SOLUTIONS - 9

CHAPTERS - 20, 21

FORMULAE

Faraday - Lenz's law

• $\mathcal{E} = \sum_c \vec{E}_{NC} \cdot \vec{\Delta l} = -\frac{\Delta \phi_B}{\Delta t}$

$\phi_B = \vec{B} \cdot \vec{A} = BA \cos(\hat{n}, \vec{B})$

INDUCTANCE $L = -\frac{\mathcal{E}}{\frac{\Delta i}{\Delta t}}$ $\left(L = \frac{N \Phi_B}{I} \right)$

Self Inductance

Solenoid $L = \mu_0 n^2 V = \frac{\mu_0 N^2 A}{l}$

$\vec{B} = \mu_0 n I \hat{n}$

$u_B = \frac{1}{2} L I^2$

$u_B = \frac{B^2}{2\mu_0}$

Generator

$\mathcal{E} = NBA\omega \cos \omega t$

EMWAVES $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$, $I = \frac{P}{4\pi R^2}$

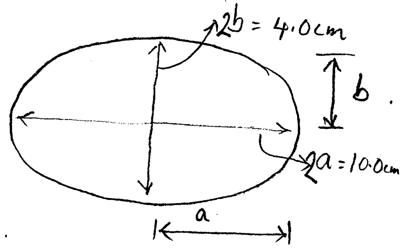
$I = \frac{1}{2} c \varepsilon_0 E_m^2 = \frac{1}{2} \frac{c B_m^2}{\mu_0} = \frac{E_m B_m}{2\mu_0}$

$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$

Solutions - 9

Homework. Chapters 20, 21

20-33



No. of turns in the coil $\Big\} = N = 10.$

Major axis : $2a = 10.0\text{ cm} = 0.1\text{ m}$

Minor axis : $2b = 4.0\text{ cm} = 0.04\text{ m}.$

Speed of rotation of this coil $\Big\} = \omega = 100\text{ rpm} = 100$ rotations per minute.

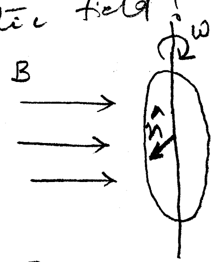
ω in rad/s :   1 rotation $= 2\pi$ radians.

$$\therefore \frac{100 \text{ rotation}}{\text{minute}} = \frac{100 \times 2\pi \text{ radian}}{60 \text{ sec}} = \frac{\cancel{20}^{10}\pi}{\cancel{6}\,3} \text{ rad/s}.$$

$$\boxed{\omega = \frac{10\pi}{3} \text{ rad/s}}$$

Magnitude of Earth's Magnetic field $\Big\} = B = 55\,\mu T = 55 \times 10^{-6}\, T.$

(a) What is the maximum voltage induced in the coil if the axis of rotation is along the MAJOR AXIS and aligned PERPENDICULAR to the Earth's magnetic field?



In this case, the flux through the coil keeps changing, reaching a maximum when the plane of the coil is perpendicular to B field

And, the maximum voltage induced is given by:

$$\boxed{\mathcal{E}_{max} = NBA\omega} \qquad A = \pi a . b = \frac{\pi(2a)(2b)}{4}.$$

$$\therefore \mathcal{E}_{max} = N.B.A\,\omega$$

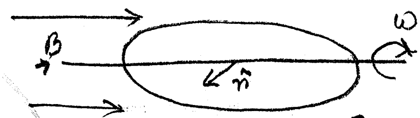
$$= (10)\left(55 \times 10^{-6}\,T\right)\left(\frac{\pi \times (0.1)(0.04)}{4}\right)m^2 \cdot \left(\frac{10\pi}{3}\ rad/s\right)$$

$$\therefore \mathcal{E}_{max} = \frac{55 \times \pi^2}{3} \times (0.1)(10)^2\left(\frac{\overset{0.01}{\cancel{0.04}}}{\cancel{4}}\right) \times 10^{-6}\ V$$

$$\boxed{\mathcal{E}_{max} = 18.1 \times 10^{-6}\ V = 18.1\ \mu V}$$

(b). When the axis of rotation is aligned along the Earth's magnetic field.

In this case, the area of the loop perpendicular to the B field is zero. So, the flux is zero always.



[$\hat{n}$ is $\perp$ to $\vec{B}$ at all times]

So, the max. induced voltage $\mathcal{E}_{max} = 0$.

20.39 Radius of the Solenoid = $r = 2.5\ cm = 0.025\ m$.

No. of turns : $N = 400$ turns.

Length of the solenoid ; $l = 20\ cm = 0.2\ m$.

(a) Inductance ; $L = \frac{\mu_0 . N^2 A}{l}$

$A = \pi . r^2$; $\mu_0 = 4\pi \times 10^{-7}\ T.m/A$

$\therefore L = \frac{(4\pi \times 10^{-7}) . (400)^2 . (\pi \times (0.025)^2)}{0.2}\ H.$

$= \frac{(4\pi)(16)(\pi)(25)^2}{2} \times 10^{-7} \times 10^4 \times 10^{-6} \times 10.$

$L = 0.002\ H$

$\boxed{L = 2mH}$.

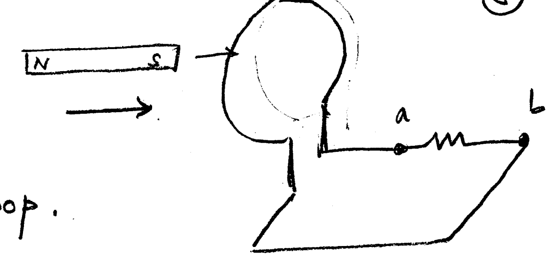
(b) We want to produce an emf of $\mathcal{E} = 75\ mV$.

Given $L = 2mH$, what is $\frac{\Delta I}{\Delta t}$, the rate of change of I?

We know: $\mathcal{E} = -L . \frac{\Delta I}{\Delta t} \Rightarrow \left|\frac{\Delta I}{\Delta t}\right| = +\frac{\mathcal{E}}{L} = \frac{-75\ mV}{2\ mH}$.

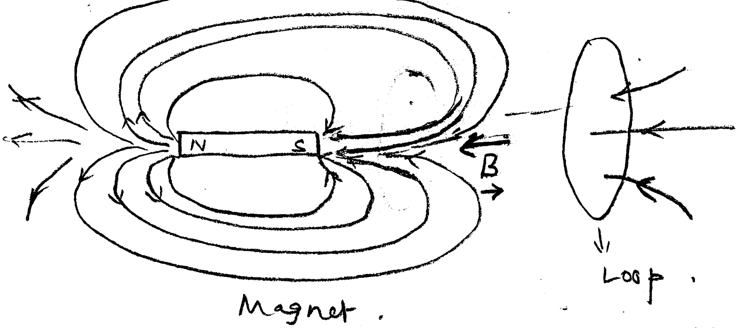
$\therefore \boxed{\frac{\Delta I}{\Delta T} = 37.5\ A/s.}$ Current changes at 37.5 A/s.

20.51. $\mathcal{E} = -\frac{\Delta\phi_B}{\Delta t}$



The South pole of the magnet is being moved toward the loop.

To answer whether $V_a - V_b$ is > 0, < 0 and $= 0$, we first look at the B field lines around a bar magnet.



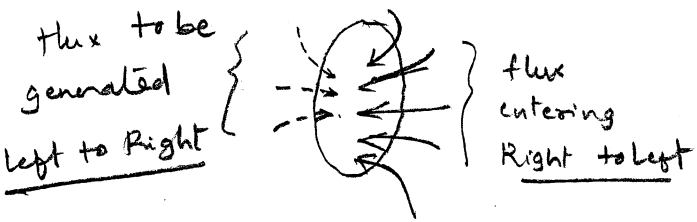
As shown in the figure, the flux through the loop in the direction opposite to the direction of motion of the magnet increases as the south pole approaches the loop. This is because magnetic field lines start at the North pole and end on the South pole.

This increase in flux, according to Faraday's law, induces a current in the loop. But LENZ's law gives the directio of this current.

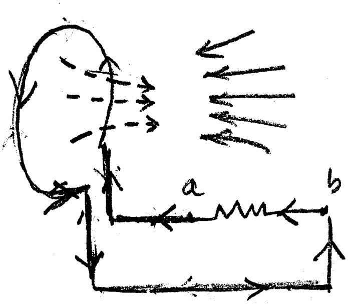
<u>Lenz's Law:</u> The direction of the current induced in a loop because of a changing magnetic flux is such that the

the change in flux is opposed.

In the present case, we apply this law by saying that:
As the flux through the loop from Right to left in the figure is increasing, the current induced in the loop opposes this by producing a magnetic flux going from Left to Right.



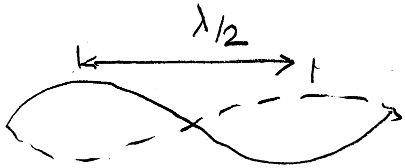
But, by the right hand rule, the current has to be as shown below to produce a flux from Left to Right.



So, current flows from b to a. But current flows from higher potential to lower.

$\therefore\ V_b > V_a \Rightarrow V_b - V_a > 0$ or $\boxed{V_a - V_b < 0}$

21-47



$f = 2.45 \times 10^9$ Hz

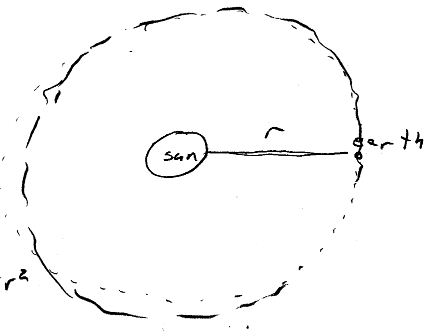
$|r_2 - r_1| = 6$ cm $= \frac{\lambda}{2}$

ANTINODES

$\lambda = 12$ cm

$v = f\lambda = .12\ m \cdot 2.45 \times 10^9\ Hz = 2.94 \times 10^8\ m/s$

1-48



$I = 1340\ W/m^2$

$P = I \cdot A$

Area encountering radiation $= 4\pi r^2$

$r = 1.49 \times 10^{11}\ m$

$P = 1340\ W/m^2 \cdot (1.49 \times 10^{11})^2\ m^2 \cdot 4\pi = 3.73 \times 10^{26}$

1-49

$I = 1340\ w/m^2$

$I = \frac{1}{2} E_{max}^2 \epsilon_0 c$

$E_{max} = \left(\frac{2I}{c\,\epsilon_0}\right)^{1/2} = \left(\frac{2 \times 1340}{3 \times 10^8 \times 9 \times 10^{-12}}\right)^{1/2} = 9.96 \times 10^2\ N/c$

$E_{max} = 1.9.96 \times 10^2\ N/c$

$B = \frac{E}{c} = \frac{9.96 \times 10^2}{3 \times 10^8} = 3.3 \times 10^{-6}\ T$

1-50

$f = 27.33 \text{ MHz} = 27.33 \cdot 10^6 \text{ Hz}$

$v = c$ (EM wave)

$v = f\lambda \qquad \lambda = \frac{v}{f} = \frac{3\times10^8 \text{ m/s}}{27.33\times10^6 \text{ Hz}}$

$\lambda = 11 \text{ m}$

1-52

$v = \frac{x}{t}$ radio waves $v = c$

$t = \frac{x}{v} = \frac{100\times10^3 \text{ m}}{3\times10^8 \text{ m/s}} = 3.3\times10^{-4} \text{ s}$

sound waves $v = 343 \text{ m/s}$

$t = \frac{3.0 \text{ m}}{343 \text{ m/s}} = 8.7\times10^{-3}$

radio listeners receive news first

1-54

$f_o = f_s\left(1 + \frac{u}{c}\right)$

space ship approaching station

$f_o = 6.0\times10^{14} \text{ Hz}\left(1 + \frac{1.8\times10^5 \text{ m/s}}{2.9979\times10^8 \text{ m/s}}\right)$

$= 6.0360\times10^{14} \text{ Hz}$

note: we get the same value approximating c as 3×10^8 m/s. This is an excellent and widely used approx.

21-63.

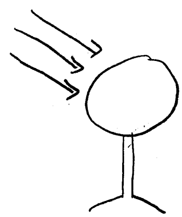
(a) $E_{max} = 0.20 \times 10^{-6}$ V/m

$$B_{max} = \frac{E_{max}}{c} = \frac{.20\times10^{-6}\ \text{V/m}}{3\times10^{8}\ \text{m/s}} = 6.67\times10^{-16}\ \text{T}$$

(b)

$$I = \frac{1}{2}\epsilon_0 c E_{max}^2 = \frac{1}{2}\times 9\times10^{-12}\times 3\times10^{8}\times(0.2)^2\times10^{-12}$$
$$= 5.3\times10^{-17}\ \text{W/m}^2$$

(c) $I = \frac{P}{A}$



Area of dish $= \pi\left(\frac{d}{2}\right)^2 = \pi\cdot 100$

$$P = 5.3\times10^{-17}\ \text{W/m}^2 \cdot 100\pi\ \text{m}^2 = 1.7\times10^{-14}\ \text{W}$$

22-2

$$c = \frac{2d}{t}$$

time for one full revolution $= \frac{1}{27.5}$ s

time between notch and neighboring tooth (same as transit time for one round trip) $= \frac{t_{rev}}{2\cdot\#\text{teeth}}$

$$t = \frac{\frac{1}{27.5}}{2\cdot 360} = 5.05\times10^{-5}$$

$$c = \frac{2\cdot 7500\ \text{m}}{5.05\times10^{-5}} = 2.97\times10^{8}\ \text{m/s}$$