SOLUTIONS — 8

Homework Chapter 19 Magnetism

Formulas

(1) $\vec{F} = \underset{\text{charge}}{q}(\underset{\text{velocity}}{\vec{v}} \times \overset{\text{magnetic field}}{\vec{B}})$ "Lorentz Force" — force on a moving charge due to a $\vec{B}$-field

(2) $\vec{F} = I\vec{l} \times \vec{B}$ force on a current-carrying wire due to $\vec{B}$-field

I: current

$\vec{l}$: length of wire, directed parallel to wire

(3) $\vec{B} = \dfrac{\mu_0 I}{2\pi r}\hat{\phi}$ magnetic field due to a current

r: distance from wire

$\hat{\phi}$: unit vector always tangent to concentric circles around wire. Direction given by Right Hand Rule (RHR)*

* Thumb in direction of current, fingers curl in same sense as the $\vec{B}$-field

(4) $\vec{B} = \mu_0 n I \hat{n}$ magnetic field due to a solenoid. $\hat{n}$ directed down solenoid axis consistent with RHR‡

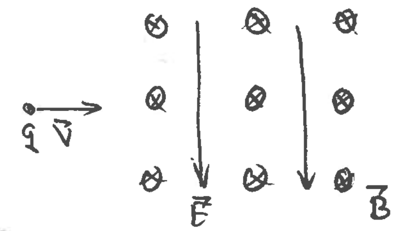
‡ Fingers curl in direction of current thumb points parallel to $\vec{B}$-field

(5) $R = \dfrac{mv}{qB}$ cyclotron radius

19-29

Total force on the particle is given by (1) + the force due to the electric field.

$$\vec{F}_T = q\vec{E} + q(\vec{v}\times\vec{B})$$



Velocity Selector!

We are looking for situation when $\vec{F}_T = 0$. This occurs if

$$\vec{E} = -\vec{v}\times\vec{B}.$$

$\vec{v}$ is perpendicular to $\vec{B}$ and by the RHR we see $\vec{v}\times\vec{B}$ is directed opposite to $\vec{E}$. This tells us which gives the

$$\boxed{E = vB}$$

condition on E, v, and B.

19-33

For an elastic collision we know that energy and momentum are conserved.

Momentum

$$M_\alpha \vec{V}_{\alpha i} = M_\alpha \vec{V}_{\alpha f} + m_p \vec{V}_{p,f}$$

initial final

Energy (velocities when particles are separated by large distance)

$$\tfrac{1}{2} M_\alpha V_{\alpha i}^2 = \tfrac{1}{2} M_\alpha V_{\alpha f}^2 + \tfrac{1}{2} m_p V_{pf}^2 \qquad (a)$$

momentum components

(x-comp) $$M_\alpha V_{\alpha i x} = M_\alpha V_{\alpha f x} + m_p V_{pfx} \qquad (b)$$

component

(y-comp) $$0 = M_\alpha V_{\alpha f y} + m_p V_{pfy} \qquad (c)$$

Because the collision is head-on we ignore components of velocity perpendicular to ray connecting two particles before they collide.

v = initial velocity of α
V_α = final velocity of α
V_p = final velocity of proton.

Our equations take a simplified form.

$m_\alpha = 4m_p$
$m_p = m$

(a) energy: $2mv^2 = 2mV_\alpha^2 + \frac{1}{2}mV_p^2 \Rightarrow 4v^2 = 4V_\alpha^2 + V_p^2$

(b) momentum: $4mv = 4mV_\alpha + mV_p \Rightarrow 4v = 4V_\alpha + V_p$

We don't care about finding the exact velocities, only about the relation between V_p and V_α. We can eliminate v from these equations.

$$4V_\alpha^2 + V_p^2 = \frac{1}{4}(4V_\alpha + V_p)^2$$

$$4V_\alpha^2 + V_p^2 = 4V_\alpha^2 + \frac{1}{4}V_p^2 + 2V_\alpha V_p$$

$$\frac{3}{4}V_p^2 = 2V_\alpha V_p \Rightarrow \boxed{\frac{3}{4}V_p = 2V_\alpha}$$

$$\boxed{V_\alpha = \frac{3}{8}V_p}$$

We know cyclotron radius of the proton

$$\boxed{R = \frac{m_p V_p}{eB}}$$

For the α-particle it will be:

$$\boxed{R_\alpha = \frac{M_\alpha V_\alpha}{2eB} = \frac{(4m_p)(\frac{3}{8}V_p)}{2eB} = \frac{3}{4}\frac{m_p V_p}{eB} = \frac{3}{4}R}$$

19-37 we know $\vec{B}$-field from a wire. The magntitude is given by (3).

$$B = \frac{\mu_0 I}{2\pi r}$$

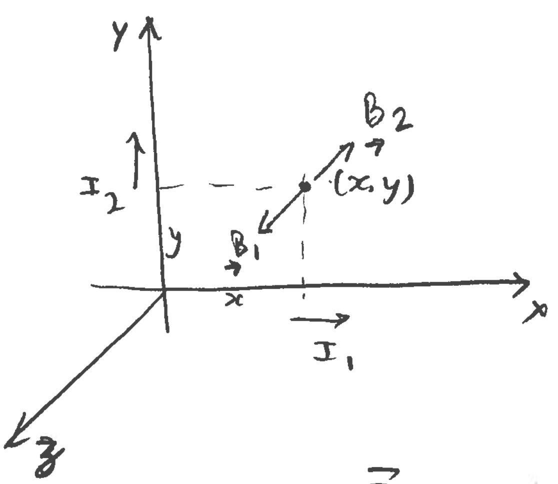
solve for distance: $r = \dfrac{\mu_0 I}{2\pi B}$

$I = 20A \qquad B = 1.7mT \qquad \mu_0 = 4\pi \times 10^{-7}\,\frac{T.m}{A}$

$$r = \frac{4\pi(20)\times 10^{-7}}{2\pi(1.7)\times 10^{-3}}\,\frac{T.m}{A}\cdot A\,\frac{1}{T}$$

$$\boxed{r = \frac{4}{1.7}\times 10^{-3}\,m = 2.35\times 10^{-3}\,m}$$

19-41



what is $\vec{B}$-field at (x, y)?

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

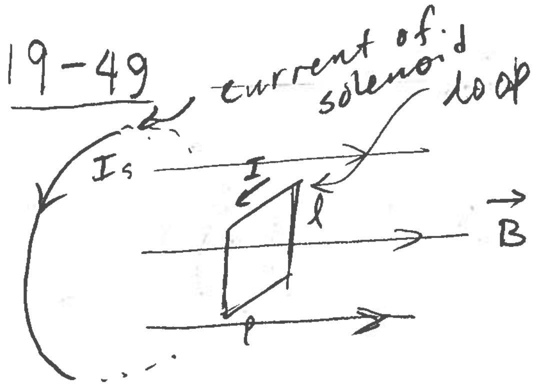
$$\left.\begin{aligned} \vec{B}_1(x,y) &= \frac{\mu_0 I_1}{2\pi y}\hat{z} \\ \vec{B}_2(x,y) &= -\frac{\mu_0 I_2}{2\pi x}\hat{z} \end{aligned}\right\} \text{Direction from RHR}$$

$$\vec{B}(x,y) = \frac{\mu_0}{2\pi}\left(\frac{I_1}{y} - \frac{I_2}{x}\right)\hat{z}$$

$\vec{B}(x,y) = 2\times10^{-7}\left(\frac{7}{3} - \frac{6}{4}\right) \frac{T.m}{A} \cdot \frac{A}{m}$

$$\boxed{\vec{B} = \frac{2(28-18)}{12} \times 10^{-7}\,T = 1.66 \times 10^{-7}\,T}$$

19-49



square loop inside a solenoid.

$\vec{B}$-field inside a solenoid is uniform.

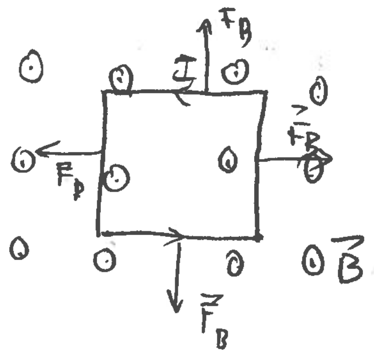
$\vec{B} = \mu_0 n I \hat{n}$

$\hat{n}$ is perpendicular to plane of loop.

Force on each side of the loop.

$\vec{F}_B = I\vec{l} \times \vec{B} = IlB\hat{r}_i$

The force on each segment works to pull the loop apart.



It's magnitude is

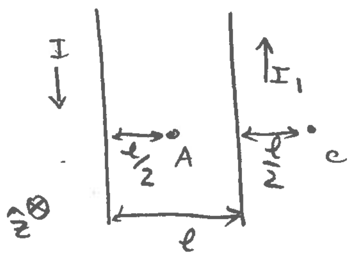
$F_B = IlB$

$= \mu_0 I l I_s n$

$F_B = 4\pi\,(2cm)(2A)(15A)(30\,/cm) \times 10^{-7} \frac{T.m}{A}$

$$\boxed{F_B \cong 9\times10^{-3}\,T.mA \simeq 9\times10^{-3}\,N}$$

First find field at c $\boxed{\text{use (3)}}$



$$\vec{B}(c) - \frac{\mu_0 I_1}{2\pi(\ell/2)}\hat{z} + \frac{\mu_0 I}{2\pi(3\ell/2)}\hat{z} = 0$$

$$\Rightarrow \quad \frac{2}{3}I - 2I_1 = 0 \Rightarrow \boxed{I = 3I_1}$$

Now find field at A

$$\vec{B}(A) = \frac{\mu_0 I_1}{2\pi(\ell/2)}\hat{z} - \frac{\mu_0 I}{2\pi(\ell/2)}\hat{z} = -\frac{\mu_0}{2\pi\ell}4I_1\,\hat{z}$$

$$\boxed{\vec{B}(A) = -\hat{z}\left[\underbrace{2\times10^{-7}\times 4\times 10\,\frac{1}{.05}}_{\frac{80\times 100}{5}\times10^{-7}}\right]\frac{T\cdot m}{A}\cdot\frac{A}{m} = -1.6\times10^{-4}\,T}$$

Chapter 20

Formulas

number of loops — mag. flux

(i) $\mathcal{E} = -N\frac{\Delta\Phi_B}{\Delta t}$ "Faraday's law" - change in magnetic flux through a surface with time is equal to the voltage around the boundary of that surface

Voltage — Lenz's contribution.

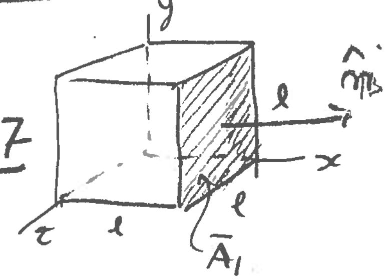
(ii) $\phi_B = \sum_S \vec{B}\cdot\Delta\vec{A}_i$ magnetic flux through surface S.

(iii) $\mathcal{E} = NAB\omega\sin\omega t$ This is the voltage induced around a coil with N turns, Area A, that is rotating with angular velocity ω. There must be a time charging magnetic flux.

More fundamental form of Field EQN. If flux of B field varies with time it generates a NON-COULOMB $\vec{E}$ field in every loop surrounding region where ϕ_B is changing. Hence lines of $\vec{E}_{NC}$ circulate around $\frac{\Delta\phi_B}{\Delta t}$. Therefore circulation of $\vec{E}_{NC}$ around a closed loop is given by $-\frac{\Delta\phi_B}{\Delta t}$ $\boxed{\sum_C(\vec{E}_{NC}\cdot\Delta\vec{\ell}) = -\frac{\Delta\phi_B}{\Delta t}}$ (Cont'd)

The minus sign on R.H.S. is crucial. It ensures that sense of $\vec{E}_{NC}$ is such as to oppose change in Φ_B

20-7



$\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$

compute flux of $\vec{B}$ through shaded face.

$\phi_B = \sum_s \vec{B}\cdot\Delta\vec{A} = \vec{B}\cdot\vec{A}_1$

$\vec{A}_1 = \hat{x}\,l^2$

$\phi_B = l^2\hat{x}\cdot(B_x\hat{x} + B_y\hat{y} + B_z\hat{z}) = l^2 B_x$

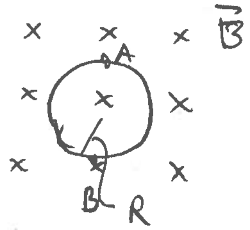
$l = 2.5\,cm \quad B_x = 5.0\,T$

$\boxed{\phi_B = 3.125\times10^{-3}\,Tm^2}$

What is total flux through cube's surface?

$\underline{\underline{0}}$ → no magnetic monopoles!

20-10



Loop is stretched from points A and B until it has zero area. This takes time T_0, what is the average induced emf around loop?

use (i)

$\Delta\phi_B = \phi_{B,i} - 0 = \pi R^2 B$

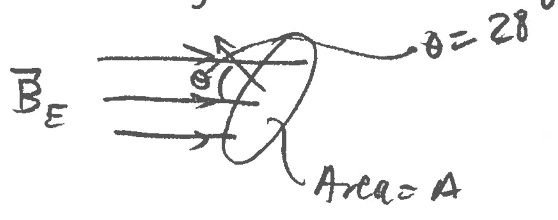
$\Delta t = T_0, \; N = 1$

Therefore, $\mathcal{E} = -\dfrac{\pi R^2 B}{T_0} = -\dfrac{\pi(12cm)^2(0.15T)}{0.2\,s}$

$$\boxed{\mathcal{E} = -\frac{\pi(1.44)(0.15)}{2}10^3\times10^{-4}\,\frac{T.m^2}{s} = 3.4\times10^{-2}\,V}$$

20-17 To find emf we use (i).

Flux through loop



$\phi_B = A(t)\,B_E\cos(28^\circ)$

$\phi_B = \vec{A}(t)\cdot\vec{B}_E$

$$\mathcal{E} = -N \frac{\Delta\phi_B}{\Delta t}$$

$$\Delta\phi_B = (A_f - A_i) B_E \cos(28^\circ)$$

$28^\circ = 0.24$ rads

$N = 200$, $B_E = 50.0 \mu T$

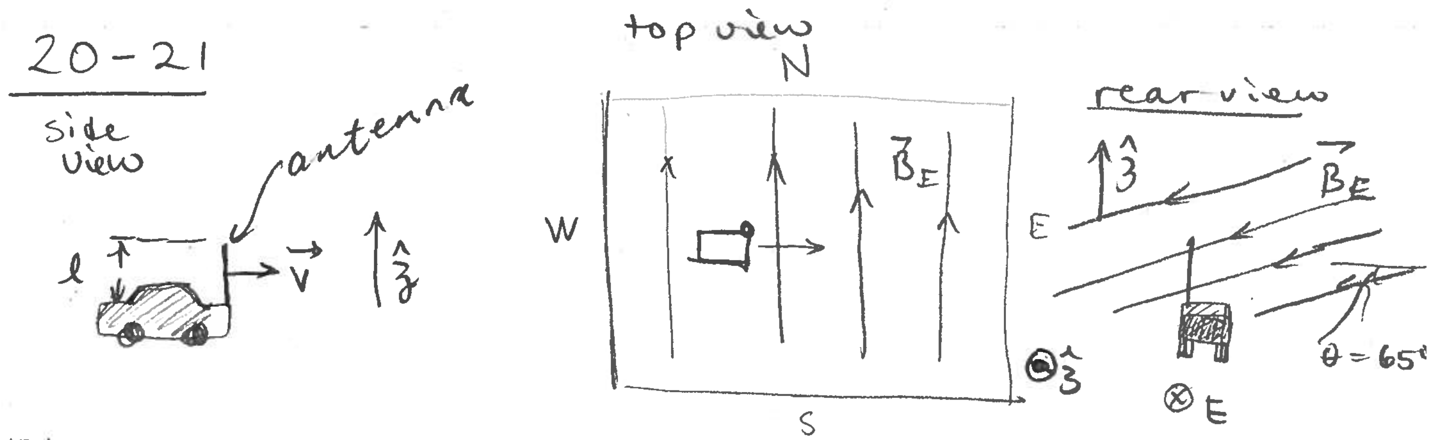
$A_f - A_i = 39.0 \text{ cm}^2$

$\Delta t = 1.8 s$

$$\mathcal{E} = -\frac{200}{1.8}(50)\cos(28^\circ)\, 39 \times 10^{-4} \text{ V}$$

$$\boxed{\mathcal{E} = -21.60 \text{ V}}$$

20-21



The car should travel East to maximize ~~maximize~~ induced voltage in the desired way.

$\vec{F} = -e(\vec{v} \times \vec{B})$ is force experienced by electrons in antenna. This reaches a maximum when $\vec{v} \perp \vec{B} \Rightarrow \vec{F} = \pm e v B \hat{3}$, but we want electrons to move to the bottom of the antenna so

$\vec{F} = -evB\hat{3}$ this occurs if $\vec{v}$ is directed <u>eastward</u>.

the magnitude of the induced emf can be computed 2 ways. First we use Faraday's Law.

north

$$\Delta\phi_B = (A_f - A_i)B_E \cos(65^\circ)$$

$$A_f - A_i = l v \Delta t$$

$$\frac{\Delta\phi_B}{\Delta t} = l v \Rightarrow \boxed{\mathcal{E} = l v B_E \cos(65^\circ)}$$

OR we can compute Force. (MOTIONAL EMF)

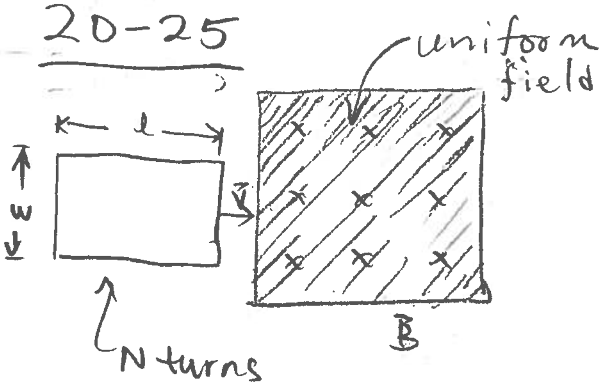
$$\vec{F} = q\vec{v}\times\vec{B} \Rightarrow q v B_E \cos(65^\circ) = F$$

$F = qE$ and $\mathcal{E} = El$ ← for a uniform field

$$\therefore \boxed{\mathcal{E} = \frac{lF}{q} = B_E v l \cos(65^\circ)}$$

$$\boxed{\begin{aligned}\mathcal{E} &= (1.2\,m)(65\,km/h)(50\,\mu T)\cos(65^\circ) \\ &= \frac{(1.2)(6.5)(5)}{3.6}\frac{\times 10^3 \times 10^2 \times 10^{-6}}{\times 10^3}\cos(65^\circ) = 4.58\times 10^{-4}\,V\end{aligned}}$$

20-25

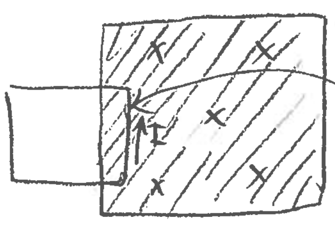


when coil enters field

$$\mathcal{E} = -N\frac{\Delta\phi_B}{\Delta t} = -NBvw$$

this will induce a current that opposes flux change (Lenz' Law). Current will go up.

$$I = \frac{|\mathcal{E}|}{R} = \frac{NBvw}{R}$$



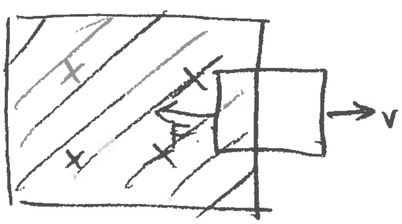
Force on this segment is only one that opposes motion.

There will be an opposing force on this coil due to this current interacting with the magnetic field.

$$\boxed{F = I\omega B = \frac{N v \omega^2 B^2}{R}}$$

Once the coil is completely inside region with $\vec{B}$-field the force vanishes because there is no longer a change in magnetic flux.

Finally, when coil begin to leave there is an opposing force (Lenz Law). The flux begins to decrease which induces a current that interacts with the magnetic field to oppose it's motion.



By symmetry
$$\boxed{F = \frac{N v \omega^2 B^2}{R}}$$

20-30

From (iii) we know

$\mathcal{E} = BNA\omega \sin\omega t$

$f = \frac{1500 \text{ rev}}{\text{min}} = \frac{1500}{60} Hz$

$\omega = 2\pi \frac{150}{6} Hz$

$$\mathcal{E} = 100(50\times10^{-6}T)(0.04 m^2) 2\pi \frac{(150)}{6} \frac{1}{s} \sin\omega t$$

$$\boxed{\mathcal{E}_{max} = \frac{5\times4\times2\pi\times1.5}{6} \times 10^{2+1-6-\cancel{2}+2} \frac{Tm^2}{s} \simeq 1\pi \times 10^{-2} V}$$