SOLUTION SET-1

FORMULAE

- TRAVELLING WAVE : A Deviation (D) from equilibrium ($\equiv m$) which is a function of both x and t such that x and t appear in the combination

$$(x \mp vt)$$

will travel as a wave with velocity

$$\vec{v} = \pm v\hat{x}$$

PERIODIC WAVE
$$D = A\sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi vt}{\lambda}\right)$$
$$= A\sin\left(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T_0}\right)$$
$$= A\sin\left(\frac{2\pi x}{\lambda} - 2\pi f t\right)$$

A = Amplitude, T_0 = period, λ = wavelength

$f = \frac{1}{T}$ = frequency

speed $v = \frac{\lambda}{T_0} = \lambda f$

If $\vec{A} \parallel \hat{x}$ Longitudinal

If $\vec{A} \perp \hat{x}$ Transverse

speed of wave on stretched string

$$v = \sqrt{\frac{T}{\mu}}$$

T = Tension in string

μ = Mass per meter

Several formulae needed from 121.

[13-38] Since a bat can detect an insect whose size is about one wavelength of the sound it emits, it is neccessary to find the wavelength of that wave. We are given

$$
\begin{aligned}
f &= 60.0 \text{ kHz} \\
&= 60.0 \times 10^3 \text{ Hz} \\
c_S &= 340 \text{ m/s}
\end{aligned}
$$

where f is the frequency and c_S is the speed of sound in air. Now

$$\lambda f = c_S$$

with λ the wavelength. Hence

$$
\begin{aligned}
\lambda &= \frac{c_S}{f} \\
&= \frac{340 \text{ m/s}}{6.00 \times 10^4 \text{ Hz}} \\
&= 56.7 \times 10^{-4} \text{ m} \\
&= 5.67 \text{ mm}
\end{aligned}
$$

[13-41] The generator that creates the wave is going to determine the frequency of that wave. We are given that this oscillator makes 40.0 vibrations in 30.0 s. So, the frequency of the wave must be

$$
\begin{aligned}
f &= \frac{40.0 \text{ vibrations}}{30.0 \text{ s}} \\
&= 1.33 \text{ Hz}
\end{aligned}
$$

We are also given the distance that a maximum travels, and the time that it takes to traverse this distance; from this, we can get the speed of the wave:

$$v = \frac{425 \text{ cm}}{10.0 s}$$

$$
\begin{aligned}
&= \frac{4.25 \text{ m}}{10.0s} \\
&= 0.425 \text{ m/s}
\end{aligned}
$$

We now have the speed of the wave and its frequency so we can get λ

$$
\begin{aligned}
\lambda &= \frac{v}{f} \\
&= \frac{0.425 \text{ m/s}}{1.33 \text{ Hz}} \\
&= 0.320 \text{ m}
\end{aligned}
$$

[13-46] The situation is depicted in Figure 2. The object is assumed at rest, and we have the Free Body Diagram for it shown in Figure 3 We then use Newton's second law, in the y-direction:

$$T - m_{obj}g_{moon} = 0$$

Again, the object is at rest so the acceleration is zero. This gives us g_{moon} in terms of the tension, T.

$$0 = T - m_{obj}g_{moon}$$

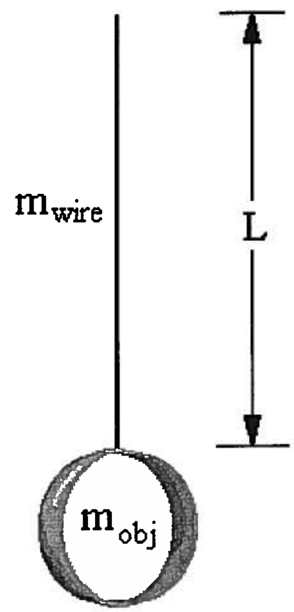


Figure 2: Object Hanging from Wire

$$\begin{aligned} T &= m_{obj} g_{moon} \\ g_{moon} &= \frac{T}{m_{obj}} \end{aligned}$$

If only we could find the tension... Oh, right. We know the speed of the wave; or rather, we can find it since we know the length of the wire ($L = 1.60$ m), and the time it takes traverse the wire (t = 36.1 ms = 36.1×10^{-3} s = 3.61×10^{-2} s), hence

$$\begin{aligned} v &= \frac{L}{t} \\ &= \frac{1.60 \text{ m}}{3.61 \times 10^{-2} \text{ s}} \\ &= 44.3 \text{ m/s} \end{aligned}$$

and we can also find μ:

$$\begin{aligned} \mu &= \frac{m_{wire}}{L} \\ &= \frac{4.00 \times 10^{-3} kg}{1.60 \text{ m}} \end{aligned}$$

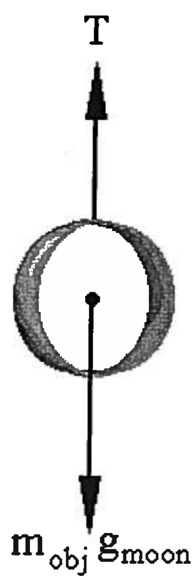


Figure 3: Free Body Diagram of Object

$$= \quad 2.5 \times 10^{-3}\ \mathrm{kg/m}$$

From the class [redacted], we found

$$T = v^2\mu$$

So that

$$\begin{aligned} g_{moon} &= \frac{T}{m_{obj}} \\ &= \frac{v^2\mu}{m_{obj}} \\ &= \frac{(44.3\ \mathrm{m/s})^2\,(2.5 \times 10^{-3}\ \mathrm{kg/m})}{3.00\ \mathrm{kg}} \\ &= 1.64\ \mathrm{m/s^2} \end{aligned}$$

13-48 Here we have two strings with same tensions but different values of μ because lengths are same but masses differ. Hence

$$\mu_1 = \frac{M_1}{L}$$

$$\mu_2 = \frac{M_2}{L} \quad \text{and} \quad M_2 = \frac{M_1}{2}$$

$$\text{so } \mu_2 = \frac{M_1}{2L} = \frac{\mu_1}{2}$$

$$V_1 = \sqrt{\frac{T}{\mu_1}}, \quad V_2 = \sqrt{\frac{T}{\mu_2}}$$

$$\frac{V_2}{V_1} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{2}{1}}$$

$$V_1 = 5.00 \, m/s$$

$$\text{So } V_2 = \sqrt{2}\, V_1 = 7.07 \, m/s.$$

13-51 Here, only the tension is being changed while μ is held constant.

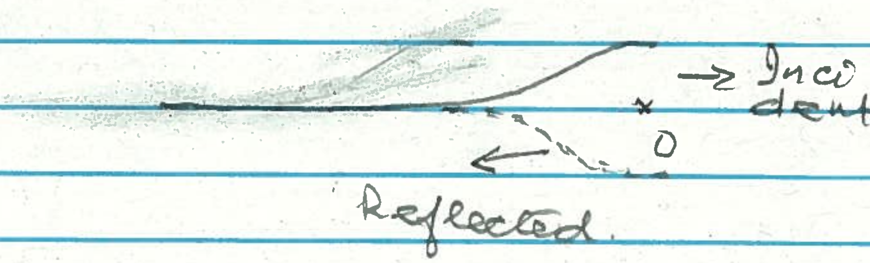
Since $V = \sqrt{\frac{T}{\mu}}$ we can write

$$T = \mu V^2 \quad \text{and} \quad T_1 = \mu V_1^2$$
$$T_2 = \mu V_2^2$$

$$\frac{T_2}{T_1} = \frac{\mu V_2^2}{\mu V_1^2} \text{ gives } T_2 = T_1 \frac{V_2^2}{V_1^2}$$

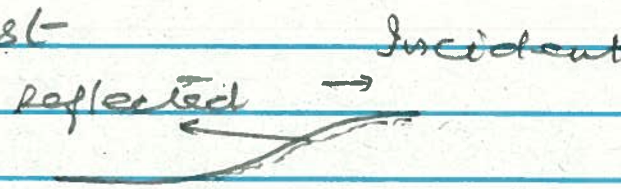
$$= \left(\frac{30}{20}\right)^2 6 = \underline{13.5 N}$$

13-52a As shown in class when the end of the string is fixed, there is a phase change of π upon reflection — a crest changes into a trough and vice versa

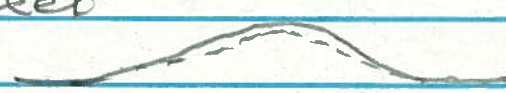


So where the two pulses meet the net amplitude becomes <u>ZERO</u>

13-52 b) If the end is free there is <u>NO</u> phase change
Crest → crest



Now when they meet



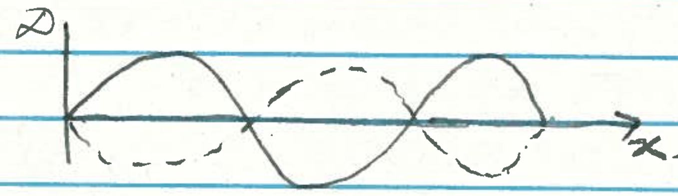
they add and amplitude is doubled

$$2A = 2(0.15) = 0.3\,m$$

13-53a The largest amplitude happens when the two waves are in phase - Maxima coincide

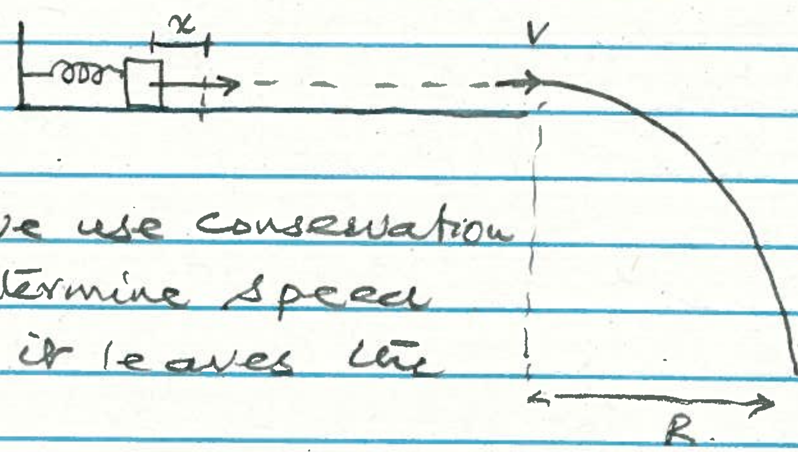
$A_{max} = A_1 + A_2 = 0.30 + 0.20 = 0.50m$

13-53b The smallest amplitude occurs when the two waves are out of phase - +ive Maximum of one coincides with -ive Maximum of the other



$A_{min} = A_1 - A_2 = 0.30 - 0.20 = 0.10\ m$

13-60



The physical situation is as shown. First, we use conservation of Energy to determine speed of pellet as it leaves the spring

$$K_f + P_f = K_i + P_i$$

$$\frac{1}{2} Mv^2 + 0 = 0 + \frac{1}{2} kx^2$$

Here $x = 0.2m$, $k = 9.80\ N/m$ $M = 10^{-3}kg$

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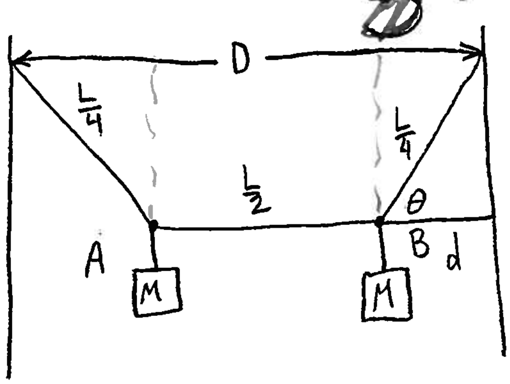
$\boxed{13-64}$



$D = 2.00\,m$
$L = 3.00\,m$
$m = 10.0\,g$
$M = 2.00\,kg$

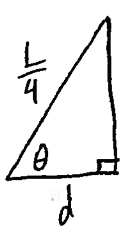
to find the travel time, we need the speed; to find the speed we need the linear mass and the tension; to find the tension we need to solve the free body diagram of the string

$$\mu = \frac{m}{L} = \frac{10.0\times10^{-3}\,kg}{3.00\,m} = 3.33\times10^{-3}\,\frac{kg}{m}$$

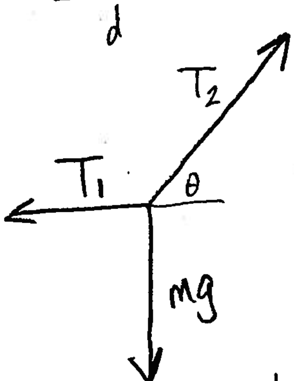
d =

need θ first

$$d = \frac{D - 2\left(\frac{L}{4}\right)}{2} = \frac{2.00 - 2\left(\frac{3.00}{4}\right)}{2} = 0.250\,m$$



$$\cos\theta = \frac{d}{L/4} \Rightarrow \theta = \cos^{-1}\left(\frac{d}{L/4}\right) = \cos^{-1}\left(\frac{0.250}{0.750}\right) = 70.5^\circ$$



free body diagram
since it is in equilibrium, sum of the forces is zero

$$\sum F_x = -T_1 + T_2\cos\theta = 0$$
$$\sum F_y = -mg + T_2\sin\theta = 0$$

two equations, two unknows; solving for $T_1 = 6.93\,N$

now we know enough to find the speed

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T_1}{\mu}} = \sqrt{\frac{6.93}{3.33\times10^{-3}}} = 45.6\,m/s$$

and the time is given by

$$t = \frac{d}{v} = \frac{L/2}{v} = \frac{\frac{3.00}{2}\,m}{45.6\,m/s} = 32.9\times10^{-3}\,s = 32.9\,ms$$

13-60 (Contd)

Hence $v = x\sqrt{\frac{k}{m}} = 0.2\sqrt{\frac{9.8}{10^{-3}}} = 19.8\,m/s.$

Next, the pellet falls under gravity

$$y = y_0 + v_{oy}t - 4.9t^2$$

Here $y_0 = 1m$

$v_{oy} = 0$

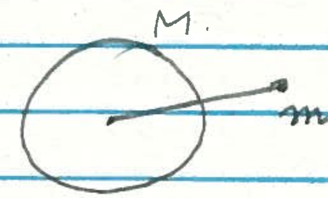
$y = 0$

Time to fall $t = \sqrt{\frac{1}{4.9}} = 0.452s.$

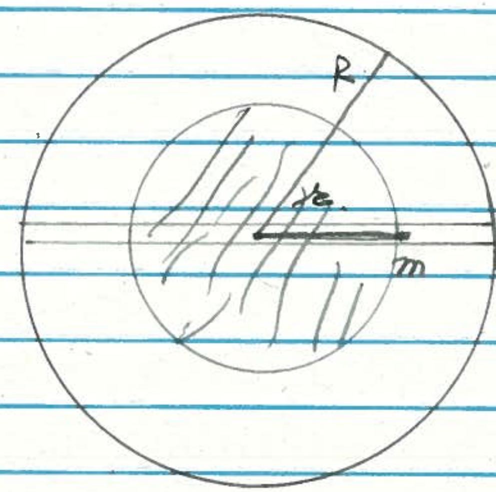
$$R = vt = (19.8)(0.452) = 8.94m$$

13-65 In Phys 121 we learnt that when a point mass is located inside a hollow spherical mass (M) the gravitational force on it is zero. while if it is outside the sphere the gravitational force is

$$\vec{F}_G = -\frac{G M m}{r^2}\hat{r}$$



So if m is inside a solid sphere the only mass that gives $\vec{F}_G$ on m is that in the shaded volume.



Now density

$$\rho = \frac{M}{V}, \quad V = \text{Vol. of sphere}$$

so mass in shaded region is

$$M_{sh} = \rho \frac{4\pi}{3} r^3$$

hence

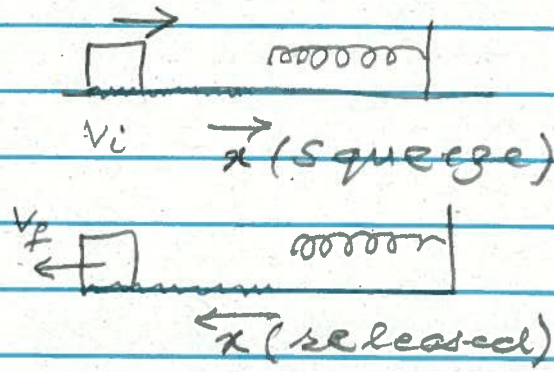
$$\vec{F}_G = -G \rho \frac{4\pi}{3} r^3 \frac{m}{r^2} \hat{r}$$

$$= -\frac{4\pi}{3} G \rho m r \hat{r}$$

which is a force proportional to displacement r and opposite to it $\vec{F}_G = -k r \hat{r}$, hence m will oscillate about $r = 0$

13–68

The physical situation is that mass M comes with velocity $v_i \hat{x}$ and leaves with velocity $-v_f \hat{x}$.



The spring is unstretched in both the initial and final states having been squeezed by an amount x and allowed to recover.

The Energy conservation law is:

$$K_f + P_f = K_i + P_i + W_{NCF}$$

W_{NCF} = work done by friction $(f_k) = -f_k(2x)$
$= -\mu_k (2x) Mg$

$P_f = P_i = 0$

$K_f = \frac{1}{2} M v_f^2$

$K_i = \frac{1}{2} M v_i^2$

$$\frac{1}{2} \times 8 \times 3^2 + 0 = \frac{1}{2} \times 8 \times 4^2 - 2 \times x \times 0.4 \times 8 \times 9.8$$

$$x = \frac{64 - 36}{2 \times 0.4 \times 8 \times 9.8} = 0.446 \text{m}$$