SOLUTIONS — Set 3.

FORMULAE

Elementary charge $\pm 1.6 \times 10^{-19}$ C

Coulomb's Law

$$\vec{F}_E = \frac{k_e Q_1 Q_2 \hat{r}}{r^2} \qquad k_e = 9\times10^{9}\ \frac{N\text{-}m^2}{C^2}$$

Note: If Q_1, Q_2 have same sign

$\vec{F}_E \parallel +\hat{r}$ Repulsion

If they have opposite signs

$\vec{F}_E \parallel -\hat{r}$ Attraction

Superposition

$$\vec{F}_E(j) = \sum_{i\neq j} \vec{F}_{ij}$$

$\vec{E}$-field due to point charge

$$\vec{E} = \frac{k_e q}{r^2}\hat{r} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}$$

Date: 19^{th} Sep '08

PHYS 122.

Solutions to Problems.

HW-3 Chapter 15.

Problem 3:

Charge of the α-particle : $q_\alpha = +2.0\,e$.

where 'e' is the charge of an electron.

$$\boxed{e = 1.6 \times 10^{-19}\ C}$$

The Charge of a gold nucleus $= q_{nu} = +79\,e$

Distance of the α-particle from the nucleus $\Big\} = d = 2.0 \times 10^{-14}$ m.

REQUIRED: To compute the electric force when the α-particle is at distance 'd' from the gold nucleus.

We know that the force between two charges q_1, q_2 at a distance 'r' apart is given by the Coulomb inverse square law:

Mag. of $\vec{F}$: $|\vec{F}| = k_e \dfrac{|q_1|\,|q_2|}{r^2}$. N (in S.I units)

$$k_e = \frac{1}{4\pi\epsilon_0} \quad \text{and} \quad \epsilon_0 = 8.85 \times 10^{-12}\ \cancel{\frac{N m^2}{C^2}}\ \frac{C^2}{m^2 N}$$

Note: It is good to remember that $k_e = \frac{1}{4\pi\epsilon_0}$ is approx. given by $k_e = 9 \times 10^{9}\ \frac{N m^2}{C^2}$.

In the present case:

$$|\bar{F}| = \frac{(9\times10^{9}).(q_{\alpha})(q_{nu})}{d^{2}} \; N$$

$$|\bar{F}| = \frac{(9\times10^{9}).(2.e)(79.e)}{(2.0\times10^{-14})^{2}}. \; N$$

$$= \frac{(9\times10^{9}).(2\times1.6\times10^{-19})(79\times1.6\times10^{-19})}{(2.0)^{2}\times(10^{-14})^{2}} \; N$$

$$= \frac{9\times2\times1.6\times79\times1.6}{4}\times\left(\frac{10^{9}\times10^{-19}\times10^{-19}}{10^{-28}}\right). \; N$$

$$= (910.08)\times10^{9-19-19+28} \; N$$

$$= (910.08)\times10^{-1} \; N$$

$$|\vec{F}| = (91.008) \; N \;=\; \underline{\underline{91.0 \; N}}.$$

$$\therefore \boxed{|\bar{F}| = 91.0 \; N}$$

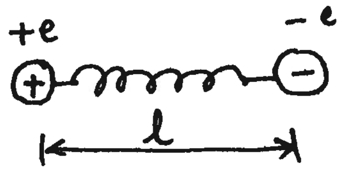
Because both charges are positive, from the rule that ~~the~~ 'like charges repel', the force is REPULSIVE.

6. Length of the DNA molecule = 2.17 μm

$$\boxed{\mu m = 10^{-6} m.}$$

$\therefore$ Length of the molecule = $l = 2.17 \times 10^{-6} m.$

The ends of the molecule become SINGLY IONISED. This means that the magnitude of charge is that of one electron, '+ve on one end and '−'ve on the other as shown in the figure :



$e = 1.6 \times 10^{-19} C$

The molecule acts like a spring. So it has been shown as a spring above. It is also given that the molecule compresses 1% in length.

$$\therefore \text{Compression} = \Delta X = \frac{1}{100} \cdot l = \frac{1}{100} \times 2.17 \times 10^{-6} m$$

$$\therefore \Delta X = 2.17 \times 10^{-8} m.$$

Let the spring constant of the 'molecule' be 'K' N/m.

Because the +e and −e charges at the end attract, the spring compresses.

But, after it compresses by Δx, the spring force which pushes both molecules away balances the electric force pulling them together.

So: $|\bar{F}_{spring}| = |\bar{F}_{electrical}|$

$$\therefore (K \cdot \Delta x) = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2} = \frac{(+e)(+e)}{4\pi\epsilon_0 \cdot l^2}$$

Taking Δx on the other side ~~k.(Δx)~~ $\Big\}$ $K = \dfrac{(+e)(+e)}{(4\pi\epsilon_0) \cdot l^2} \cdot \dfrac{1}{(\Delta x)}$

$$\therefore K = \frac{(9\times 10^{9}) \cdot (1.6\times 10^{-19})(1.6\times 10^{-19})}{(2.17\times 10^{-6})^2} \cdot \frac{1}{(2.17\times 10^{-8})} \frac{N}{m}$$

$$\therefore K = \frac{9\times 1.6 \times 1.6}{(2.17)^2 \times (2.17)} \cdot \frac{10^{+9}\times 10^{-19}\times 10^{-19}}{(10^{-6})^2 \times (10^{-8})} \frac{N}{m}$$

$$\therefore K = (2.25) \times 10^{9-19-19+12+8}$$

$$= 2.25 \times 10^{29-38}$$

$$\boxed{K = 2.25\times 10^{-9} \frac{N}{m}}$$

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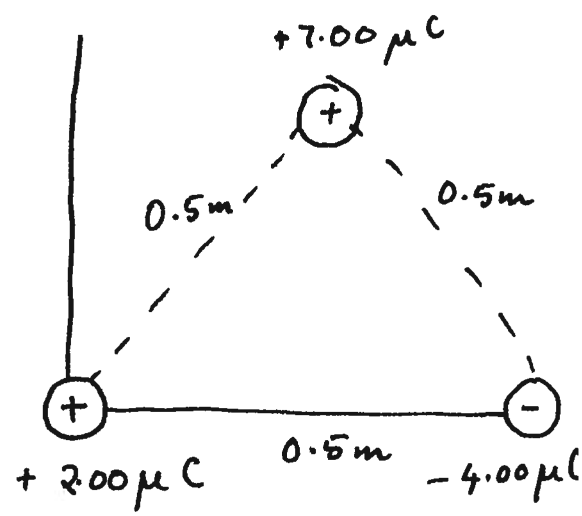
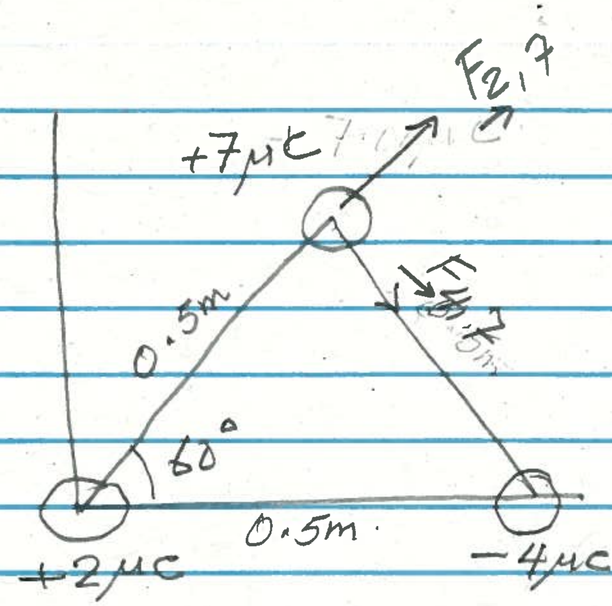
Calculate the net electric force at the $+7.00\mu C$ charge:



Principle of Superposition tells us that the force due to a group of point charges on a test charge is equal to the

The two forces acting on the $7\mu C$ charge are as shown $\vec{F}_{2,7}$ and $\vec{F}_{4,7}$



Magnitudes are $F_{2,7} = \dfrac{9\times10^{9} \times 2\times7\times10^{-12}}{(0.5)^2}$ N.

$$F_{4,7} = \frac{9\times10^{9}\times4\times7\times10^{-12}}{(0.5)^2} N$$

$$\vec{F}_{2,7} = \left[(0.504 \cos 60)\hat{x} + 0.504 \sin 60\, \hat{y}\right] N.$$

$$\vec{F}_{4,7} = \left[1.008 \cos 60 \hat{x} - 1.008 \sin 60\, \hat{y}\right] N$$

Total Force on $7\mu C$ charge becomes

$$\vec{F} = \left[0.75\hat{x} - 0.43\hat{y}\right] N$$

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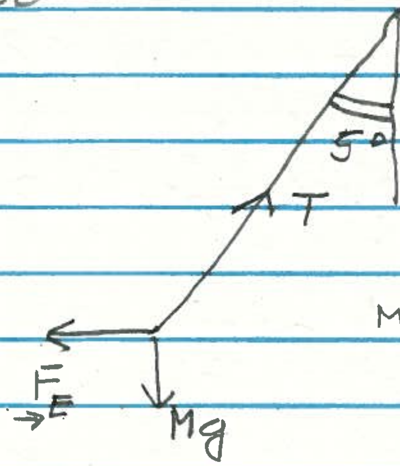
Total Force must be zero so

$T \sin 5° - F_E = 0$



$T \cos 5° - Mg = 0$

$M = 0.2\ \text{gm}$
$= 0.2 \times 10^{-3}\ \text{kg}$

$T \sin 5 = F_E$

$T \cos 5 = Mg$

$$\frac{F_E}{Mg} = \tan 5 \qquad F_E = Mg \tan 5°$$

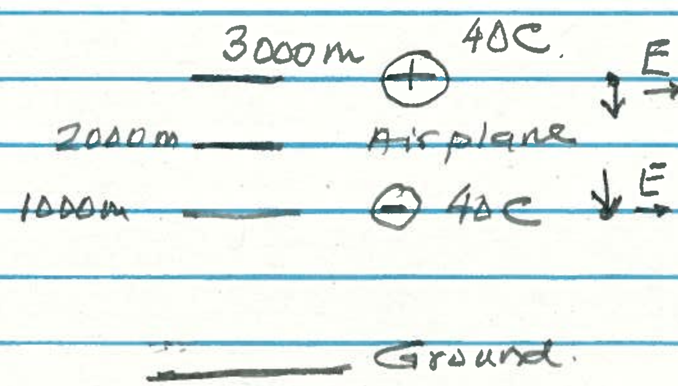
$$F_E = \frac{k_e q^2}{d^2}$$

$$\frac{9 \times 10^9 \times q^2}{(0.05)^2} = 0.2 \times 10^{-3} \times 9.8 \times \tan 5°$$

$$q = \left[\frac{(0.05)^2 \times 0.2 \times 10^{-3} \times 9.8 \times \tan 5°}{9 \times 10^9}\right]^{1/2}$$

$$\simeq 7 \times 10^{-9}\ \text{C}$$

15-19 The picture is because we are assuming point charges



Net Electric field at a point = Vector Sum of electric fields due to individual charges.

Magnitude of the Electric field due to a point charge q at a distance 'r' is given by $\left.\right\} = |\bar{E}| = \frac{1}{4\pi\epsilon_0} \cdot \frac{|q|}{r^2}$.

Direction : towards the charge if it is $-$ve.

away from charge if it is $+$ve.

Clearly in the figure, the $+40$ C gives an electric field pointing downward, and -40C also gives it in the same direction.

$|\bar{E}_1|$ due to $+40$C $= |\bar{E}_1| = \frac{1}{4\pi\epsilon_0} \cdot \frac{(+40\,C)}{(3000-2000)^2}$ N/C

$$\therefore \quad |\bar{E}_1| = \frac{1}{4\pi\epsilon_0} \cdot \frac{40\,C}{(1000)^2}$$

$$|\bar{E}_1| = 9 \times 10^9 \times \frac{40\,C}{(1000)^2} = \frac{9 \times 40 \times 10^9}{10^6}$$

$$|\bar{E}_1| = 360 \times 10^3 = 3{,}60{,}000 \frac{N}{C}.$$

$|\bar{E}_2|$ due to -40 C is : $|\bar{E}_2| = \frac{1}{4\pi\epsilon_0} \cdot \frac{|(-40\,C)|}{(2000-1000)^2}$ N/C

$$\therefore \quad |\bar{E}_2| = 3{,}60{,}000 \frac{N}{C}.$$

$\therefore$ If we call the direction up as $\hat{z}$, then

$\bar{E}_1 = (-360{,}000\hat{y})\ N/C$

$\bar{E}_2 = (-360{,}000)\hat{y}\ N/c$

$\therefore$ E_{net} at airplane $= \bar{E}_1 + \bar{E}_2 = (-360{,}000 - 360000)\hat{z}\ N/c$

$$\boxed{\therefore\ \bar{E}_{net} = (-720{,}000\ \hat{y})\ \frac{N}{C}.}$$

or $720{,}000\ N/c$ downward.

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(23). Magnitude of the uniform electric field $\Big\} = |E| = 640\ N/C$.

the charge on a proton $= q_p = +e = +1.6 \times 10^{-19}\ C$.

Mass of a proton $= m_p = 1.67 \times 10^{-27}\ kg$.

At some time 't' sec, Speed of proton $= v_p = 1.20 \times 10^{6}\ m/s$

(a) Let $a\ m/s^2$ be the acceleration of proton.

From Newton's II law: $\bar{F}_{net} = m\bar{a}$. at that pt at that time

$\therefore\ |\bar{F}_{net}| = m|\bar{a}|$.

But as there is an electric field, $|\bar{F}_{net}| = q_p|\bar{E}|$.

$\therefore \quad |q_p E| = m|\bar{a}|$.

$$\therefore \quad |\bar{a}| = \frac{|q_p E|}{m} = \frac{(1.6 \times 10^{-19}) \times (640)}{(1.67 \times 10^{-27})} \cdot m/s^2$$

$$\therefore \boxed{|\bar{a}| = 6.13 \times 10^{10} \ m/s^2}$$

(b) Initial speed of proton = $v_i = 0 \ m/s$.

Final speed of proton = $v_f = v_p = 1.2 \times 10^6 \ m/s$

Time taken to reach this speed = $t = t$ sec.

~~Assume~~ Since the acceleration is constant, we have

$$V_f = V_i + at$$

$$\therefore V_p = 0 + (6.13 \times 10^{10})\, t$$

$$\therefore (1.2 \times 10^6)\ m/s = (6.13 \times 10^{10})\ m/s^2 \cdot (t)$$

$$\therefore \boxed{t = \frac{1.2 \times 10^6}{6.13 \times 10^{10}} = 1.96 \times 10^{-5} \text{ sec.}}$$

(c) How far has it moved in time t sec?

$S = v_i t + \frac{1}{2} a t^2$ $\qquad v_i = 0, \quad t = 1.96 \times 10^{-5}$ sec.

$a = 6.13 \times 10^{10} \ m/s^2$.

$\therefore$ distance covered $\Big\} = S = \frac{1}{2} \times (6.13 \times 10^{10}) \frac{m}{\cancel{s^2}} \cdot (1.96 \times 10^{-5})^2 \cancel{s}$

$$= \frac{6.13 \times 1.96}{2} \times 10^{10-10} \; m$$

$$= \frac{6.13 \times 1.96}{2} \, m = 12.01 \, m$$

$\therefore$ $\boxed{\text{It travels } \underline{12.01 \text{ m}}}$

(d) Kinetic energy at final velocity v_p is:

$$K = \frac{1}{2} m_p \cdot v_p^2 = \frac{1}{2} \times (1.67 \times 10^{-27} \, kg) \times (1.2 \times 10^6)^2 \frac{m^2}{s^2}$$

$$K = \frac{1}{2} \times 1.67 \times 1.44 \times 10^{-27} \times 10^{12} \; J$$

$$\boxed{K = 1.20 \times 10^{-15} \; J}$$

25. Three identical charges ($q = -5.0\ \mu C$) lie along a circle of radius 2.0 m at angles of 30°, 150°, and 270°, as shown in Figure P15.25. What is the resultant electric field at the center of the circle?

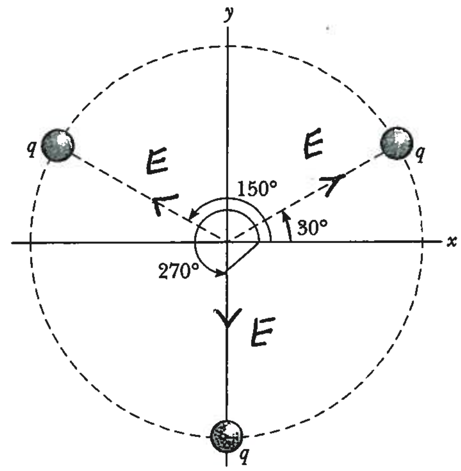


Figure P15.25

All the fields have the same magnitudes

$$E = \frac{9\times10^{9} \times 5\times10^{-6}}{(2)^2}\ N/C$$

$$= 11.25\times10^{3}\ N/C.$$

Along X $\quad E_x = E\cos\theta - E\cos\theta$

$$= E\cos 30 - E\cos 30 = 0.$$

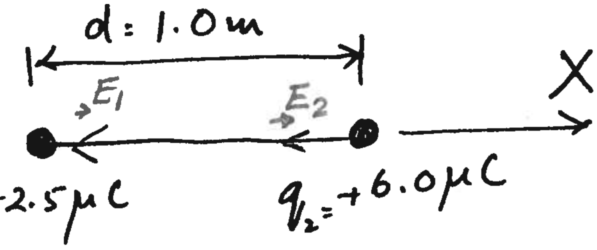
Along Y $\quad E_y = E\cos 60 + E\cos 60 - E$

$$= 0.5E + 0.5E - E = 0.$$

Hence $\vec{E}$ at the center is zero!

You could, of course, invoke symmetry to justify the answer

#27. Determine the point where the net electric field is zero.



The electric field cannot be ZERO at a point between q_1 and q_2 along the line joining the charges. This can be seen easily from the fact that at every point in between the charges, the directi is from q_2 to q_1. So there is no cancellation of the fields due to each charge.

To find the point P at which the net field is ZERO,

Let the distance of the point P from q_1 be r_1 m

and let the distance of the point P from q_2 be r_2 m

At the point P, the net electric field = $\bar{E}_{net}$

$$\therefore \quad \bar{E}_{net} = \bar{E}_1 + \bar{E}_2 .$$

$$|\bar{E}_1| = \frac{+|q_1|}{4\pi\epsilon_0 \cdot r_1^2} . \qquad |E_2| = \frac{+|q_2|}{4\pi\epsilon_0 r_2^2}$$

$\therefore$ At that point where $\bar{E}_{net} = 0$, $\quad \bar{E}_1 = -\bar{E}_2$

$$\therefore \quad |\bar{E}_1| = |-\bar{E}_2| = |\bar{E}_2| .$$

$$\therefore \frac{|q_1|}{4\pi\epsilon_0 r_1^2} = \frac{|q_2|}{4\pi\epsilon_0 r_2^2}.$$

$$\therefore \frac{(2.5\,\cancel{\mu C})}{(\cancel{4\pi\epsilon_0})\cdot(r_1^2)} = \frac{(6.0\,\cancel{\mu C})}{(\cancel{4\pi\epsilon_0})(r_2^2)}.$$

$$\therefore \frac{2.5}{6.0} = \frac{r_1^2}{r_2^2}. \qquad \boxed{\frac{r_1}{r_2} < 1}$$

$$\therefore \boxed{r_1 = 0.64\, r_2}.$$ $\therefore r_2 > r_1$. So it lies to the ~~right of q_2~~. left of q_1.

We also know that, $r_2 - r_1 = d = 1.0\,m$

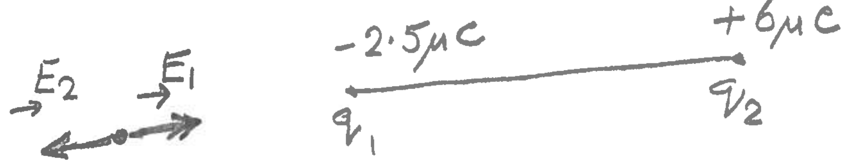
$$\therefore r_2 - r_1 = d = 1.0\,m$$

$$r_2 - 0.64\,r_2 = 1.0\,m$$

$$\therefore r_2(0.36) = 1.0\,m$$

$$\boxed{r_2 = 2.78\,m}$$

Therefore the point lies 2.78 m from q_2, to the left of q_1; OR it is $(2.78 - 1) = 1.78$ m to the left of q_1 in the figure.



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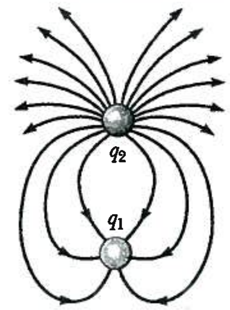


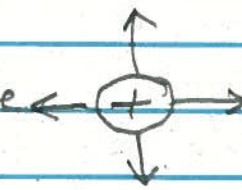
Figure P15.28

The number of field lines coming out of a charge is proportional to the magnitude of that charge because the field is proportional to the charge.

So we simply need to count the # of lines

$$\frac{q_1}{q_2} = \frac{\text{\# of lines out of } q_1}{\text{\# of lines out of } q_2}$$

$$= \frac{6}{18} = \frac{1}{3}$$

Now +ive charge is a source 

−ive charge is a sink 

So here q_1 is negative

q_2 is positive

ratio $\frac{q_1}{q_2} = -\frac{1}{3}$.

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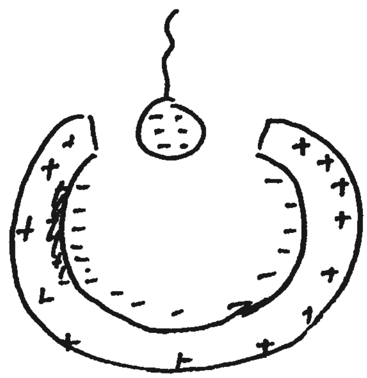
Magnitude of charge lowered into the centre of the hollow cylinder $\Big\}$ = $|q| = 5\mu C$

Sign of the charge = −ve.

$\therefore$ charge $q = -5\mu C$.

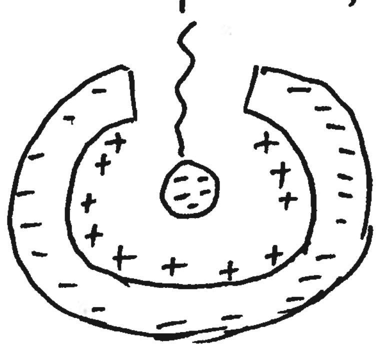
Case (a):



THE $\vec{E}$ field inside the conductor must be zero.

When the charge is just outside the ~~sphere~~ conductor, because the charge is negative, the outer surface of the ~~sphere~~ conductor has an induced positive charge of $+5\mu C$, and the inside surface has a negative induced charge of total $-5\mu C$. The sphere, overall, remains Neutral. [charge is conserved]

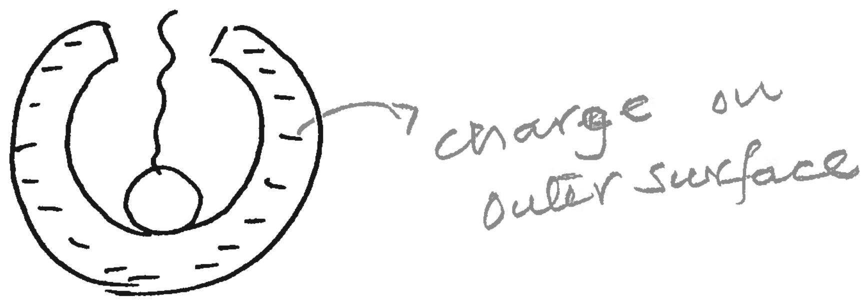
Case b:



When the ~~sphere~~ charge is inside the conductor, the inner surface gets an induced +ve charge, and the total magnitude of this is $+5\mu C$. The outer surface ~~of~~

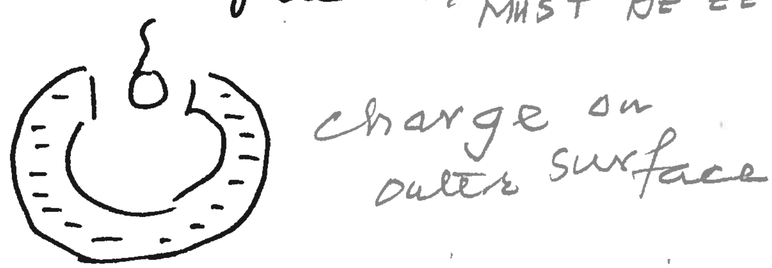
of the sphere gets a total $-5\mu C$ charge. The sphere, as a whole, remains neutral.

Case (c):



The charge is now brought into contact with the conductor. The charge gets transferred to the Conductor. So the conductor gets an over-all $-5\mu C$ of charge. As we know that the charge has to reside on the outside of a conductor, all the -ve charge will appear on the outer surface. There is no charge on the inner surface [$\vec{E}$ FIELD INSIDE MUST BE ZERO]

Case(d):



Now the chargeless sphere is pulled out. This does not change any charge arrangement. So we still have the same charge distribution as Case(c). 0 charge on the inside surface and $-5\mu C$ on the outside.

#36. Electric field magnitude $= |E| = 3\times10^{4}\ N/C$.

charge on the electron $= e = 1.6\times10^{-19}\,C$.

Density of the oil drop $= \rho = 858\ \frac{kg}{m^3}$.

The weight of this drop is balanced by the electric force of the field on one electron in the drop.

What is the radius of the drop?

Let the radius be 'r m'..

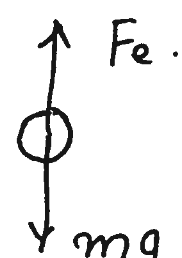
The volume of the drop, assuming the drop to be a sphere, is $V = \left(\frac{4}{3}\pi r^3\right) m^3$.

~~As~~ Its mass $= (m)$. kg.

Density $= \rho = \frac{m}{V} \quad \therefore\ m = \rho . V$

$\therefore\ m = \rho\left(\frac{4}{3}.\pi r^3\right)$.

Now, force of gravity acting on the drop $= mg$ downward. $g = 9.8\ m/s^2$



Electric force $= F_e = e.|E|$ upward to balance the force of gravity.

$\therefore\ mg = e|E|$

$\therefore\ (\rho.V)\,g = e|E|$

$\therefore\ V = \frac{e|E|}{g.\rho} = \frac{1.6\times10^{-19}\times 3\times10^{4}}{9.8\times(858)}\ m^3$

$\therefore \quad V = 5.71 \times 10^{-19} \ m^3$

$\therefore \quad \frac{4\pi}{3} \cdot r^3 = 5.71 \times 10^{-19} \ m^3$

$\therefore \quad r^3 = \frac{3}{4\pi} \times 5.71 \times 10^{-19} \ m^3$ $\quad \boxed{\pi \approx 3.14}$

$\therefore \quad r^3 = 1.36 \times 10^{-19} \ m^3$

or $r^3 = \cancel{13} \ 0.136 \times 10^{-18} \ m^3$

Taking the cubeth root on both sides,

$r = \sqrt[3]{(0.136 \times 10^{-18})} \ m$

$= (0.136 \times 10^{-18})^{1/3}$

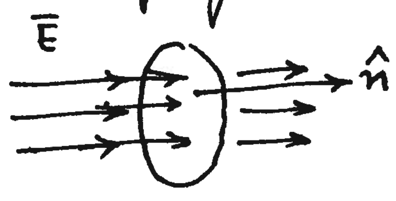
$\boxed{r = 0.514 \times 10^{-6} \ m}$

#38 Area of the Flat surface $= A = 3.2 \ m^2$.

Magnitude of the uniform Electric field $= E = 6.2 \times 10^5 \ N/C$.

What is the electric flux through the area A when:

(a) Electric field is perpendicular to the surface?



If $\bar{E}$ is the electric field, and if $\hat{n}$ is the normal vector to the surface, in the figure, $\bar{E}$ and $\hat{n}$ are parallel to each other. In other words, the angle θ made between $\vec{E}$ and $\hat{n}$ is $0°$.

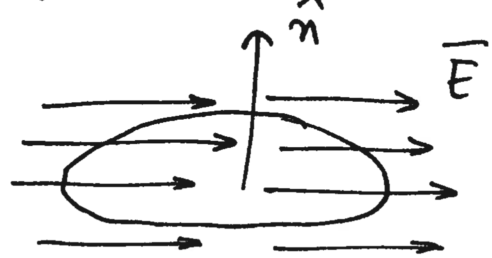
$\therefore$ The flux: $\Phi = E \cdot A \cos\theta$

$$\Phi = \left(6.2 \times 10^5 \frac{N}{C}\right) \cdot (3.2\, m^2)\, (\cos 0°)$$

$$= 6.2 \times 10^5 \times 3.2 \quad \frac{N m^2}{C}$$

$$\boxed{\Phi = 19.84 \times 10^5 \frac{N m^2}{C}}.$$

(b) When the electric field is parallel to the surface:



In this case, the normal to the surface makes an angle $\theta = 90°$ with the Electric field.

$\therefore$ $\Phi = EA \cos 90° = 0$ because $\cos 90° = 0$.

$\therefore$ $\boxed{\Phi = 0 \frac{N m^2}{C}}$.