FIELDS: GRAVITATIONAL, COULOMB $\vec{E},\vec{B}$

$\vec{G}$: A mass m located in a Gravitational field feels a force $\vec{F}_G = m\vec{G}_F$

Measure $\vec{F}_G$, map out $\vec{G}_F$.

A mass M located at the origin creates a $\vec{G}_F$

$$\vec{G}_F = -\frac{-GM}{r^2}\hat{r}$$

consequently, $\Sigma_c \vec{G}_F \bullet \Delta\vec{A} = -4\pi G \Sigma M_i$

FLUX OF $\vec{G}_F$ Through a closed surface is determined solely by the masses enclosed by the surface.

$\vec{E}$: A charge q located in an $\vec{E}$-field feels a force $\vec{F}_E = q\vec{E}$.

Measure $\vec{F}_E$, map out $\vec{E}$.

A stationary charge Q located at r=0 generates a coulomb $\vec{E}$-field

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

[+Q (source), -Q (sink)] consequently, $\Sigma_c \vec{E} \bullet \Delta\vec{A} = \frac{1}{\varepsilon_0}\Sigma Qi$.

FLUX OF $\vec{E}$ THROUGH A CLOSED SURFACE IS DETERMINED SOLELY BY THE CHARGES ENCLOSED BY THE SURFACE.

ε

The Devices resulting from this are: (i) Battery

(ii) Capacitor $C = \frac{Q}{V}$ which leads to energy density $\eta_E = \frac{1}{2}\varepsilon_0 E^2$

That is the energy contained in $1m^3$ vol. of $\vec{E}$-field

(iii) Resistor $R = \frac{V}{I}$ which leads to $\vec{J} = \sigma\vec{E}$, because $I = \vec{J} \bullet \vec{A}$

That is, if you apply $\vec{E}$ to a Conductor it responds by setting up a current density $\vec{J}$ whose magnitude is determined by the electrical conductivity σ.

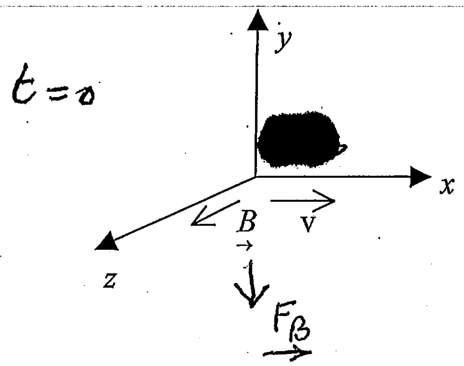
Magnetic ($\vec{B}$): If a $\vec{B}$-field is present, a charge q moving with velocity $\vec{v}$ will experience a

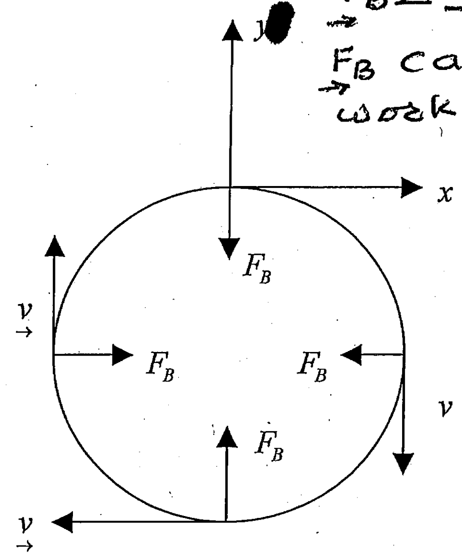
force $\vec{F}_B = q[\vec{v} \times \vec{B}]$

Magnitude of $F_B = qvB\,Sin(\vec{v},\vec{B})$ [Equivalently, if a moving charge experiences a force which is always perpendicular to its velocity, and there is no visible agency applying the force, then the charge must be moving in a $\vec{B}$-field].

Direction of $\underset{\to}{F_B} \to$ *rt hand rule* $\left\{ q\,\underset{\to}{v} \parallel Thumb, \underset{\to}{B} \parallel fingers, \underset{\to}{F_B} \perp Palm \right\}$

Problem I: At t=0, charge q is at origin and has velocity $\underset{\to}{v} = v\hat{x}$. Turn on a field $\underset{\to}{B} = B\hat{z}$ Immediately, it experiences $\underset{\to}{F_B}$ along $-\hat{y}$. Makes $\underset{\to}{v}$ turn, but $\underset{\to}{F_B}$ turns also. Net result is as shown in Figure. q goes around in circle, $\underset{\to}{F_B} \perp \underset{\to}{v}$ always so Kinetic Energy fixed,* magnitude of $\underset{\to}{v}$ does not change.*





* $\underset{\to}{F_B} \perp \underset{\to}{v}$

$\underset{\to}{F_B}$ cannot do work on q. Dir. of $\underset{\to}{v}$ changes, magnitude of $\underset{\to}{v}$ does not!

Particle moves under influence of $\underset{\to}{F_B} = -qvB\hat{r}$ [$\underset{\to}{v}$ & $\underset{\to}{B}$ *are* $\perp$ *to one another*]

Note: Plane of orbit $\perp$ to $\underset{\to}{B}$ field.

Note: Uniform circular motion needs a centripetal force.

$$\underset{\to}{F_C} = \frac{-Mv^2}{r}\hat{r}$$

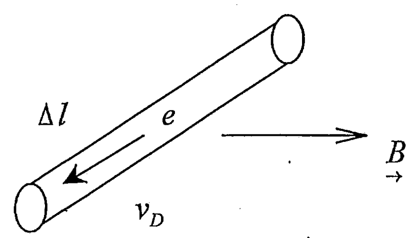
$\underset{\to}{F_B}$ provides it.

$$\underset{\to}{F_B} = \underset{\to}{F_C} \quad so \quad r = \frac{Mv}{qB}$$

angular velocity $\underset{\to}{w} = \frac{-qB}{m}\hat{z}$ (see picture above)

Note: w independent of v.

Problem II: Force on Current Carrying conductor of length l; Cross. Sec A, charge density n_e each electron feels $\vec{F}_B = (-e)[\vec{v}_D \times \vec{B}]$



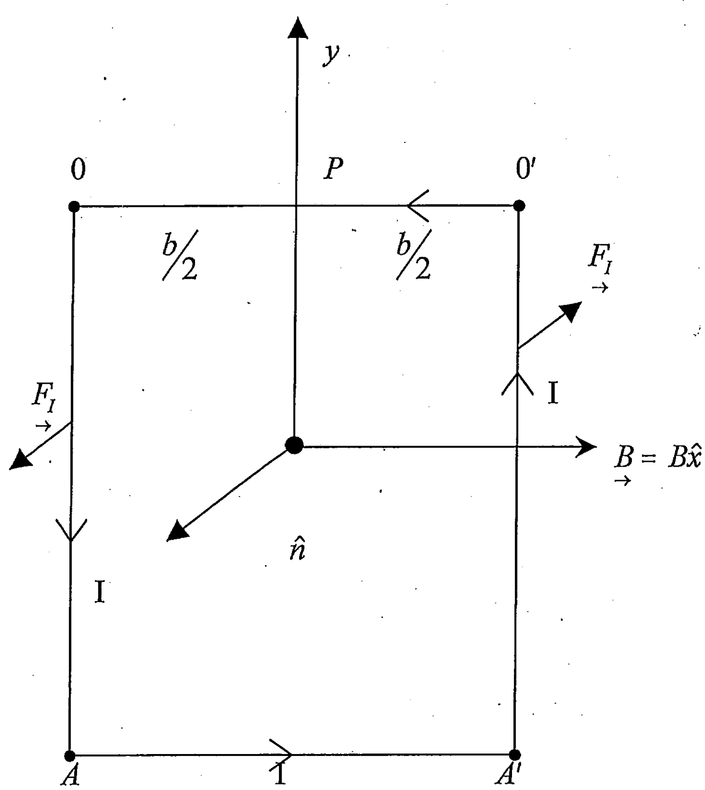
\# of $electrons = n_e A \Delta l$

so total force on conductor $\vec{F}_I = n_e A_e \Delta l[\vec{v}_D \times \vec{B}]$

electrons constrained to move along Δl so $\vec{F}_I = n_e A_e v_D \left[\Delta \vec{l} \times \vec{B}\right] = I \Delta \vec{l} \times \vec{B}$

Problem III

Rectangular loop of wire suspended in a $\vec{B}$-field with current in loop as shown, start with loop in xy-pl, at t=0.



$\vec{F}_I = IlB\hat{z}$ on $0A$

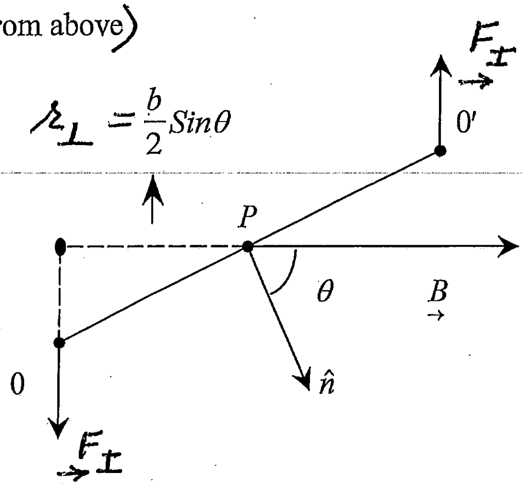
$\vec{F}_I = -IlB\hat{z}$ on $0'A'$

Net force is zero. However, torque is given by

~~Torque~~ $$\underset{\rightarrow}{\tau} = IlB\frac{b}{2}\hat{y} + IlB\frac{b}{2}\hat{y} = IlBb\hat{y} \qquad (1)$$

<u>Rotate</u> (look from above)

$r_{\perp} = \frac{b}{2}Sin\theta$



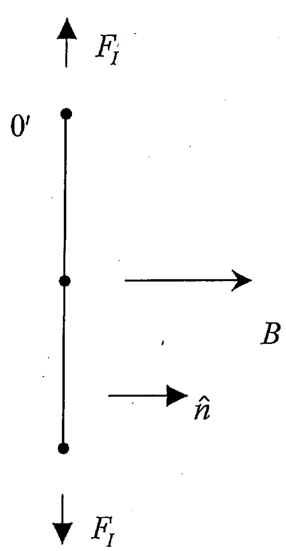
$\underset{\rightarrow}{\tau} = [\underset{\rightarrow}{r} \times \underset{\rightarrow}{F}]$

$\tau = r_{\perp} F$

Note I and B still at right angles to one another, F_I does not change but now $r_{\perp} = \frac{b}{2}Sin\theta$.

[Direction of $\hat{n}$ also fixed by right hand rule] $\underset{\rightarrow}{\tau} = IlBb\,Sin\theta\,\hat{y}$ (2)

Rotate further:



$\underset{\rightarrow}{\tau} = 0$ (3)

$\hat{x} \times \hat{y} = \hat{z}$
$\hat{y} \times \hat{z} = \hat{x}$
$\hat{z} \times \hat{x} = \hat{y}$

Equations (1), (2), (3) combine to give $\underset{\rightarrow}{\tau} = Ilb\hat{n} \times \underset{\rightarrow}{B}$

Define Magnetic (Dipole) moment $\vec{\mu} = Ilb\hat{n} = IA\hat{n}$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

If the coil has N turns $\vec{\mu} = IAN\hat{n}$!

Note: The top $(00')$ and bottom (AA') wires have equal and opposite Forces. They will make the coil out of shape but have no other effect.

Generation of $\vec{B}$– field

We have seen that a stationary charge experiences a force in an $\vec{E}$-field and a stationary charge creates a (coulomb) $\vec{E}$-field. Now we know that a moving charge experiences a force in a $\vec{B}$–field so it is natural to expect that a moving charge will generate a $\vec{B}$-field. This is indeed the content of the so-called Biot Savart Law.

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \vec{r}}{r^3}.$$

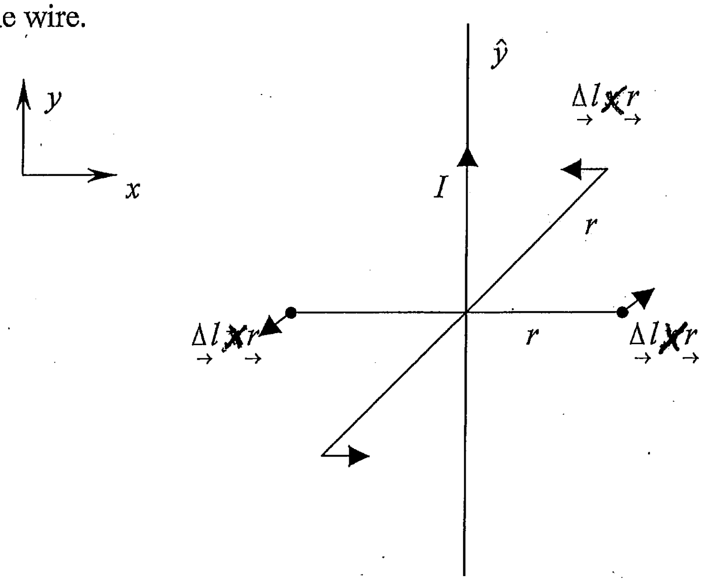
where $\mu_0 = 4\pi x 10^{-7} \frac{T-m}{A}$ is a universal constant. This equation has the immediate consequence that for a current I in a conductor of length Δl.

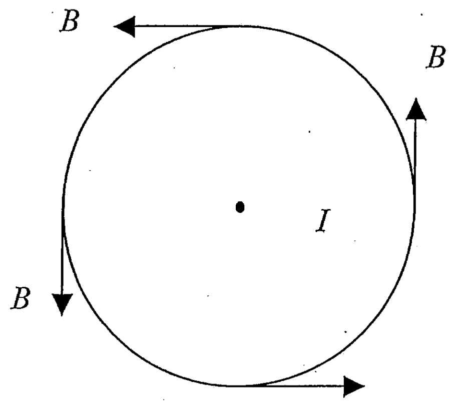
$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \frac{\Delta \vec{l} \times \vec{r}}{r^3}$$

We will not use these equations in detail.

CASES OF SPECIAL INTEREST.

<u>Single current</u> – I in a long wire: What can we say about $\vec{B}$-field at a distance r from the wire? Notice that $\Delta l || + \hat{y}$. And the vector $\Delta \vec{l} \times \vec{r}$ is perpendicular to $\vec{r}$ so we can say that $\vec{B}$ must curl around the wire.





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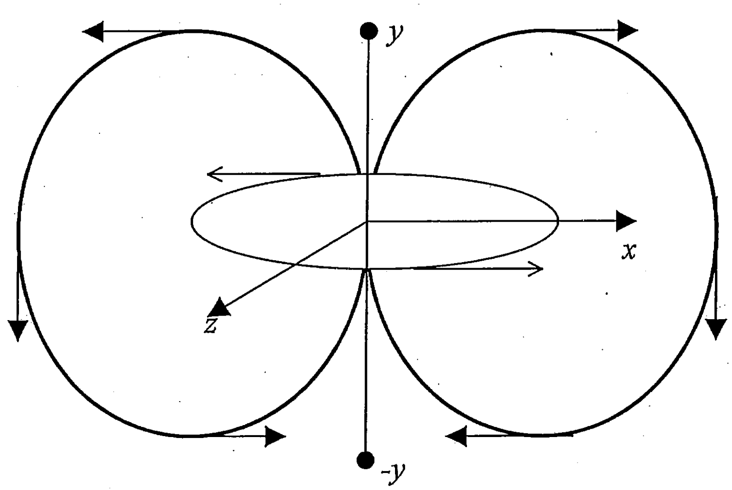
Looking end on (see picture) we have cylindrical symmetry so $\underset{\rightarrow}{B}$ can be a function of r only. It turns out that $B = \dfrac{\mu_0 I}{2\pi r}$

so, $\underset{\rightarrow}{B} = \dfrac{\mu_0 I}{2\pi r}\hat{\varnothing}$

Thus, $\underset{\rightarrow}{B}$ is said to be Azimuthal, $\hat{\varnothing}$ is the direction which curls around I. [check with the sheet on right hand rules].

Next, take the wire and make a circular loop out of it, put it in *xz-pl.* with center at the origin. What is the $\underset{\rightarrow}{B}$-field at y or $-y$?



The $\underset{\rightarrow}{B}$-field lines are shown schematically, it turns out that

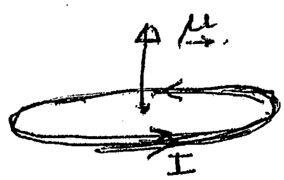
$$\underset{\rightarrow}{B}(y) = \underset{\rightarrow}{B}(-y) = \frac{\mu_0}{4\pi}\frac{2I\pi a^2}{(a^2+y^2)^{3/2}}\hat{y}$$

Once again, we encounter $IA\hat{n}$ so we can write using magnetic (dipole) moment

$$\vec{B}(y) = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{(a^2+y^2)^{3/2}}$$

Far away from $\vec{\mu}, y >> a$

$$\vec{B}(y) = \frac{\mu_0}{4\pi}\cdot\frac{2\vec{\mu}}{y^3} \leftarrow \text{Magnetic Dipole}$$

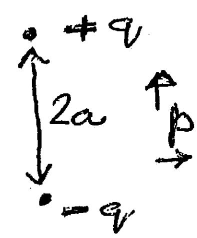


Recall that for an Electric Dipole

$$\vec{p} = 2qa\hat{y}$$

and the $\vec{E}$-field at y is

$$\vec{E}(y) = \frac{1}{4\pi\varepsilon_0}\frac{4qay}{\left(y^2-a^2\right)^2}\hat{y}$$



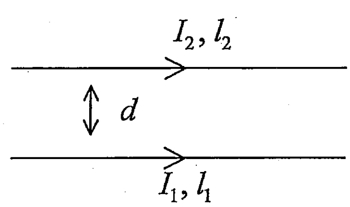
so that at $y>>a$

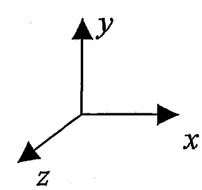
$$\vec{E}(y) = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{y^3} \leftarrow \text{Electric Dipole}$$

very Imp.

However, there is a major difference here: along y the magnetic dipole has no "size" while electric dipole has length ($2a$). You can split the latter but not the former. This has the extremely important consequence that whereas electric-field lines start at $+q$ and end at $-q$, magnetic field lines <u>close on themselves</u> there is no beginning and no end.

<u>Current-Current force</u>

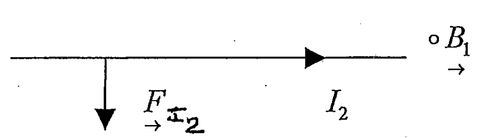




Two wires of lengths l_1, l_2 carry currents I_1, I_2. Separation d along y, wires parallel to x. Force on I_2 due to I_1. To calculate this first. Write $\vec{B}_1$ at location of I_2

$$\vec{B}_I = \frac{\mu_0 I_1}{2\pi d}\hat{z}$$

So I_2 is located in $\vec{B}_1$: $\vec{F}_{I_2}$ looks like

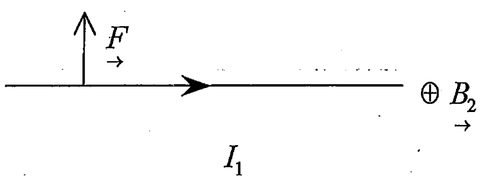


and is given by

$$\vec{F}_{I_2 I_1} = I_2\, \Delta \vec{l}_2 \times \vec{B}_1$$
$$= -\frac{\mu_0 I_1 I_2 l_2}{2\pi d}\hat{y}$$

Force is attractive
Force on I_1 due to I_2

$$\vec{B}_2 = \frac{-\mu_0 I_2}{2\pi d}\hat{z}$$
$$\vec{F}_{I_1 I_2} = I_1\, \Delta \vec{l}_1 \times \vec{B}_2$$
$$= \frac{\mu_0 I_1 I_2 l_1}{2\pi d}\hat{y}$$



Force is attractive. If $l_1 = l_2 = 1\,meter$ the forces/meter $\vec{F} = \frac{-\mu_0 I_1 I_2}{2\pi d}\hat{d}$ are an action-reaction pair. The –sign with $\hat{d}$ ensures force is attractive if I_1 I_2 parallel $\rightarrow$ $\rightarrow$ and repulsive when they are anti-parallel $\rightarrow$ $\leftarrow$ • [You will do an Expt. to check this equation]

Incidentally, this is a very fundamental equation as it is used to define the unit of current- The Ampere. That is,

If

$I_1 = I_2 = 1\,amp$

and $d = 1\,meter$

Force per meter is $2 \times 10^{-7}\,N$ $\quad \left(\frac{\mu_0}{2\pi}\right)$

And the claim is that I should be regarded as a "DIMENSION" in place of Q.
So we can write $L, T, M, \theta, [IT]$
rather than
L, T, M, θ, Q