SOLUTIONS – 4

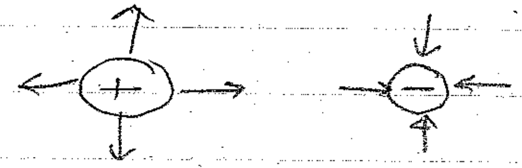
FORMULAE (Chap 15)

$$\underset{\rightarrow}{F_E} = \frac{k_e q_1 q_2}{r^2}\hat{r}$$

$$= \frac{q_1 q_2}{4\pi \varepsilon_0 r^2}\hat{r}$$

$k_e = 9 \times 10^{9} \frac{N\text{-}m^2}{C^2}$ $\varepsilon_0 = 9 \times 10^{-12}\ F/m$

Coulomb

$\underset{\rightarrow}{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$ 

$\underset{\rightarrow}{F_E} = q\,\underset{\rightarrow}{E}$

FLUX $\Delta\phi_E = \underset{\rightarrow}{E}\cdot\underset{\rightarrow}{\Delta A} = E\,\Delta A\cos(\hat{n}, \underset{\rightarrow}{E})$

Gauss' Law $\Sigma_c\, \underset{\rightarrow}{E}\cdot\underset{\rightarrow}{\Delta A} = \frac{1}{\varepsilon_0}\Sigma Q_i$

Total flux of Coulomb $\underset{\rightarrow}{E}$ through a closed surface is determined solely by the enclosed charges

FORMULAE (Chap 16) $\underset{\rightarrow}{F_E}$ IS A CONSERVATIVE FORCE SO POTENTIAL ENERGY IS DEFINABLE

$\Delta P_E = -\underset{\rightarrow}{F_E}\cdot\underset{\rightarrow}{\Delta S}$

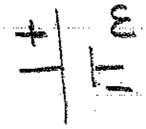
Potential $\Delta V = -\underset{\rightarrow}{E}\cdot\underset{\rightarrow}{\Delta S}$

conservation of Energy equation

$$K_f + (P_G + P_{sp} + P_E)_f = K_i + (P_G + P_{sp} + P_E)_i + W_{NCF}$$

Point charge $V(r) = \frac{q}{4\pi\varepsilon_0 r}$

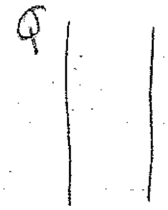
Battery: Device to generate Coulomb $\underline{E}$ field using chemical energy



Output is in volts

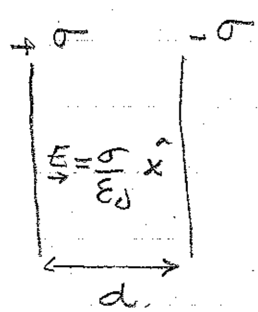
<u>Capacitor</u> Device to "store" an $\vec{E}$ field

Parallel plate (air filled)



area A

$\sigma = \frac{Q}{A}$



Potential Difference

$$V = \frac{\sigma}{\varepsilon_0} d$$

Capacitance $\quad C = \frac{Q}{V} = \frac{\sigma A}{(\sigma/\varepsilon_0) d} = \frac{\varepsilon_0 A}{d}$

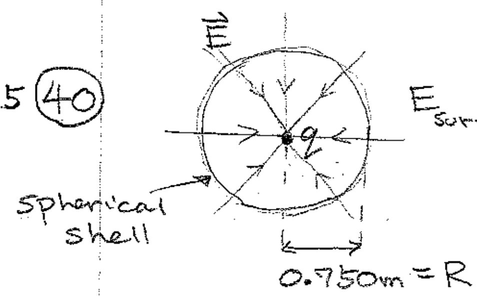
Week 4 Homework Solutions

Phys 122, Fall 2008

Ch. 15 : 40, 43, 45, 50, 57

Ch. 16 : 3, 9, 11, 15, 19, 27, 31, 37

Ch. 15 (40)



$E_{surface} = 890\ N/C$, directed inward

- Since the electric field points radially inward, and electric fields go from positive to negative, there must be a negative charge distribution at the sphere's center.

(a)

- $E = \frac{kq}{r^2} \xrightarrow{\text{When } r=R} E = \frac{k|q|}{R^2}$

$$\left(\frac{R^2}{k}\right)E = \frac{k|q|}{R^2}\left(\frac{R^2}{k}\right)$$

$$|q| = \frac{R^2}{k}E = \frac{(0.75\,m)^2(890\,N/C)}{(8.99\times10^9\,N\cdot m^2/C^2)}$$

$$\boxed{|q| \cong 5.57\times10^{-8}\,C = 55.7\,nC}$$

$$\therefore \boxed{q \cong -55.7\,nC}$$

(b) Since the field points radially inward everywhere, the charge distribution inside the sphere must be spherically symmetric (such as a point charge).

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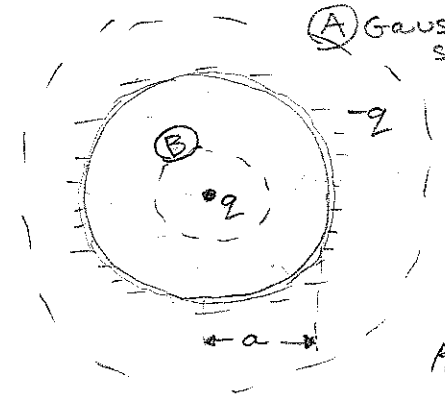


(a) Find $\vec{E}$ outside the shell.

(b) Find $\vec{E}$ for a point inside at distance r from the center of shell, a

FIELD HAS SPHERICAL SYMMETRY AROUND CENTER OF SHELL SO $\vec{E}$ CAN BE A FUNCTION OF r only and be along $\hat{r}$

(a) Use Gaussian surface A:

$$\sum_c \vec{E}\cdot\Delta\vec{A} = \Phi = \frac{Q_{IN}}{\epsilon_0} = \frac{(q+(-q))}{\epsilon_0} = 0$$

$$\Phi = EA\cos\theta = EA\underbrace{\cos(0^\circ)}_{1}$$

$$= E(4\pi a^2) = 0$$

$$\therefore \boxed{E = 0}$$

Note: Σ_c denotes SUM OVER CLOSED SURFACE!

(b) Use Gaussian surface B:

$$\sum \vec{E}\cdot\Delta\vec{A} = \Phi = \frac{Q_{IN}}{\epsilon_0} = \frac{q}{\epsilon_0}$$

$$\Phi = EA\cos\theta = EA\underbrace{\cos(0^\circ)}_{1}$$

$$= E(4\pi r^2)$$

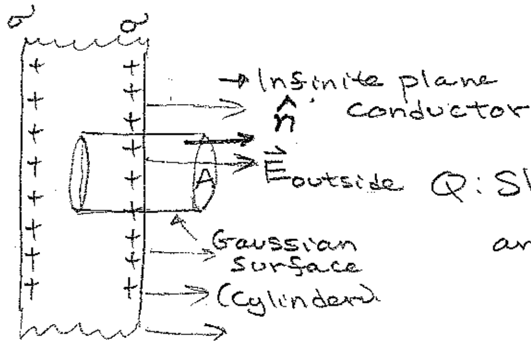
$$\therefore 4\pi r^2 E = \frac{q}{\epsilon_0} \rightarrow E = \frac{q}{4\pi\epsilon_0 r^2} \quad \text{since } \frac{1}{4\pi\epsilon_0} = k$$

$$\boxed{E = \frac{kq}{r^2}}$$

→ E is directed radially <u>outward</u> because q is positive ($\vec{E}$ goes from positive to negative

σ = charge per unit area.

(45)



Q: Show that $E = \sigma/\varepsilon_0$ at any point outside the conductor.

$\underset{\rightarrow}{E} \parallel \hat{n}$

UNDER STATIONARY CONDITIONS

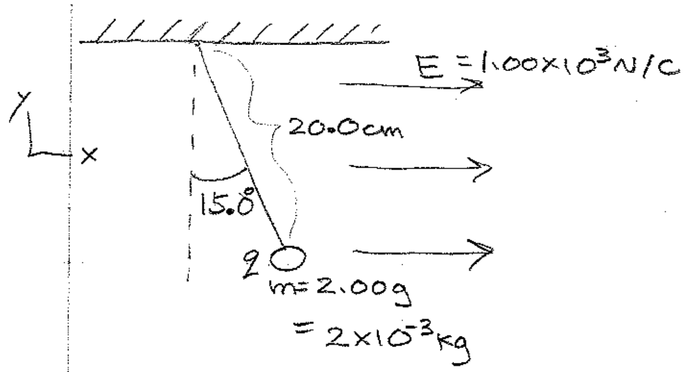
- $E = 0$ inside a conductor.
- $\vec{E}_{outside}$ points to the right. Since $\Delta\Phi = EA\cos\theta$, the only contribution to Φ comes from the right surface the plate where $\theta = 0°$.

$$\therefore EA = \frac{Q}{\varepsilon_0}, \quad \sigma = \frac{Q}{A} \rightarrow Q = \sigma A$$

$$E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0} \rightarrow \boxed{E = \frac{\sigma}{\varepsilon_0}}$$

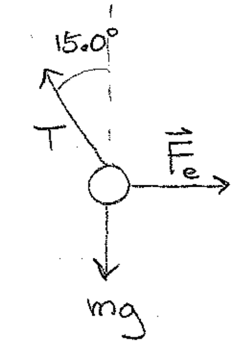
(50)

The suspended charge below is in equilibrium. Find q of the charge.



$\underset{\rightarrow}{F_e} = q\underset{\rightarrow}{E}$

Free-body diagram:



$$\Sigma F_y = T\cos(15.0°) - mg = 0$$
$$+mg \quad +mg$$

$$T\cos(15.0°) = mg$$

→ Divide both sides by $\cos(15.0°)$

$$T = mg/\cos(15.0°)$$

Plug T in.

$$\Sigma F_x = F_e - T\sin(15.0°) = 0$$

$$T\sin(15.0°) = F_e = qE$$

$$\left(\frac{mg}{\cos(15.0°)}\right)\sin(15.0°) = qE$$

$$mg\tan(15.0°) = qE$$

$$q = \frac{mg}{E}\tan(15.0°)$$

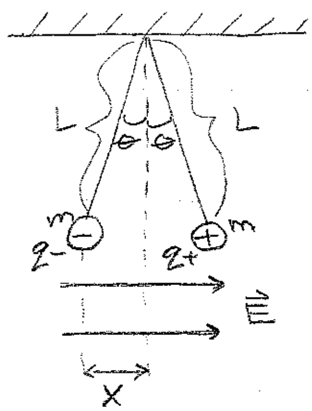
$$q = \frac{(2\times10^{-3}\,kg)(9.8\,m/s^2)}{(1.00\times10^{3}\,N/C)}\tan(15.0°)$$

$$q \cong 5.25\times10^{-6}\,C$$

$$\boxed{q \cong 5.25\,\mu C}$$

∴ Since the charge was deflected <u>in the direction of $\vec{E}$</u>, the charge must be <u>positive</u>.

(57)



$m = 2.0\,g = 2.0\times10^{-3}\,kg$

$L = 10.0\,cm = 0.100\,m$

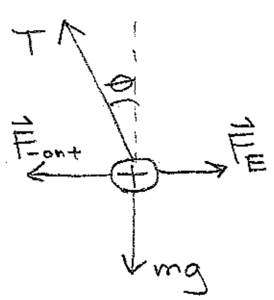
$q_- = -5.0\times10^{-8}\,C$

$q_+ = +5.0\times10^{-8}\,C$

$\theta = 10°$ Q: $E = ?$

$\sin\theta = \frac{X}{L}$

free-body diagram:



$\vec{F}_{-on+}$ = force of q_- on q_+.

$\vec{F}_E$ = force exerted on q_+ by the electric field.

• r = distance between spheres

$r = 2x = 2L\sin\theta$

$$F_{-on+} = \frac{K|q_-||q_+|}{r^2}$$

$$F_E = |q_+|E$$

$\Sigma F_y = T\cos(10.0^\circ) - \cancel{mg} = 0 \rightarrow T\cos(10.0^\circ) = mg$ Divide by $\cos(10.0^\circ)$ both sides.

$+\cancel{mg} \quad +mg \qquad T = \dfrac{mg}{\cos(10.0^\circ)}$

plug in

$\Sigma F_x = F_E - F_{cont} - T\sin(10.0^\circ) = 0$

$|q_+|E - \dfrac{k|q_-||q_+|}{r^2} - \dfrac{mg}{\cos(10.0^\circ)}\sin(10.0^\circ) = 0$ $\quad \tan(10.0^\circ)$

$\dfrac{1}{|q_+|}\left(|q_+|E\right) = \left(\dfrac{k|q_-||q_+|}{r^2} + mg\tan(10.0^\circ)\right)\dfrac{1}{|q_+|}$

$E = \dfrac{k|q_-|}{r^2} + \dfrac{mg}{|q_+|}\tan(10.0^\circ)$

$\rightarrow r^2 = 2^2L^2\sin^2\theta = 4(L\sin\theta)^2$

$E = \dfrac{(8.99\times10^9\ N\cdot m^2/C^2)(5\times10^{-8}C)}{4[(0.100m)\sin(10.0^\circ)]^2} + \dfrac{(2.0\times10^{-3}kg)(9.8m/s^2)\tan(10.0^\circ)}{(5\times10^{-8}C)}$

$\boxed{E \cong 4.4\times10^5\ N/C}$

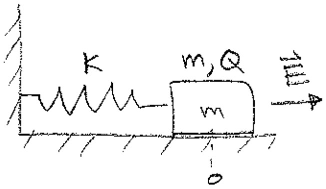
CHAPTER 16

(3) $\Delta V = 90mV$ $\quad \Delta V = \Delta PE/q \rightarrow \Delta PE = q\Delta V$

$\Delta PE = -W_{field} = W_{input}$

$W_{input} = q\Delta V = (1.6\times10^{-19}C)(90\times10^{-3}J/C)$

$\boxed{W_{\substack{input\\needed}} \cong 1.4\times10^{-20}J}$



$m = 4.00\ kg$
$Q = 50.0\ \mu C$
$k = 100\ N/m$
$E = 5.00 \times 10^5\ V/m$

$\Delta PE_{elec.} = -F\Delta x$
$\Delta PE_{electrical} = -qE\Delta x$

(9)

(a) $\Delta KE = \Delta PE$

Note: The block is at rest at start, & also when the spring is stretched to its maximum distance.

$\Delta KE = \frac{1}{2}m\cancel{v_f^2}^{0} - \frac{1}{2}m\cancel{v_i^2}^{0}$

$\boxed{\Delta KE = 0}$

$0 = \Delta PE = \Delta PE_{spring} + \Delta PE_{electric}$

$\Delta PE_{spring} = \frac{1}{2}kx_f^2 - \frac{1}{2}k\cancel{x_i^2}^{0}$, where $x_f = x_{max}$

$\Delta PE_{spring} = \frac{1}{2}kx_{max}^2$

$\Delta PE_{electrical} = -QEx_{max}$

$\Delta PE = \left(\frac{1}{2}kx_{max}^2 - QEx_{max}\right)\frac{1}{x_{max}} = 0\ \frac{1}{x_{max}}$

$2\frac{1}{k}\left(\frac{1}{2}kx_{max}\right) = (QE)\left(\frac{2}{k}\right)$

$x_{max} = \frac{2QE}{k} = \frac{2(50.0 \times 10^{-6}\ C)(5.00 \times 10^5\ V/m)}{(100\ N/m)}$

$\boxed{x_{max} \cong 0.50\ m}$

(b) At equilibrium, $\vec{F}_e + \vec{F}_s = 0$

$\vec{F}_s \leftarrow \boxed{m} \rightarrow \vec{F}_e$ Free-body diagram

$\Sigma F = \vec{F}_e + \vec{F}_s = 0$

$F_e = F_s$

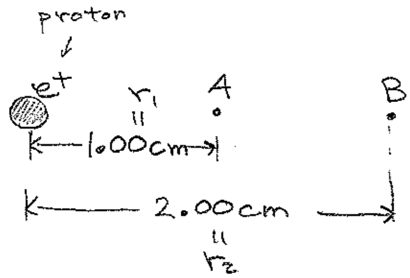
$\frac{1}{k}(QE) = (kx_{eq})\frac{1}{\cancel{k}}$

$x_{eq} = \frac{QE}{k}$ → remember that $x_{max} = 2\frac{QE}{k}$

$x_{eq} = \frac{x_{max}}{2} = \frac{0.50\ m}{2}$

$\boxed{x_{eq} = 0.25\ m}$

(11)



(a) find: Electrical potential at A.

$$V = \frac{kq}{r_1}$$

$$V = \frac{(8.99\times10^9\ N\cdot m^2/C^2)(1.6\times10^{-19}C)}{(0.01m)}$$

$$\boxed{V \cong 1.44\times10^{-7}\,V}$$

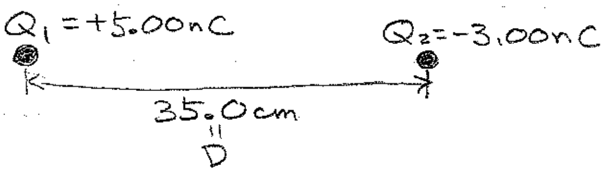
(b) $V_B - V_A = \Delta V_{AB} = ?$

$$\Delta V_{AB} = \frac{kq}{r_2} - \frac{kq}{r_1} \rightarrow \Delta V_{AB} = kq\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$\Delta V_{AB} = (8.99\times10^9\ N\cdot m^2/C^2)(1.6\times10^{-19}C)\left[\frac{1}{(0.02m)} - \frac{1}{(0.01m)}\right]$$

$$\boxed{\Delta V_{AB} \cong -7.19\times10^{-8}\,V}$$

(15)



35.0cm = 0.35m

(a) $V_{center} = \frac{KQ_1}{r} + \frac{KQ_2}{r}$, where r is half the distance between the charges.

$\therefore r = .175m$

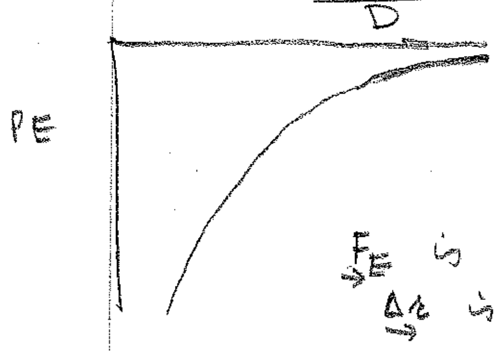
$$V_{center} = \frac{K}{r}(Q_1 + Q_2)$$

$$V_{center} = \frac{(8.99\times10^9\ N\cdot m^2/C^2)}{(0.175m)}\left[(5\times10^{-9}) + (-3\times10^{-9})\right]$$

$$\boxed{V_{center} \cong 103V}$$

(b) $PE = \frac{KQ_1Q_2}{D} = W_{input}$

$\Delta P_E = -\vec{F}_E \cdot \Delta \vec{r}$

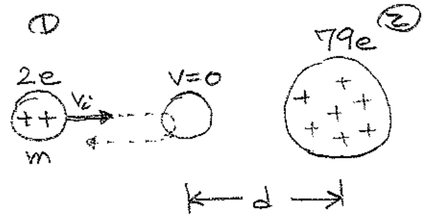


$$= \frac{(8.99\times10^9\ N\cdot m^2/C^2)(5\times10^{-9})(-3\times10^{-9})}{(0.35m)}$$

$$\boxed{PE \cong -3.85\times10^{-7}J}$$

$\vec{F}_E$ is along $-\hat{r}$ so $-\vec{F}_E$ is along $+\hat{r}$

$\Delta\vec{r}$ is -ive so ΔP_E from $r = \infty$ to $r = 0.35m$ is -ive



$2e = 2(1.6\times10^{-19}C)$

$79e = 79(1.6\times10^{-19}C)$

(19) $m = 6.64\times10^{-27}\,kg$

$v_i = 2.00\times10^{7}\,m/s$

$-\Delta KE = \Delta PE_e$

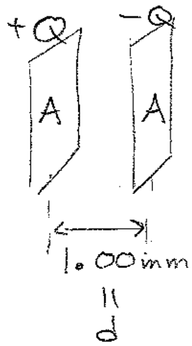
$-(\cancel{\tfrac{1}{2}mv_f^2}^{\,0} - \tfrac{1}{2}mv_i^2) = \dfrac{KQ_1Q_2}{d} - \cancel{\dfrac{KQ_1Q_2}{r}}$ 0, assume particle comes in from infinity (r = very large).

$\left(\dfrac{2}{mv_i^2}\right)(d)\tfrac{1}{2}mv_i^2 = \dfrac{KQ_1Q_2}{d}(d)\left(\dfrac{2}{mv_i^2}\right)$

$$d = \frac{2KQ_1Q_2}{mv_i^2} = \frac{2(8.99\times10^9\,N\cdot m^2/C^2)(2)(79)(1.6\times10^{-19}C)^2}{(6.64\times10^{-27}kg)(2.00\times10^7)^2}$$

$\boxed{d \cong 2.74\times10^{-14}\,m}$

(27)



$A = 5.00\,cm^2 \rightarrow 5cm^2\cdot\dfrac{1m}{100cm}\,\dfrac{1m}{100cm}$

$Q = 400pC$

$5cm^2 = 5\times10^{-4}m$

(a) ΔV across capacitor:

$\Delta V = \dfrac{Q}{C}$, $C = \dfrac{\epsilon_0 A}{d}$

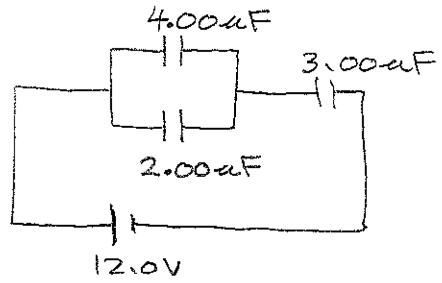
$$\Delta V = \frac{Qd}{\epsilon_0 A} = \frac{(400\times10^{-12}C)(1\times10^{-3}m)}{(8.85\times10^{-12}C^2/N\cdot m^2)(5\times10^{-4}m^2)}$$

$\boxed{\Delta V \cong 90.4V}$

(b) $\Delta V = Ed$ Q: Find E, the uniform electric field between the plates.

$E = \dfrac{\Delta V}{d} = \dfrac{90.4V}{1\times10^{-3}m} \rightarrow \boxed{E \cong 9.04\times10^4\,V/m}$

(31)

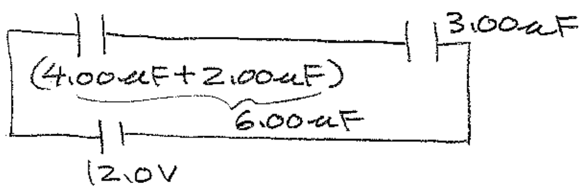


(a) Find the equivalent capacitance

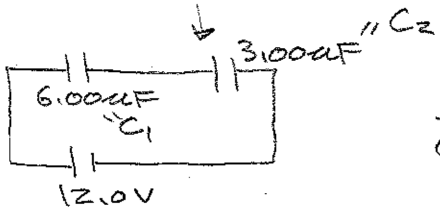
(b) Find the charge on each capacitor, and the potential difference across each.

(a) Parallel capacitors add:

(i)



(ii)



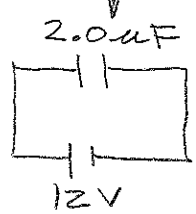
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{6.00\times10^{-6}\,F} + \frac{1}{3.00\times10^{-6}\,F}$$

$$= \frac{1}{10^{-6}}\left(\frac{1}{6.00\,F} + \frac{2}{6.00\,F}\right)$$

$$= \frac{3}{6.00\times10^{-6}\,F}$$

$$\therefore \frac{1}{C_{eq}} = \frac{1}{2\times10^{-6}\,F} \rightarrow \boxed{C_{eq} = 2.0\,\mu F}$$

(iii)



(b) $Q = C\Delta V$ → For case (iii), $Q = C\Delta V = (2.0\times10^{-6}\,C)(12\,V)$

$Q = 24\,\mu C$

→ Since the 3.00 μF is in series with the rest of the configuration, it charges with the full charge value $Q = 24\,\mu C$.

$$\therefore \boxed{Q_{3.00\mu F} = 24\,\mu C}$$

→ $$\Delta V_{\substack{\text{parallel}\\\text{capacitors}}} = \frac{Q_{3.00\mu F}}{C_{eq,\text{parallel}}} = \frac{24\,\mu C}{6\,\mu F} = \boxed{4.0\,V}$$

$$\Delta V_{3.00\mu F} = \frac{Q_{3.00\mu F}}{C_{3.00\mu F}} = \frac{24\,\mu C}{3.00\,\mu F} = \boxed{8.0\,V}$$

→ Since parallel capacitors share the same voltage:

$$\Delta V_{\substack{parallel\\cap}} = \Delta V_{2\mu F} = \Delta V_{4\mu F} = 4.0\,V$$

$$Q_{4\mu F} = (C_{4\mu F})\Delta V_{4\mu F}$$
$$Q_{4\mu F} = (4\,\mu F)(4.0\,V)$$
$$Q_{4\mu F} = 16.0\,\mu C$$

$$Q_{2\mu F} = (C_{2\mu F})\Delta V_{2\mu F}$$
$$Q_{2\mu F} = (2\,\mu F)(4.0\,V)$$
$$Q_{2\mu F} = 8.0\,\mu C$$

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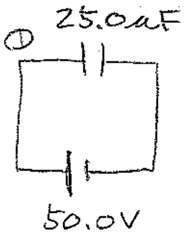
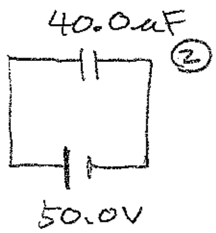




(a) $Q_1 = C_1(\Delta V)$

$Q_1 = (25.0\,\mu F)(50.0\,V)$

$Q_1 = 1.25 \times 10^3\,\mu C$

$Q_1 = 1.25\,mC$

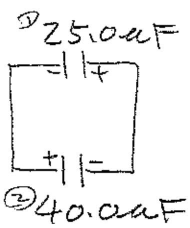
$Q_2 = C_2(\Delta V)$

$Q_2 = (40.0\,\mu F)(50.0\,V)$

$Q_2 = 2.0 \times 10^3\,\mu C$

$Q_2 = 2.0\,mC$

(b)



$$C_{eq} = C_1 + C_2 = 25.0\,\mu F + 40.0\,\mu F$$
$$C_{eq} = 65\,\mu F$$

$$Q_{total} = Q_2 - Q_1 = (2.0\,mC) - (1.25\,mC) = 750\,\mu C$$

→ We subtract to get the total charge because we connect negative plates to positive plates

$$\Delta V = \frac{Q_{total}}{C_{eq}} = \frac{750\,\mu C}{65\,\mu F}$$

$$\Delta V = 11.5\,V$$ across each capacitor (since they are in the parallel configuration).

$Q_1 = C_1 \Delta V$

$Q_1 = (25\,\mu F)(11.5\,V)$

$Q_1 \cong 288\,\mu C$

$Q_2 = C_2 \Delta V$

$Q_2 = (40\,\mu F)(11.5\,V)$

$Q_2 \cong 460\,\mu C$