

Name: SOLUTION

(Sign in ink, print in pencil)

Notes

- 1) There are six (6) problems in this exam. Please make sure that your copy has all of them.
- 2) Please show your work indicating clearly what formula you used and what the symbols mean. Just writing the answer will not get you full credit. In stating vectors give both magnitude and direction.
- 3) Write your answers on the sheets provided.
- 4) Do not forget to write the units.
- 5) Do not hesitate to ask for clarification at any time during the exam. You may buy a formula at the cost of one point.

Take Care! God Bless You!

$$k_e = 9 \times 10^9 \frac{N \cdot m^2}{C^2}, \mu_0 = 4\pi \times 10^{-7} \frac{T \cdot m}{A}$$

$$\epsilon_0 = 9 \times 10^{-12} \frac{F}{m}$$

$$\text{Mass of proton} \quad m_p = 1.6 \times 10^{-27} \text{ kg}$$

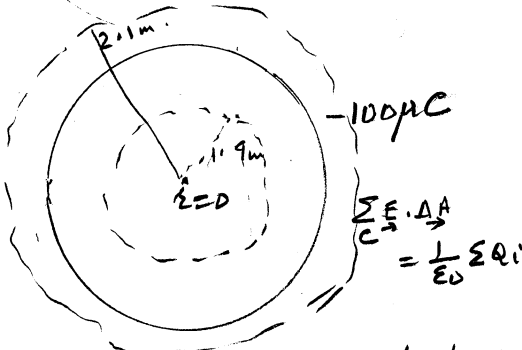
$$\text{Mass of electron} \quad m_e = 9 \times 10^{-31} \text{ kg}$$

$$\text{Elementary Charge} \quad e = 1.6 \times 10^{-19} \text{ C}$$

Problem 1a

A conducting sphere of radius 2m is placed with its center at $r=0$ and carries a charge of $-100\ \mu\text{C}$. Calculate the \vec{E} -field at $r=1.9\text{m}$ and $r=2.1\text{m}$. (16)

Conducting sphere - charge can sit only on the surface



We have spherical symmetry about $r=0$ so \vec{E} is a function of r only and along \hat{r} .

For using Gauss's law: choose sphere of radius r centered at $r=0$.

If $r = 1.9\text{m}$

$$\sum \vec{E} \cdot \Delta \vec{A} = E(1.9) 4\pi(1.9)^2 = 0 \quad (\text{No charge inside})$$

So $\underline{\underline{\vec{E} = 0}}$

If $r = 2.1\text{m}$

$$\sum \vec{E} \cdot \Delta \vec{A} = E(2.1) 4\pi(2.1)^2 = -\frac{100 \times 10^{-6}}{9 \times 10^{-12}}$$

So $\vec{E} = -\frac{100 \times 10^{-6}}{9 \times 10^{-12} \times 4\pi \times (2.1)^2} \text{ N/C } \hat{r}$
 $= -2 \times 10^5 \text{ N/C } \hat{r}$

Problem 2a

Why is there a minus sign on the right side of the equations

$$\Delta P = -\underline{F}_E \cdot \underline{\Delta S}$$

$$\Delta V = -\underline{E} \cdot \underline{\Delta S}$$

where ΔP is change in potential energy and ΔV is change in potential. (2, 2)

Potential Energy measures the work done to assemble a system in the presence of a conservative force such as \underline{F}_E . The force which does the work has to be opposite to the conservative, hence the minus.

Sign.
$$\Delta V = \frac{\Delta P}{q} = - \frac{\underline{F}_E \cdot \underline{\Delta S}}{q} = - \underline{E} \cdot \underline{\Delta S}$$

Problem 2b

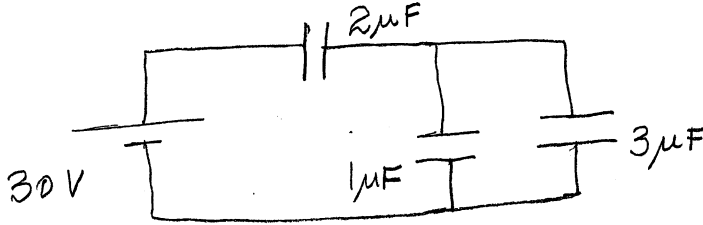
In order to place a charge Q on a capacitor C one has to put in $\frac{Q^2}{2C}$ Joules of work. Where does this energy go? Why? (4)

This energy is stored in the \underline{E} field inside the capacitor.

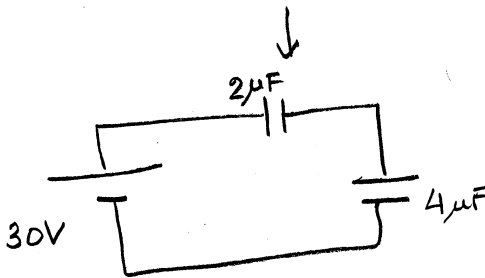
Problem 2c

In the circuit shown which capacitor has i) the highest potential difference and ii) which capacitor has the smallest charge? Why?

(8)

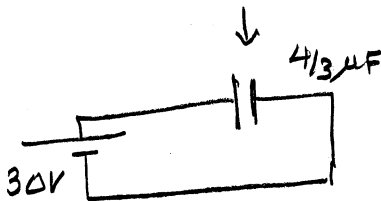


↓ Parallel.
 $C_p = \sum C_i$
 $\frac{1}{C_s} = \sum \frac{1}{C_i}$
 ↓ Series



$$\frac{1}{2} + \frac{1}{4} = \frac{1}{C_s}$$

$$C_s = \frac{8}{6} \mu F$$



$$Q = CV \text{ so } Q = \frac{4}{3} \times 30 = 40 \mu C.$$

In Series Q is common so
 $Q_2 = Q_4 = 40 \mu C.$

$$V = \frac{Q}{C}, \quad V_2 = \frac{40}{2} = \underline{\underline{20V}} \leftarrow \text{highest pote. Diff.}$$

In Parallel V is common so

$$V_1 = V_3 = \frac{40}{4} = 10V.$$

$$Q_1 = 1 \times 10 = 10 \mu C \rightarrow \text{Smallest Charge}$$

Problem 3a

Discuss the physical bases of Kirchhoff's Rules.

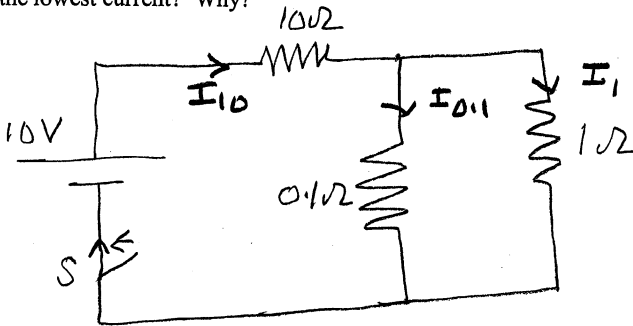
(3,3)

Loop Rule. Change of potential is independent of the path $A \rightarrow B$ $\Delta V_{AB} = -\Delta V_{BA}$
So total change in closed loop is zero

JUNCTION RULE: Current is flow of charge, charge is conserved so at a junction
 $\Sigma I_{out} = \Sigma I_{in}$.

Problem 3b

In the circuit shown, when you close the switch which resistor has i) the highest current and ii) the lowest current? Why? (10)



Jn. Rule $I_1 + I_{0.1} = I_{10}$
 I_{10} is the highest current.

Loop Rule $V_{0.1} = V_1$
 $I_1 = \frac{V_1}{1}$ is the lowest current.

Problem 4a

In a RC circuit, (seen below) the characteristic time is $T = RC$. Why do both R and C appear in T?

The process involves transport of charge. ⁽⁶⁾

→ R controls rate of transfer of charge, larger R slower transfer.

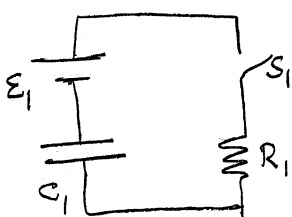
→ C controls total amount of transfer

$T_{max} = CE$ so larger C, longer it takes.

Problem 4b

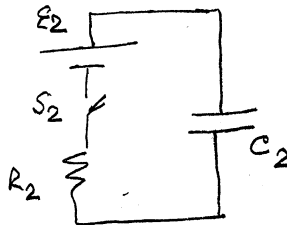
Shown are two RC circuits.

If both switches are closed at the same time which capacitor potential will reach 5 volts first if i) $\epsilon_1 = \epsilon_2 = 10V$ or ii) $\epsilon_1 = 9V, \epsilon_2 = 10V$? Why? (10)



$R_1 = 10k\Omega$

$C_1 = 10\mu F$



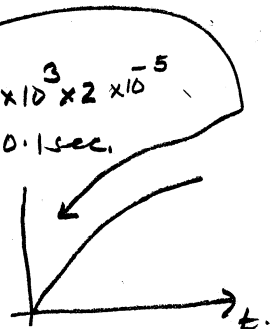
$R_2 = 5k\Omega$

$C_2 = 20\mu F$

$V_C = E[1 - e^{-t/RC}]$

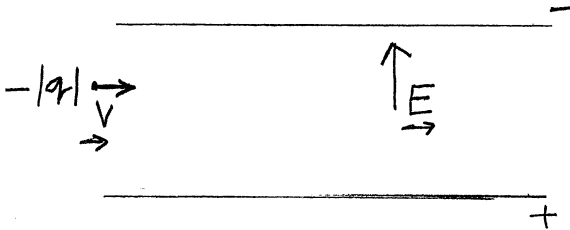
Here $R_1 C_1 = 10^5 \times 10^4 = 0.1 \text{ sec}$, $R_2 C_2 = 5 \times 10^3 \times 2 \times 10^{-5} = 0.1 \text{ sec}$.

- i) If $\epsilon_1 = \epsilon_2$ both arrive together.
- ii) If $\epsilon_2 > \epsilon_1$, C_2 will arrive first b/c parameters value can be smaller.



Problem 5

Two parallel plates have uniform $\underline{E} = 100\text{N}/\text{C}\hat{y}$ between them. Introduce a particle of charge $-q$ traveling at $v = 10^4\text{m}/\text{s}\hat{x}$. What \underline{B} field, magnitude and direction, would you apply so that the particle goes through the plates undeflected. (16)



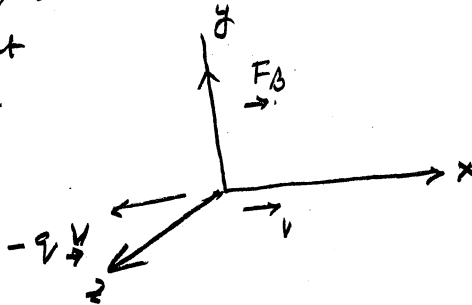
$$\underline{F}_E = q \underline{E} = -100\text{N}\hat{y}$$

So to go undeflected \underline{F}_B must be along $+\hat{y}$.

Now $\underline{F}_B = q [\underline{v} \times \underline{B}]$.

so by right hand rule

we need $\underline{B} \parallel +\hat{z}$.



$$\underline{F}_B = qvB\hat{y}$$

$$\underline{F}_E + \underline{F}_B = 0$$

$$-100q + q \times 10^4 B = 0$$

$$\underline{B} = +\frac{100}{10^4} \hat{z} = 10^{-2}\text{T}\hat{z}$$

Problem 6

- a) A moving charge in a \underline{B} field goes on a circular path. Why is its kinetic energy constant? (5)

The force that makes the charge go on a circle is $\underline{F}_B = q [\underline{v} \times \underline{B}]$.

Since $\underline{F}_B \perp \underline{v}$, it cannot do any work on the charge so it cannot change kinetic energy.

- b) What is the difference between a Coulomb \underline{E} -field and a Non-Coulomb \underline{E} -field? (5)

Coulomb \underline{E} is generated by a stationary charge

$$\underline{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$$

Non-Coulomb \underline{E}_{NC} is generated when flux of \underline{B} varies with time and \underline{E}_{NC} appears in every loop surrounding the region where ϕ_B is changing

$$\sum_c \underline{E}_{NC} \cdot \underline{\Delta l} = - \frac{\Delta \phi_B}{\Delta t}$$

Problem 6 (cont'd)

c) What is a Conservative Force?

(4)

A force is conservative if the work done by it is independent of the path.

d) How do mass (m) and charge (q) help you to discover the Gravitational field (\underline{G}_g), the \underline{E} -field and the \underline{B} field?

(6)

m is needed b/c it experiences a force $\underline{F}_G = m \underline{G}_g$ in \underline{G}_g

stationary q is needed b/c it experiences a force $\underline{F}_E = q \underline{E}$ in \underline{E} .

Moving q is needed b/c it experiences a force $\underline{F}_B = q [\underline{v} \times \underline{B}]$ in \underline{B} .