

Name: SOLUTION

(Sign in ink, print in pencil)

Notes

- 1) There are six (6) problems in this exam. Please make sure that your copy has all of them.
- 2) Please show your work indicating clearly what formula you used and what the symbols mean. Just writing the answer will not get you full credit. In stating vectors give both magnitude and direction.
- 3) Write your answers on the sheets provided.
- 4) Do not forget to write the units.
- 5) Do not hesitate to ask for clarification at any time during the exam. You may buy a formula at the cost of one point.

Take Care! God Bless You!

$$k_e = 9 \times 10^9 \frac{N \cdot m^2}{C^2}, \mu_0 = 4\pi \times 10^{-7} \frac{T \cdot m}{A}$$

$$\epsilon_0 = 9 \times 10^{-12} \frac{F}{m}$$

Mass of proton  $m_p = 1.6 \times 10^{-27} \text{ kg}$

Mass of electron  $m_e = 9 \times 10^{-31} \text{ kg}$

Elementary Charge  $e = 1.6 \times 10^{-19} \text{ C}$

Problem 1a

Fill in the blanks in the equation

$$D = \sin(x-vt)$$

where  $D$  is the deviation from equilibrium. Explain the physical meaning of each of the symbols that you write. (10)

Complete eqn is  $D = A \sin \frac{2\pi(x-vt)}{\lambda}$

$A$  is necessary because  $D$  has dimensions/units while  $\sin$  is dimensionless. Indeed, unit of  $A$  tells us nature of wave.

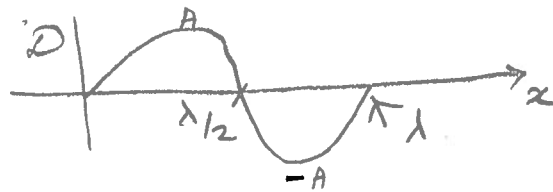
$\lambda$ , a length, is necessary because argument of  $\sin$  cannot have dimensions.

$2\pi$  is included to indicate that  $\sin$  repeats every  $2\pi$ .

$A$  is amplitude, gives largest value of  $D$ .

$\lambda$  is repeat distance hence wavelength

That is, at  $t=0$

Problem 1b

What is the difference between a longitudinal wave and a transverse wave? (6)

Write down the sine wave

$$D = A \sin \frac{2\pi(x-vt)}{\lambda}$$

Longitudinal:  $\vec{A} \parallel \hat{x}$

Transverse:  $\vec{A} \perp \hat{x}$

Problem 2a

What is sound? (6)

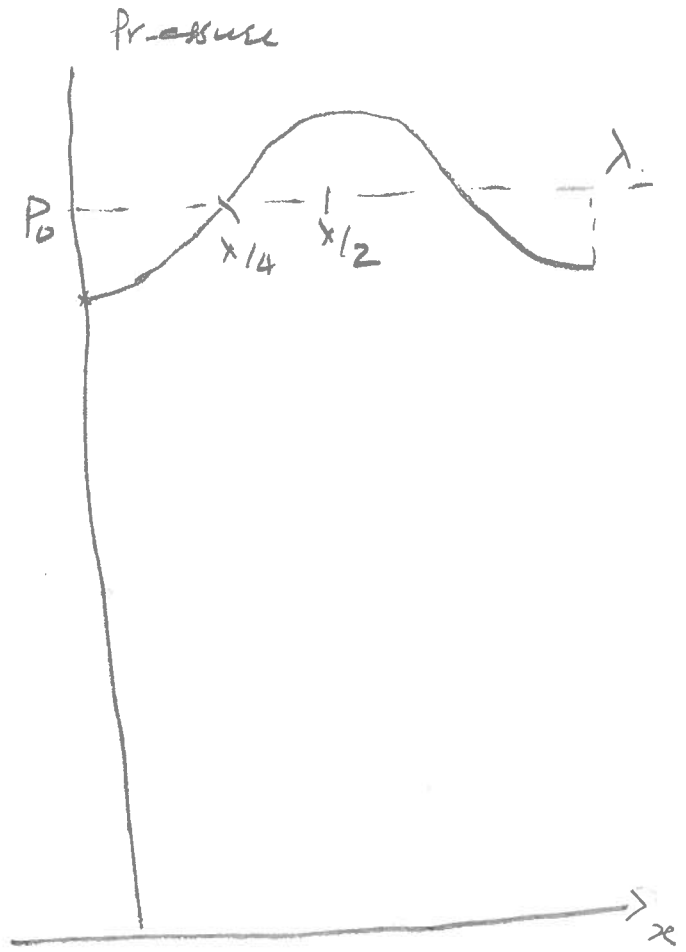
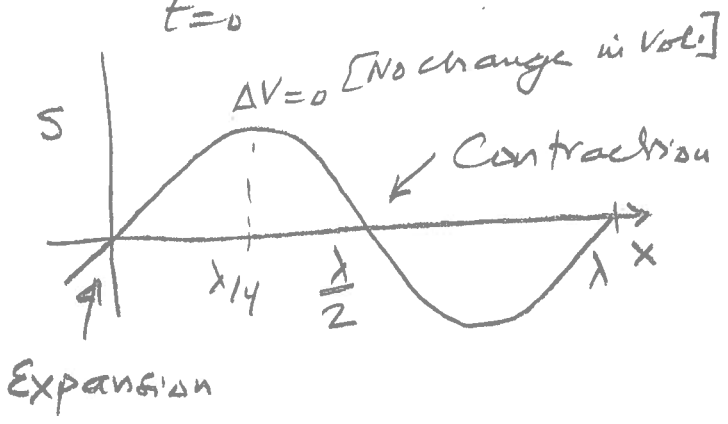
Any mechanical wave whose frequency is between 20 Hz and 20 kHz. We call it SOUND because our EARS detect it.

Problem 2b

Draw a periodic (sine) sound wave as a (i) displacement wave and (ii) as its corresponding pressure wave. (10)

Displacement  
 $S = S_m \sin(kx - \omega t)$

$t = 0$



Problem 3

The intensity of a sound wave in air is given by

$$I = \frac{1}{2} \gamma \frac{P_0 S_m^2 \omega^2}{v_s} \quad (16)$$

Calculate the amplitude  $S_m$  of a sound wave of a frequency  $\omega = 1000 \text{ rad/s}$  if

$$I = 10^{-12} \text{ watt/m}^2$$

$$\gamma = 1.4$$

$$P_0 = 10^5 \text{ N/m}^2$$

$$v_s = 340 \text{ m/sec}$$

$$10^{-12} = \frac{1}{2} \times \frac{1.4 \times 10^5 \times S_m^2 \times (1000)^2}{340}$$

$$S_m^2 = \frac{2 \times 340 \times 10^{-12}}{1.4 \times 10^5 \times 10^6} \text{ m}^2$$

$$= \frac{2 \times 34}{1.4} \times 10^{-22} \text{ m}^2$$

$$\underline{S_m = 7 \times 10^{-11} \text{ m}}$$

which is roughly equal to the radius of a hydrogen atom.

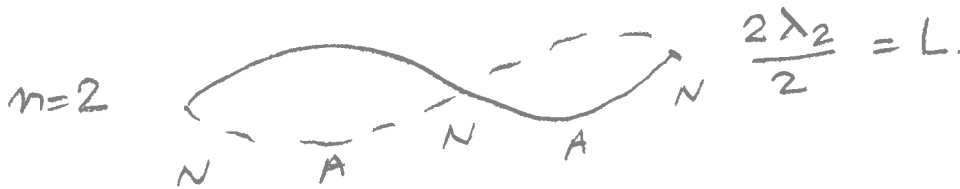
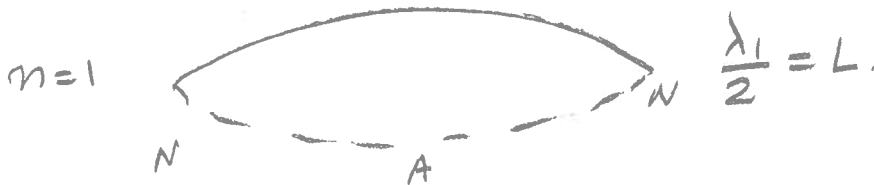
Problem 4

Draw the first three modes of vibration of a wire fixed at both ends. If the wire is 1m long, has a mass of 1 gram and a tension of 10N in it, what is the frequency of the fundamental mode? (16)

If wire is fixed at both ends  
modes obey

$$\frac{n \lambda_n}{2} = L$$

where  $n$  is an integer.



$$L = 1\text{m}$$

$$v = \sqrt{\frac{10}{10^{-3}}} = 100\text{m/s}$$

$$\lambda_1 = 2L = 2\text{m}$$

$$f_1 = \frac{100}{2} = 50\text{Hz}$$

$$1\text{gm} = 10^{-3}\text{kg}$$

$$\mu = 10^{-3}\text{kg/m}$$

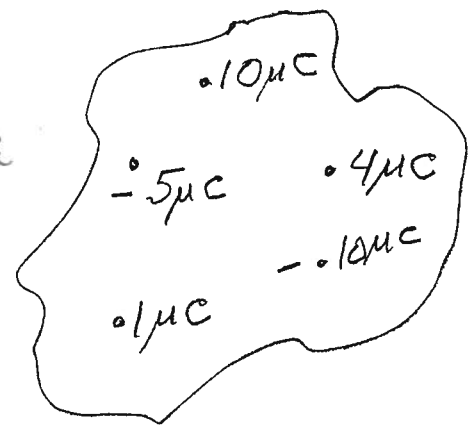
Problem 5a

The picture shows a closed surface and the charges enclosed. What is the Total flux of the  $\underline{E}$ -field through this surface? Why? (10)

Gauss' law

$$\sum_{\vec{c} \rightarrow} \underline{E} \cdot \underline{\Delta A} = \frac{1}{\epsilon_0} \sum Q_i$$

Total flux of  $\underline{E}$  through a closed surface is determined solely by enclosed charges.



Here

$$\sum Q_i = 10 + 4 - 10 - 5 - 1 = 0$$

hence  $\sum_{\vec{c} \rightarrow} \underline{E} \cdot \underline{\Delta A} = 0.$

Problem 5b

What is the  $\underline{E}$ -field at any point of the closed surface? (6)

We cannot calculate the  $\underline{E}$ -field as the geometry is not specified. We neither know the location of the charges nor of the surface.

Problem 6a

Which force is larger, the force between  $1\mu\text{C}$  and  $-1\mu\text{C}$  which are  $1\text{m}$  apart or the force between  $1\mu\text{C}$  and  $1\mu\text{C}$  which are also  $1\text{m}$  apart? Why? (5)

$$\vec{F}_E = k_e \frac{q_1 q_2}{r^2} \hat{r}$$

$$\vec{F}_{1,-1} = -k_e \frac{10^{-6} \times 10^{-6}}{1} \hat{r} = -\frac{9 \times 10^9 \times 10^{-12}}{1} \text{N} \hat{r} = -0.009 \text{N} \hat{r}$$

$$\vec{F}_{1,1} = +\frac{9 \times 10^9 \times 10^{-12}}{1} \text{N} \hat{r} = +0.009 \text{N} \hat{r}$$

Both forces have SAME magnitude. However, one is attractive and the other repulsive.

Problem 6b

What is the speed of sound on the moon? (3)

There is NO SOUND ON THE  
MOON AS THERE IS NO ATMOSPHERE

Problem 6c

Do you think that the following wave can exist?

$$P = 100 \text{ N/m}^2 + 200 \text{ N/m}^2 \cos(kx - \omega t)$$

Justify your answer. (5)

No. ~~press~~ This is a pressure wave and pressure can never be -ive. In this wave  $P$  will become -ive when  $\cos$  becomes more negative than  $0.5$ .

Problem 6d

Charge  $-|q|$  at  $x = -a$ ,  $+q$  at  $x = +a$ . Show that the  $\vec{E}$ -field at  $P(0, y)$  is along  $-\hat{x}$ . (7)

$$\vec{E} = \frac{k_e q \hat{r}}{r^2}$$

The directions of  $\vec{E}_+$  &  $\vec{E}_-$  at pt.  $P$  are as shown. Magnitudes of  $E_+$  &  $E_-$  are equal being

$$E_+ = E_- = \frac{k_e q}{(a^2 + y^2)}$$

Hence  $y$ -components cancel and only  $x$ -component survives along  $-\hat{x}$  as shown.

