SOLUTIONS – Set 2.

FORMULAE

Intensity $\quad I = \dfrac{P_{AV}}{4\pi R^2}$

decibel

$$\beta = 10 \log \frac{I}{I_0}$$

$$I_0 = 10^{-12} \text{ watt/m}^2$$

$$V_s = \sqrt{\frac{\gamma k_B T}{m}} \qquad T \text{ in Kelvin}$$

DOPPLER EFFECT –

Observer moves $\quad f_o = f_s \left[1 \pm \dfrac{V_{obs}}{V_s} \right]$

Source moves $\quad f_o = \dfrac{f_s}{1 \mp \dfrac{V_{sou}}{V_s}}$

INTERFERENCE

Maxima $\quad (r_1 - r_2) = n\lambda, \; n = 0, \pm 1, \pm 2, \ldots$

Minima $\quad (r_1 - r_2) = (n + \tfrac{1}{2})\lambda; \; n = 0, \pm 1, \pm 2$

NORMAL MODES

BOTH ENDS Fixed $\quad \dfrac{n\lambda_n}{2} = L, \quad n = 1, 2, 3, \ldots$

$$f_n = \frac{V}{\lambda_n}$$

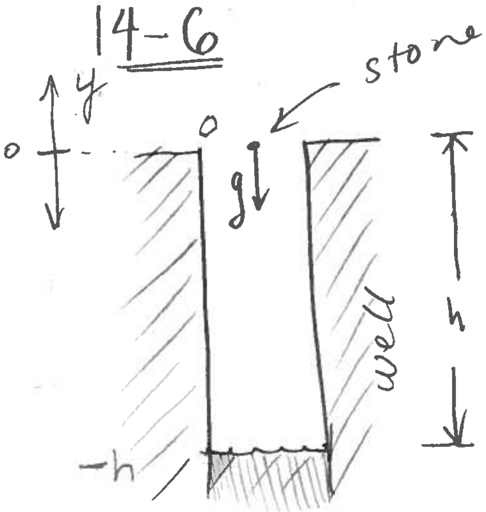
$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

Beat Freq

$$f_b = |f_1 - f_2|$$

PLUS FORMULAE FROM 121

14-6



Phys121

Basic kinematics tells us

$$\boxed{\underset{\text{position with time}}{\vec{y}(t)} = (\underset{\text{initial position}}{y_0} + \underset{\text{initial velocity}}{v_0} t + \tfrac{1}{2}\underset{\text{acceleration}}{a} t^2)\,\hat{y}}$$

Let down be in the negative x-direction and the y-coordinate of ground level be $y=0$. The rock will uniformly accelerate from rest at ground level, so the position of the rock inside the well is given by $\vec{a} = -9.8\,m/s^2\,\hat{y}$

$$\boxed{y(t) = -\tfrac{1}{2} g t^2 = -4.9t^2}$$

When the rock hits the bottom of the well ($t = t_b$)

$$\boxed{-h = -\tfrac{1}{2} g t_b^2 \text{, where } h \text{ is well depth}}$$

Once the rock hits the well bottom a sound wave will propagate out of the well at constant velocity, V_s. If t^* is the time it takes the sound to reach the top of the well then

$$\boxed{h = V_s t^*}$$

V_s is velocity of sound

We know how long it takes from when we drop the rock to when the sound of the splash reaches us, T.

$$T = t_* + t_b = 2\text{sec.}$$

But we also know

$$\frac{1}{2} g t_b^2 = V_s t^* \Big\} \text{ both equal the depth of well}$$

Thus $\frac{1}{2} g t_b^2 = V_s (2 - t_b) \Rightarrow \frac{1}{2} g t_b^2 + V_s t_b - 2V_s = 0$

$$\frac{1}{2} \frac{g t_b^2}{V_s} + t_b - 2 = 0$$

$$g = 9.8\, m/s^2, \quad V_s = 331\sqrt{\frac{283}{273}} = 337\, m/s$$

so we get

$$\frac{9.8\, t_b^2}{674} + t_b - 2 = 0$$

Solve the quadratic in t_b

$$t_b = \frac{-1 \mp \sqrt{1 + 4\left(\frac{9.8}{674}\right)2}}{2\left(\frac{9.8}{674}\right)}$$

$$= 1.95\,s.$$

$$h = \frac{1}{2} g\, t_b^2 = 4.9(1.95)^2$$

$$= \underline{\underline{18.5\,m}}$$

14-7

Let speed of sound in water be V_W and let speed in air be V_A. The time it takes a signal travelling at constant velocity to travel a distance w is

$$\underbrace{W}_{\text{distance}} \over \underbrace{V}_{\text{velocity of signal}}$$

$$\frac{W}{V} = t$$

distance, velocity of signal, travel time

Let d be width of inlet.

$$\frac{d}{V_A} = \text{travel time for sound signal in air} = t_A$$

$$\frac{d}{V_W} = \text{travel time for signal in water} = t_W$$

We know $t_W = t_A + \Delta t$ where Δt is a known quantity. Thus,

$$\frac{d}{V_W} - \frac{d}{V_A} = t_W - t_A = \Delta t = 4.5s$$

$$\boxed{d = \frac{V_W V_A}{V_A - V_W} \Delta t}$$

plug in appropriate velocities to get numerical value.

$V_A = 331\sqrt{\frac{293}{273}} = 343 m/s, \quad V_W = 1530 m/s$

Hence $d = 4.5\left[\frac{1530 \times 343}{1530-343}\right] = 1.99$ km

14-12

Intensity is defined by

I is the intensity of sound

$$\beta \equiv 10 \log\left(\frac{I}{I_0}\right)$$

in terms of decibels, where I_0 is a constant reference.

$$\beta_1 - \beta_2 = 10 \log\left(\frac{I_1}{I_0}\right) - 10 \log\left(\frac{I_2}{I_0}\right)$$

Using properties of logarithms we can write

$$\beta_1 - \beta_2 = 10 \log\left(\frac{I_1}{I_2}\right)$$

$$\boxed{\beta_1 - \beta_2 = 10 \log\left(\frac{100}{200}\right) = 10 \log\left(\tfrac{1}{2}\right)}$$

14-17

We know $\beta \equiv 10 \log\left(\frac{I}{I_0}\right)$ and we know $\log(10^n) = n$. So at 10 km away $\beta = 50 = 10 \log\left(\frac{I}{I_0}\right)$, thus

$I = I_0 \times 10^5 = 10^{-7}$ Watt/m^2

The intensity can be related to the average power. The power of the source remains constant, and the intensity is the power per area. So as the wavefront travels outward the intensity will fall off inversely proportional the area of the wavefront.

The wavefront is spherical because the speed of sound is the same in all directions.

$$\underbrace{I}_{\text{intensity}} = \frac{\overbrace{P_{av}}^{\text{average power of source}}}{\underbrace{4\pi R^2}_{\text{area of spherical surface}}}$$

R = distance from source

$$\boxed{\begin{array}{l} \text{Thus,} \quad P_{av} = 4\pi R^2 I \\ P_{AV} = 4\pi \times (10\times10^3)^2 \times 10^{-7} = 1.3\times10^2 \text{ watt} \end{array}}$$

14-19

We know the intensity in decibals is $\beta = 10 \log\left(\frac{I}{I_0}\right)$.

so $\beta_1 - \beta_2 = 10\log\left(\frac{I_1}{I_0}\right) - 10\log\left(\frac{I_2}{I_0}\right)$

, but $\log(x) - \log(y) = \log(x/y)$.

$$\boxed{\beta_1 - \beta_2 = 10 \log\left(I_1/I_2\right)}$$

The intensity is equal to power per area.

$$I = P/A = P/4\pi R^2$$

If I_1 and I_2 are measured from the same source, then the power producing the sound is the same.

$$\beta_1 - \beta_2 = 10 \log\left(\frac{P/4\pi R_1^2}{P/4\pi R_2^2}\right) = 10 \log\left(\frac{R_2^2}{R_1^2}\right)$$

$$= 20\log \frac{R_2}{R_1}$$

Because $\log(a^x) = x \log a$

$$\boxed{\beta_1 - \beta_2 = 20 \log\left(R_2/R_1\right)}$$

14-21

The Doppler effect can be quantified by the formula

$$f_o = f_s \left(\frac{v + v_o}{v - v_s}\right)$$

velocity of the observer

velocity of source

observed frequency

frequency of source

speed of sound.

For approaching $v_o, v_s > 0$, and when receding $v_o, v_s < 0$. Both v_s and v_o are measured with respect to the air.

For our case $v_o = 0$. When the train approaches

$$f_o = f_s \frac{v}{v - v_s}$$

, and when the train moves away

$$f_o = f_s \frac{v}{v+v_s}.$$

Thus, the difference in the frequencies is

$$\Delta f = f_s v \left(\frac{1}{v-v_s} - \frac{1}{v+v_s} \right)$$

$$\Delta f = f_s v \frac{v+v_s-(v-v_s)}{v^2-v_s^2}$$

part a

$$\boxed{\Delta f = \frac{2 v v_s f_s}{v^2 - v_s^2}}$$

part b wave length as train approaches,

$$f\lambda = v, \quad \lambda = \frac{v}{f_o} = \frac{v}{f_s v}(v-v_s)$$

f_o ↑ frequency observed when approaching

$$\boxed{\lambda = \frac{v-v_s}{f_s}}$$

$f_s = 320\,Hz$

$v_s = 40\,m/s$

Let motion be along x-axis

sound vel. $V = 331\sqrt{\frac{T}{273}} = 331\sqrt{\frac{24+273}{273}} = 345\,m/s$

source moving

$$(f_o)_{approach} = \frac{320 \times 345}{345-40} = 362\,Hz$$

$$(f_o)_{recede} = \frac{320 \times 345}{345+40} = 287\,Hz$$

$$\Delta f = 362 - 287 = 75\,Hz$$

$$\lambda_{app} = \frac{V}{(f_o)_{app}} = \frac{345}{362} = 0.953\,m.$$

14-27

10

We know the velocity when the tuning fork produces f_o.

$$f_o = f_s \frac{V}{V + V_s}$$

sound speed

velocity of tuning fork

observed freq.

natural freq.

receding

We know how long it take for tuning fork to reach V_s.

$$V_s = g t_1$$

time elapsed until fork reaches velocity to produce f_o.

But, that sound has to travel back to observer, and that takes time.

$$\frac{d}{V} = t_{back}$$

distance travelled in time t_1

time for signal to reach observer

velocity of sound.

First, calculate V_s

$f_s = 512 \text{ Hz}$

$f_o = 485 \text{ Hz}$

$$f_o = \frac{f_s V}{V + V_s} = \frac{340 \times 512}{340 + V_s}$$

$$V_s = 18.9 \text{ m/s}$$

The distance it has fallen before reaching this speed is $\left[v^2 = \underset{\overset{\parallel}{0}}{v_0^2} - 19.6(y-y_0)\right]$

$$\Delta y_1 = \frac{(18.9)^2}{19.6}$$

$$= 18.3\,m.$$

Hence $t_{back} = \frac{18.3}{340} = 0.054$ sec.

In t_{back} it drops

$$\Delta y_2(t) = 4.9\,t_{back}^2 + V_s\,t_{back}$$

$$= 4.9\left(\frac{18.3}{340}\right)^2 + (18.9)\left(\frac{18.3}{340}\right)$$

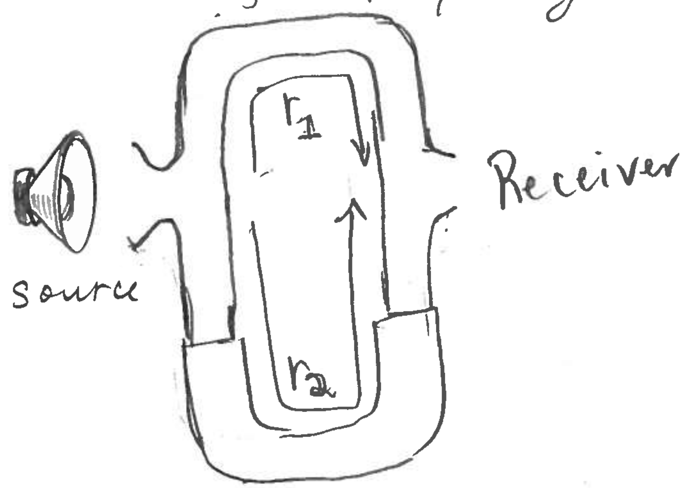
$$= 1.03\,m$$

Total $\Delta y = \Delta y_1 + \Delta y_2$

$$= 18.3 + 1.03 = 19.3\,m$$

14-30

f = frequency of source = 400Hz



r_1 = length of upper path

r_2 = length of lower path

The condition for destructive interference is that two waves arrive at a source 180° out of phase, or the their difference in path lengths is $\lambda(n+\frac{1}{2})$ where λ is wavelength and n is an integer.

Constructive interference occurs if difference in path lengths is $n\lambda$.

Initially the waves destructively interfere. Therefore, $r_2 - r_1 = \lambda(n+\frac{1}{2}) = \frac{v}{f}(n+\frac{1}{2})$.

To hear contstructive interference we must add $\boxed{\frac{v}{2f}}$ to r_2, giving $r_2 - r_1 = \frac{v}{f}n$

For Destructive add a complete wave length or $\frac{v}{f} = \lambda$ to r_2.

a) If at the start there is destructive interference we must increase path difference by $\frac{\lambda}{2}$ to get maximum

wave length is

$$\lambda = \frac{345}{400} = 0.863\,m$$

So increase $(r_2 - r_1)$ by $0.431\,m$ by moving tube by $\frac{0.431}{2}\,m$.

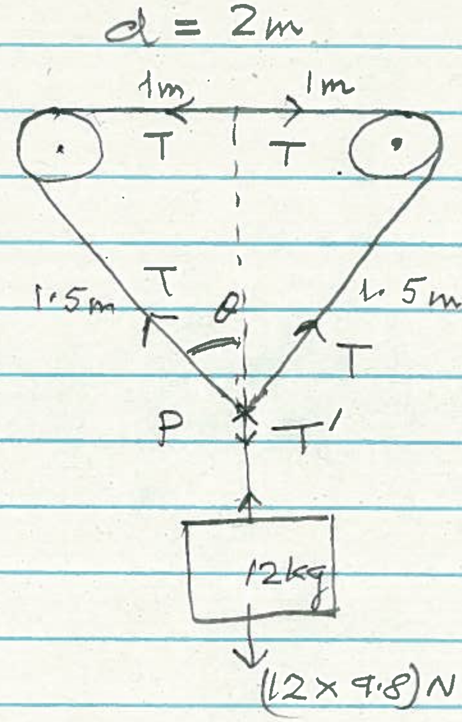
b) to go from minimum to minimum change $(r_2 - r_1)$ by λ $(0.863\,m)$

& move tube by $0.431\,m$

14-39

$L = 5.0\,m$

$\mu = 0.001\,kg/m$



$\sin\theta = 1/1.5$, $\theta = 41.75°$

Pt. P is in ≡m

y- component

$2T\cos\theta - T' = 0$

$T' = 12 \times 9.8\,N.$

$$T = \frac{T'}{2\cos\theta}$$

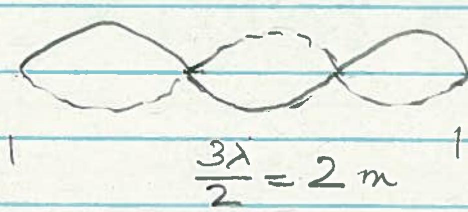
$$= \frac{12 \times 9.8}{2 \times 0.745} = 78.9\,N = \underline{79\,N}$$

Speed of wave is

$$V = \sqrt{\frac{78.9}{10^{-3}}}\ m/s = 280.9\ m/s$$

The mode shown is

$$\lambda = \frac{4}{3}\ m$$

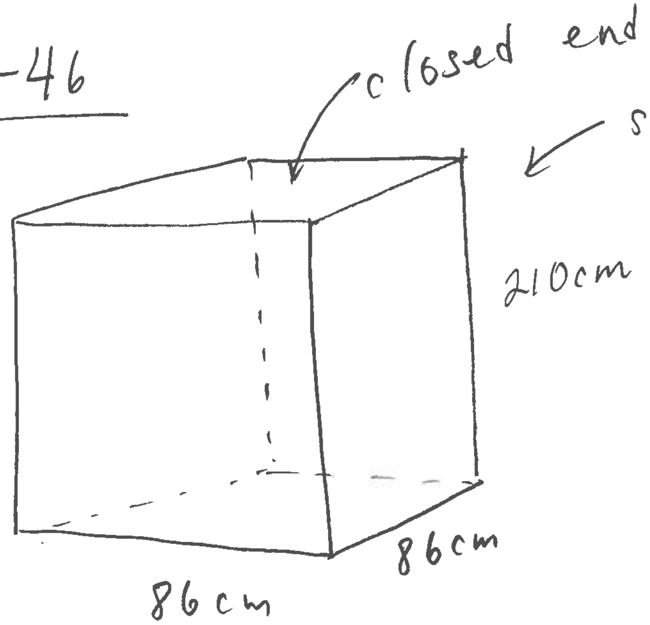


So

$$f = \frac{V}{\lambda} = \frac{280.9 \times 3}{4}$$

$$= 210\,Hz$$

14-46



The frequencies which resonate in this shower will sound richest.

Resonance will occur if an integer number of half wavelengths fit between the walls of the stall.

Possibilities

$$\lambda = \begin{cases} 2(86\text{cm})/n \\ 2(210\text{cm})/m \end{cases}$$

n, m are integers

we know $\boxed{f\lambda = v}$

So corresponding frequencies are

$$f = \frac{v}{\lambda} \begin{cases} \dfrac{vn}{2(86)} \dfrac{1}{\text{cm}} \\ \dfrac{vm}{2(210)} \dfrac{1}{\text{cm}} \end{cases}$$

v is speed of sound taken to be 355 m/s

$1\,\text{m} = 10^2\,\text{cm}$ so $v = 3.55 \times 10^4\,\text{cm/s}$

$$f = \begin{cases} n \dfrac{3.55}{2(86)} \times 10^4 \text{ Hz} = \{206, 413, 619, 826, 1031, \ldots & \boxed{\begin{matrix} n \times 206.4 \text{ Hz} \\ 1 \le n \le 9 \end{matrix}} \\ m \dfrac{3.55}{2(210)} \times 10^4 \text{ Hz} = \{169, 254, \ldots & \boxed{\begin{matrix} 85.5 \times m \\ 2 \le m \le 23 \end{matrix}} \end{cases}$$

14-49

frequency $\times \lambda =$ speed

- wavelength for both mandolins is the same. (identical geometry)

$$\lambda = \frac{\text{speed}}{f} = \frac{c}{528 \text{ Hz}}$$

- Speed $= c = \sqrt{\dfrac{T}{\mu}}$ ← tension, ← linear mass density

- The linear mass density should also be the same between both instruments. Therefore

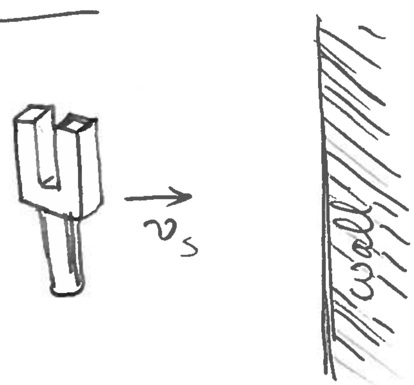
$$\frac{\sqrt{T_1}}{f_1} = \frac{\sqrt{T_2}}{f_2} \qquad \boxed{f_2 = \sqrt{\frac{T_2}{T_1}}\, f_1}$$

The beat frequency is the absolute value of the frequency difference.

$$f_B = |f_1 - f_2| = f_1 \left| 1 - \sqrt{\frac{T_2}{T_1}} \right|$$

$$\boxed{f_B = 523\,Hz \left| 1 - \sqrt{\frac{196}{200}} \right| = 5.3\,Hz}$$

14-53



- frequency of tuning fork, f_s

$f_s = 256\,Hz$

- velocity toward wall, v_s

$v_s = 1.33\ m/s$

a) what is beat frequency between tuning fork and echo?

— we need doppler shift formula.

(*) $$f_o = f_s \left(\frac{V \pm V_o}{V \mp V_s} \right) \quad \left(\begin{matrix} \text{approaching} \\ \text{receding} \end{matrix} \right)$$

There are two doppler shifts that occur here, so well break calculation into steps.

(*) for explanation of this formula see 14-21

Step 1 what frequency does wall hear?

$$f_{o,w} = f_s \left(\frac{V}{V - V_s} \right)$$

frequ. heard by wall

Step 2 This sound reflects from wall with the frequency $f_{o,w}$ and becomes the source for the sound observed by tuning fork.

$$f_{o,tf} = f_{o,w} \left(\frac{V + V_s}{V} \right)$$

although I have put V_s here this is the velocity of observer i.e. tuning fork.

The final expression is

$$\boxed{f_{o,tf} = f_s \left(\frac{V + V_s}{V - V_s} \right) = 1.007 f_s \simeq 258 \, Hz}$$

The beat frequency $f_B \cong 2 Hz$.

To get $f_B = 5 Hz$ what must $V_s = ?$

$$f_B = f_s \left| 1 - \frac{V + V_s}{V - V_s} \right| = f_s \frac{2 V_s}{V - V_s}$$

$$f_B (V - V_s) = 2 V_s f_s \Rightarrow f_B V = V_s (2 f_s + f_B)$$

$$\boxed{V_s = \frac{V f_B}{2 f_s + f_B} = 3.33 \, m/s}$$

velocity to achieve $f_B = 5 \, Hz$