SOLUTIONS – 5 – FORMULAE

Battery $\mathcal{E}$ Capacitor $C = \frac{Q}{V}$

Parallel Plate (air filled) $C_0 = \frac{\varepsilon_0 A}{d}$ $\vec{E}_0 = \frac{\sigma}{\varepsilon_0}\hat{x}$

Parallel Plate (with dielectric) $C_k = k\frac{\varepsilon_0 A}{d}$, $\vec{E}_k = \left(\frac{\sigma - \sigma_b}{\varepsilon_0}\right)\hat{x}$

$E_k = \frac{E_0}{k}$

dielectric Const. $k = \frac{\sigma}{\sigma - \sigma_b}$

Capacitors in series: Q's are common
V's add

Hence $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$ or $\frac{1}{C_s} = \Sigma\left(\frac{1}{C_i}\right)$

Capacitors in parallel: V's are common
Q's add

Hence $C_p = C_1 + C_2 + C_3$ or $C_p = \Sigma C_i$

Energy stored in Capacitor — Resides in $\underline{E}$-field.

Air filled $P_E = \frac{Q^2}{2C_0} = \frac{1}{2} C_0 V^2$ hence $\eta_E = \frac{1}{2}\varepsilon_0 E_0^2$

With Dielectric $P_E = \frac{Q^2}{2C_k}$ hence $\eta_E = \frac{1}{2}\varepsilon_0 k E_k^2$

η is energy per m^3 of $\vec{E}$ field

FORMULAE - CHAP 17

Current $I = \dfrac{\Delta Q}{\Delta t}$

Note: electrons carry -ive charge so electrons moving left yield current going right

$$\overleftarrow{\quad e^{-} \quad} \qquad \longrightarrow I$$

$$I = \vec{J} \cdot \vec{A}$$

$$I = n_e e A v_D$$

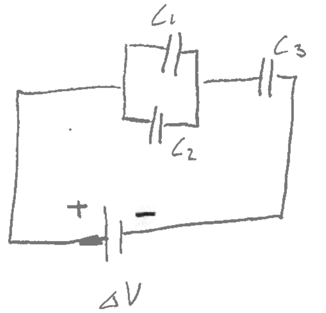
Definition of $R = (V/I)$

$$V = IR \qquad R = \frac{\rho \ell}{A} = \frac{\ell}{\sigma A}$$

$$V = \frac{\ell J A}{\sigma A}$$

$$J = \sigma \frac{V}{\ell} \qquad \vec{J} = \sigma \vec{E}$$

16-31 (a)



$C_1 = 4.00\ \mu F$

$C_2 = 2.00\ \mu F$

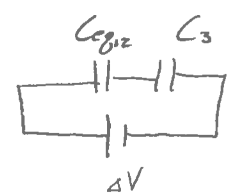
$C_3 = 3.00\ \mu F$

$\Delta V = 12\ V$

C_1 and C_2 are in parallel

$C_{eq_{12}} = C_1 + C_2 = 6.00\ \mu F$

$C_{eq_{12}}$ and C_3 are in series



$$\frac{1}{C_{eq_{123}}} = \frac{1}{C_{eq_{12}}} + \frac{1}{C_3} = \frac{1}{2.00\mu F} \qquad C_{eq_{123}} = 2.00\ \mu F$$

(b) $C_{eq_{12}} = \dfrac{Q_{12}}{\Delta V_{12}}$ $\qquad Q_{12} = Q_3$ for capacitors in series

$C_3 = \dfrac{Q_3}{\Delta V_3}$ $\qquad \Delta V_{12} + \Delta V_3 = \Delta V$

solve capacitance equations for respective ΔV and add

$$\Rightarrow \frac{Q_3}{C_3} + \frac{Q_{12}}{C_{eq_{12}}} = \Delta V \qquad Q_3 = Q_{12} = \frac{C_3 C_{eq_{12}}}{C_3 + C_{eq_{12}}} \Delta V = 24.0\ \mu C$$

$$\Delta V_3 = \frac{Q_3}{C_3} = 8.00\ V$$

$C_1 = \dfrac{Q_1}{\Delta V_1}$ $\qquad \Delta V_1 = \Delta V_2$ capacitors in parallel

$C_2 = \dfrac{Q_2}{\Delta V_2}$ $\qquad Q_1 + Q_2 = Q_{12}$

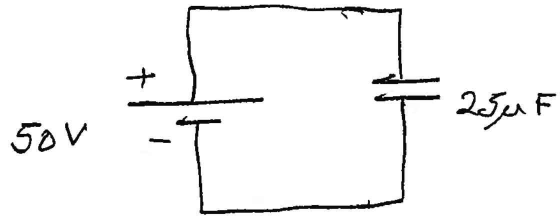
$$C_1 \Delta V_1 + C_2 \Delta V_2 = Q_{12} \qquad \Delta V_1 = \Delta V_2 = \frac{Q_{12}}{C_1 + C_2} = 4.00\ V$$

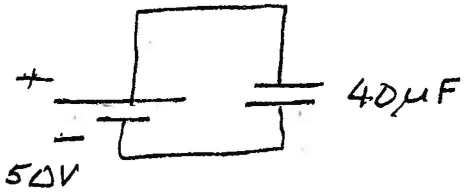
$Q_1 = C_1 \Delta V_1 = 16.0\ \mu C$

$Q_2 = C_2 \Delta V_2 = 8.00\ \mu C$

16-37 FIRST CALCULATE CHARGE ON 25 μF and 40 μF

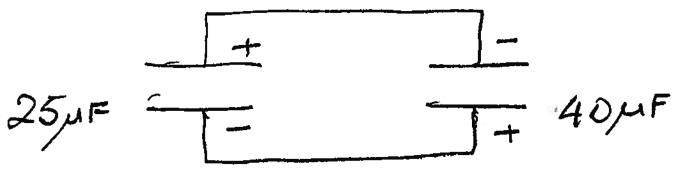
$$Q = CV$$





$$Q_{25} = (50 \times 25 \times 10^{-6})\,C = 1.25 \times 10^{-3}\ C$$

$$Q_{40} = (50 \times 40 \times 10^{-6})\,C = 2.00 \times 10^{-3}\,C.$$

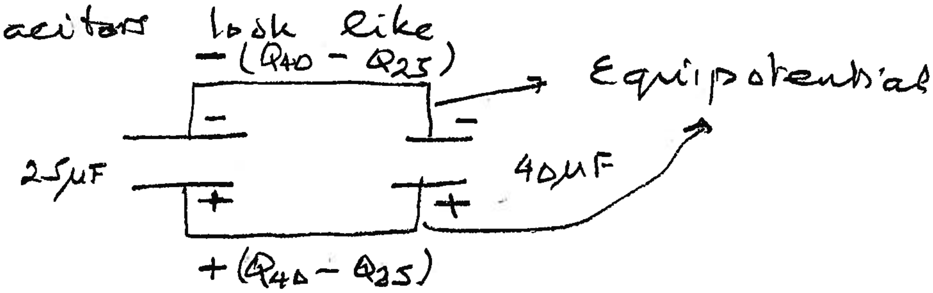


Note: + to −
− to +.

Charge is conserved so Total charge now is

$$Q_{40} - Q_{25} = (2 \times 10^{-3} - 1.25 \times 10^{-3})\,C = 7.5 \times 10^{-4}\,C = 750\,\mu C$$

and the capacitors look like



Because conductor is Equipotential $V_{25} = V_{40}$

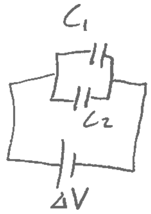
$$\frac{Q_{25}}{25 \times 10^{-6}} = \frac{Q_{40}}{40 \times 10^{-6}} = V_{25} = V_{40}$$

$$\frac{7.5 \times 10^{-4} - Q_{40}}{25 \times 10^{-6}} = \frac{Q_{40}}{40 \times 10^{-6}} \qquad Q_{40} = 462\,\mu C.$$

$$V_{40} = \frac{Q_{40}}{40} = 11.5V.$$

$$Q_{25} = 750 - 462\,\mu C = 288\,\mu C,$$

16 44



$C_1 = 25.0\ \mu F$

$C_2 = 5.00\ \mu F$

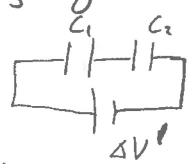
$\Delta V = 100\ V$

(a) $E = \frac{1}{2} C (\Delta V)^2$

we want energy stored in both capacitors, so use equivalent capacitance

$C_1 + C_2 = C_{eq} = 30.0\ \mu F = 30.0 \times 10^{-6}\ F$

$E = \frac{1}{2}(30.0 \times 10^{-6} F)(100\ V)^2 = 0.15\ J$



(b) $E = \frac{1}{2} C (\Delta V)^2$

now E_{tot} is known $(.15\ J)$ and we want the equivalent capacitence in series

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{25.0\mu F} + \frac{1}{5.00\mu F} = \frac{6}{25.0\ \mu F}$

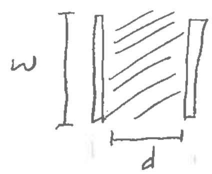
$C_{eq} = \frac{25.0\ \mu F}{6} \approx 4.17\ \mu F = 4.17 \times 10^{-6}\ F$

solve for ΔV

$\sqrt{\frac{2E}{C}} = (\Delta V) = \sqrt{\frac{2(.15)\ J}{4.17 \times 10^{-6}\ F}} \approx 268\ V$

16-50

$C = K \varepsilon_0 \dfrac{A}{d}$ plate capacitor with dielectric



$K = 3.70$ $\qquad C = 9.50 \times 10^{-8}$ F

$d = .0250$ mm

$w = 7.00$ cm

"width · length = Area of each plate

$w \cdot L = A$

$C = K \varepsilon_0 \dfrac{w \cdot L}{d} \Rightarrow L = \dfrac{C\, d}{K \varepsilon_0 w}$

$$L = \frac{(9.50 \times 10^{-8} F)(.0250 \times 10^{-3} m)}{(3.70)(8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2})(.07 m)} \approx 1.04 \text{ m}$$

16-54 $\quad C_p = C_1 + C_2 \rightarrow (1)$

$\dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$, $\quad C_s = \dfrac{C_1 C_2}{C_1 + C_2} \rightarrow (2)$

Multiply (1) and (2)

$C_s C_p = C_1 C_2$, $\quad C_1 = \dfrac{C_s C_p}{C_2}$

substitute in (1) $\quad C_p = \dfrac{C_s C_p}{C_2} + C_2$.

or $\quad C_p C_2 = C_s C_p + C_2^2$

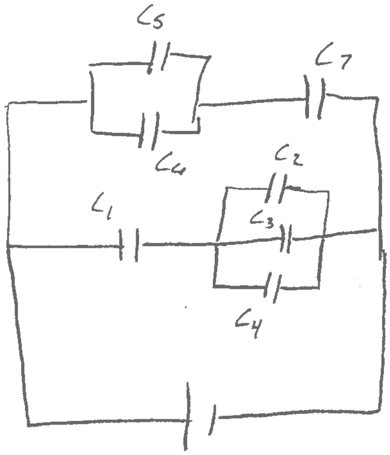
$C_2^2 - C_p C_2 + C_s C_p = 0$

Solve quadratic $\quad C_2 = \dfrac{C_p \pm \sqrt{C_p^2 - 4 C_s C_p}}{2}$

$C_1 = C_p - C_2 = \dfrac{2C_p - C_p \mp \sqrt{C_p^2 - 4 C_s C_p}}{2}$

$= \dfrac{C_p \mp \sqrt{C_p^2 - 4 C_s C_p}}{2}$.

16-57.



$C_1 = 6.00\ \mu F$

$C_2 = 2.00\ \mu F$

$C_3 = 3.00\ \mu F$

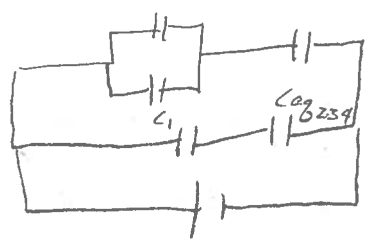
$C_4 = 7.00\ \mu F$

$C_5 = 5.00\ \mu F$

$C_6 = 4.00\ \mu F$

$C_7 = 3.00\ \mu F$

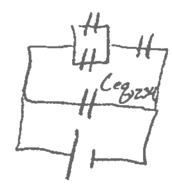
This problem involves finding many equivalent capacitors. Start with the first parallel set

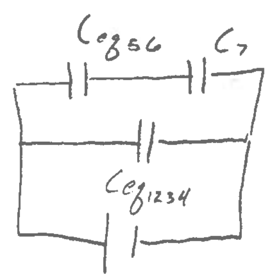


$C_{eq_{234}} = C_2 + C_3 + C_4 = 12.0\ \mu F$

$$\frac{1}{C_{eq_{1234}}} = \frac{1}{C_1} + \frac{1}{C_{eq_{234}}} = \frac{1}{4.00\ \mu F}$$

$C_{eq_{1234}} = 4.00\ \mu F$

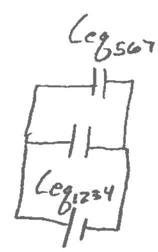




$C_{eq_{56}} = C_5 + C_6 = 9.00\ \mu F$

$$\frac{1}{C_{eq_{567}}} = \frac{1}{C_{eq_{56}}} + \frac{1}{C_7} = \frac{4}{9.00\ \mu F}$$

$C_{eq_{567}} = 2.25\ \mu F$



total equivalent capacitance $C_{eq} = C_{eq_{567}} + C_{eq_{1234}} = 6.25\ \mu F$

Ch. — 17

17-1 $I = \frac{\Delta Q}{\Delta t}$ total charge ΔQ = charge of electrons · number of electrons

let n = number of electrons, e = charge of electron

$$I = \frac{en}{\Delta t} \Rightarrow n = \frac{I\Delta t}{e} = \frac{(80\times10^{-3}\ A)(600\ s)}{1.6\times10^{-19}\ C} = 3\times10^{20} \text{ electr}$$

current $\longrightarrow$

$\leftarrow v_e$ ~ electron velocity

electrons move opposite current direction

17-5. $\Delta Q = e \cdot 1$ one electron

Δt can be found from distance and velocity

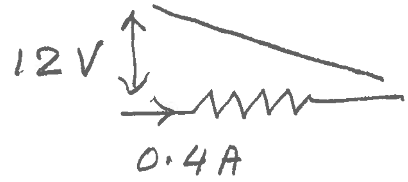
distance traveled $= 2\pi r$ (electron orbit)

$$2\pi r = v\Delta t \Rightarrow \Delta t = \frac{2\pi r}{v} = \frac{2\pi\ 5.29\times10^{-11}\ m}{2.19\times10^{6}\ m/s} = 1.52\times10^{-16}$$

$$I = \frac{\Delta Q}{\Delta t} = \frac{1.60\times10^{-19}}{1.52\times10^{-16}} \approx 1.05\ mA$$

17-15

(a) $\Delta V = IR$



$$R = \frac{\Delta V}{I} = \frac{12\,V}{.40\,A} = 30\ \Omega$$

(b)

$$R = \rho \frac{l}{A} \qquad \rho \equiv \text{resistivity}$$

$$A = \pi r^2 = \pi(.004m)^2 = 5.02 \times 10^{-6}\ m^2$$

$$l = 3.2\ m$$

$$\rho = \frac{RA}{l} = \frac{30\ \Omega \quad 5.02\times10^{-6}\ m^2}{3.2\ m} = 4.7\times10^{-4}\ \Omega m$$

17-19. We need to know the resistivity of the wire to see how a changing length will effect the resistance

$$\rho = \frac{RA}{l} = \frac{1.0\ \Omega \quad \pi r_0^2}{L_0}$$

now wire is stretched so that $r = 0.25\, r_0$

How did the length change?

we had $r_0^2 L_0$ cubic meters of material, the amount of materiel cannot change during stretching, so $r^2 L = r_0^2 L_0$

$$\Rightarrow L = \frac{r_0^2}{r^2} L_0$$

$$R = \rho \frac{L}{\pi r^2} = \frac{1.0\ \Omega \quad \cancel{\pi} r_0^2}{\cancel{L_0}} \cdot \frac{r_0^2}{r^2} \cancel{L_0} \cdot \frac{1}{\cancel{\pi} r^2} = \frac{1.0\ \Omega \quad r_0^4}{(.25\, r_0)^4} = 256\ \Omega$$