.$$

$$\frac{p}{q} = \frac{2p-R}{R} = \frac{> 2R - R}{R} = > 2 - 1 = > 1.$$

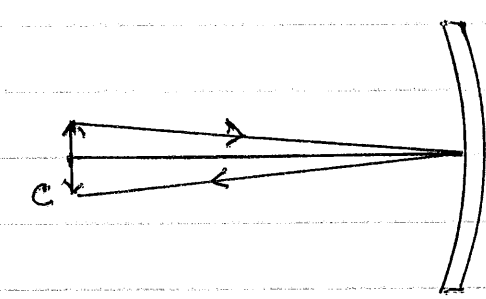
so $q < R$.

$m = -\frac{q}{p}$, $|m| < 1$.

Inverted, Real, Reduced Image.



III $p = R$, $q = R$, $m = -1$. [Lamp Expt.].
Object lies at C.
All light falls on mirror at $i = 0$, so it leaves at $r = 0$ and goes right back to C.



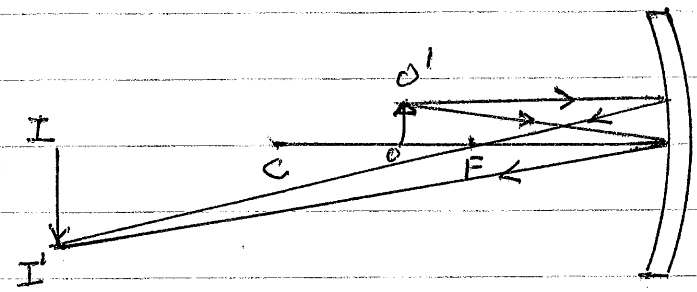
REAL, INVERTED IMAGE SAME SIZE AS OBJECT.

$\overline{\text{IV}}$ $p < R$, : object ~~at~~ lies between C and F

$$\frac{p}{q} = \frac{2p-R}{R} = \frac{<2R-R}{R} = <1$$

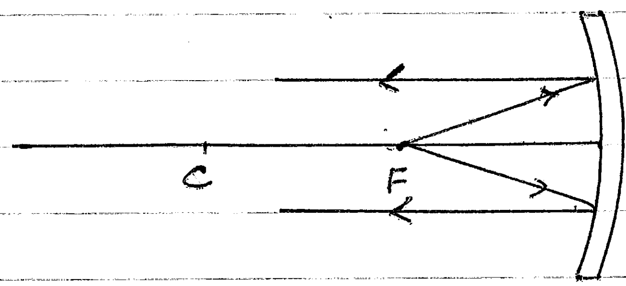
$q > R$, $\quad m = -\frac{q}{p}$, $\quad |m| > 1$.

Inverted, Real, Enlarged Image



$\overline{\text{V}}$ $p = f$, object at F, $q \to \infty$, $m \to \infty$

Point Source
at F on
reflection
produces
a parallel
beam.

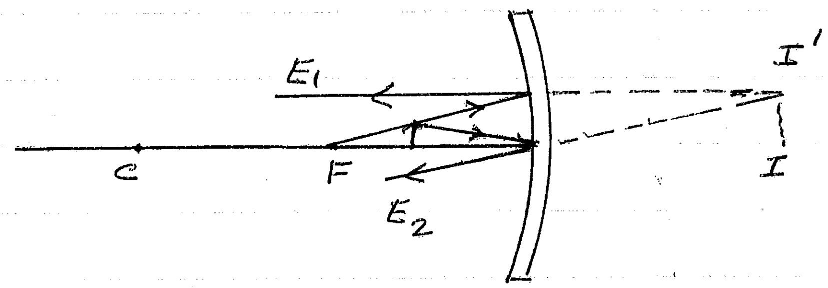


$\overline{\text{VI}}$: $p < f$, object closer to mirror than F,

Since $\quad \frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

this equation cannot be satisfied unless
q becomes $\underline{-ive}$; IMAGE IS BEHIND MIRROR,
ON THE DARK SIDE.

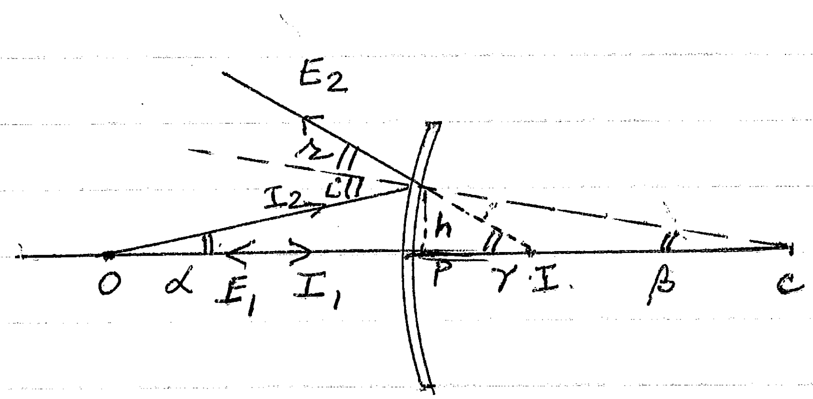
We get a VIRTUAL, ENLARGED IMAGE



To Summarize: Start O far away, I at f (closest to mirror), bring O closer I moves away from mirror, gets bigger but remains inverted real, O at C, I at C, $m = -1$, O between C and F, I beyond C, enlarged, inverted, real, O at F, $I \to \infty$, Parallel light; O closer than F, I goes BEHIND mirror becomes Virtual and upright.

B Images in Convex Mirror

R is -ive



$OP = p$

$IP = q$ [q is -ive].

Eqns

$$\gamma = i + \beta.$$

$$i = \alpha + \beta.$$

$$\gamma = \alpha + 2\beta.$$

$$\alpha - \gamma = -2\beta$$

$$\tan\alpha - \tan\gamma = -2\tan\beta.$$

$$\frac{h}{p} - \frac{h}{q} = -\frac{2h}{R}$$

or

$$\frac{1}{p} - \frac{1}{q} = -\frac{2}{R}$$

However, both q and R are -ive so one should write

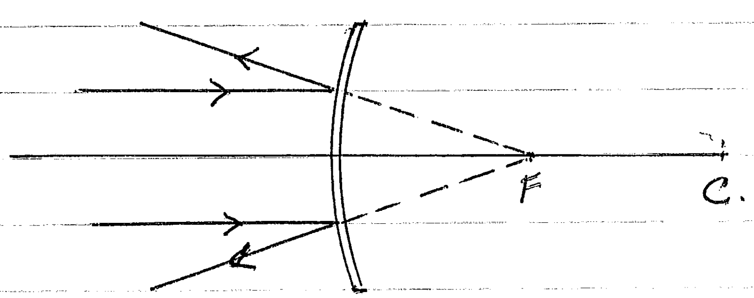
$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R}. \qquad (3)$$

and supplement it with

$$m = -\frac{q}{p} \qquad (4)$$

SPECIAL CASES

I $p \to \infty$, Parallel light falls on mirror
$q = R/2 = f$ but f is -ive, f is behind mirror on dark side.



A parallel beam of light incident on a convex mirror, on reflection appears to start from F and becomes divergent.

Hence

CONVEX MIRROR $\Rightarrow$ DIVERGENT MIRROR.

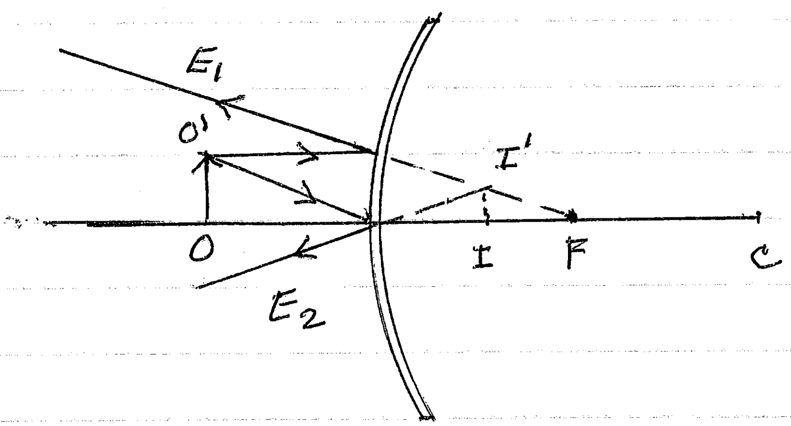
II Since q is −ive all Images are VIRTUAL and upright.

Also $\frac{1}{p} - \frac{1}{q} = -\frac{2}{R}$.

or $\frac{1}{q} = \frac{1}{p} + \frac{2}{R}$. $\left[\frac{1}{q} > \frac{1}{p}\right]$.

So q is always less than p.
and always less than f.

All Images are Virtual, upright and reduced.



Right rear view mirror of automobiles!!