SOUND-WIND INSTRUMENTS

In a wind instrument sound is generated by causing vibrations in a column of air contained in a tube which may be open at both ends or open at one end and closed at the other. In analogy with the case of the wire of finite length we again set up a series of MODES of vibration in the air column. So, we begin by recalling that for a wire (transverse displacement wave) when reflection occurs at a fixed end there is a phase change of π so that wire of length L, fixed at both ends can have only those modes for which the wavelength λ_n satisfies the Equation:

$$n\left(\frac{\lambda_n}{2}\right) = L \qquad (1)$$

or frequencies

$$f_n = \frac{n}{2L}\sqrt{\frac{F}{\mu}} \qquad (2)$$

Before we use these Equations to write down the modes in an air column. We should ask what happens when the end is not fixed. Recall that if we send an incident wave

$$y = A_i Sin(kx - \omega t)$$
$$\omega = vk$$

to a point at x=0 where v changes to v' we will get a reflected wave

$y_r = A_r Sin(kx + \omega t)$ and a transmitted wave $\begin{aligned} y_t &= A_t Sin(k'x - \omega t) \\ \omega &= v'k' \end{aligned}$

Note again ω does not change!

Also

$$\frac{A_r}{A_i} = \frac{v - v'}{v + v'}, \quad \frac{A_t}{A_i} = \frac{2v'}{v + v'}$$

To simulate an open end we go to the other limit $\mu' \to 0, v' >> v$. Then $\frac{A_r}{A_i} = -1,\ A_t = 2A_i$

N.B. It may appear that A_t being $2A_i$ transmitted power will be large but that is not true, because on a wire $\langle P \rangle = \frac{1}{2} A^2 w^2 \frac{F}{v}$

Since $v' \to \infty, \langle P_t \rangle \to 0$.

$$y_i = A_i Sin(kx - \omega t)$$

So now on the wire at left we have $\frac{A_r}{A_i} = -1$

$$y_r = -A_i Sin(kx + \omega t)$$

We compare phases at x=0

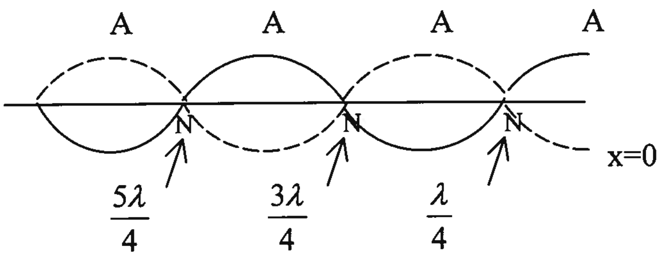
$y_i = A_i Sin(-\omega t) = -A_i Sin\omega t$

$y_r = -A_i Sin\omega t$

so now there is NO phase change on reflection-incoming crest gives rise to outgoing crest.
By superposition,

$y = y_i + y_r = 2A_i Cos kx\ Cos\omega t$ And this is also NOT a traveling wave. Indeed, I) now we have an ANTINODE at x=0

II) The first node is at $\frac{\lambda}{4}$



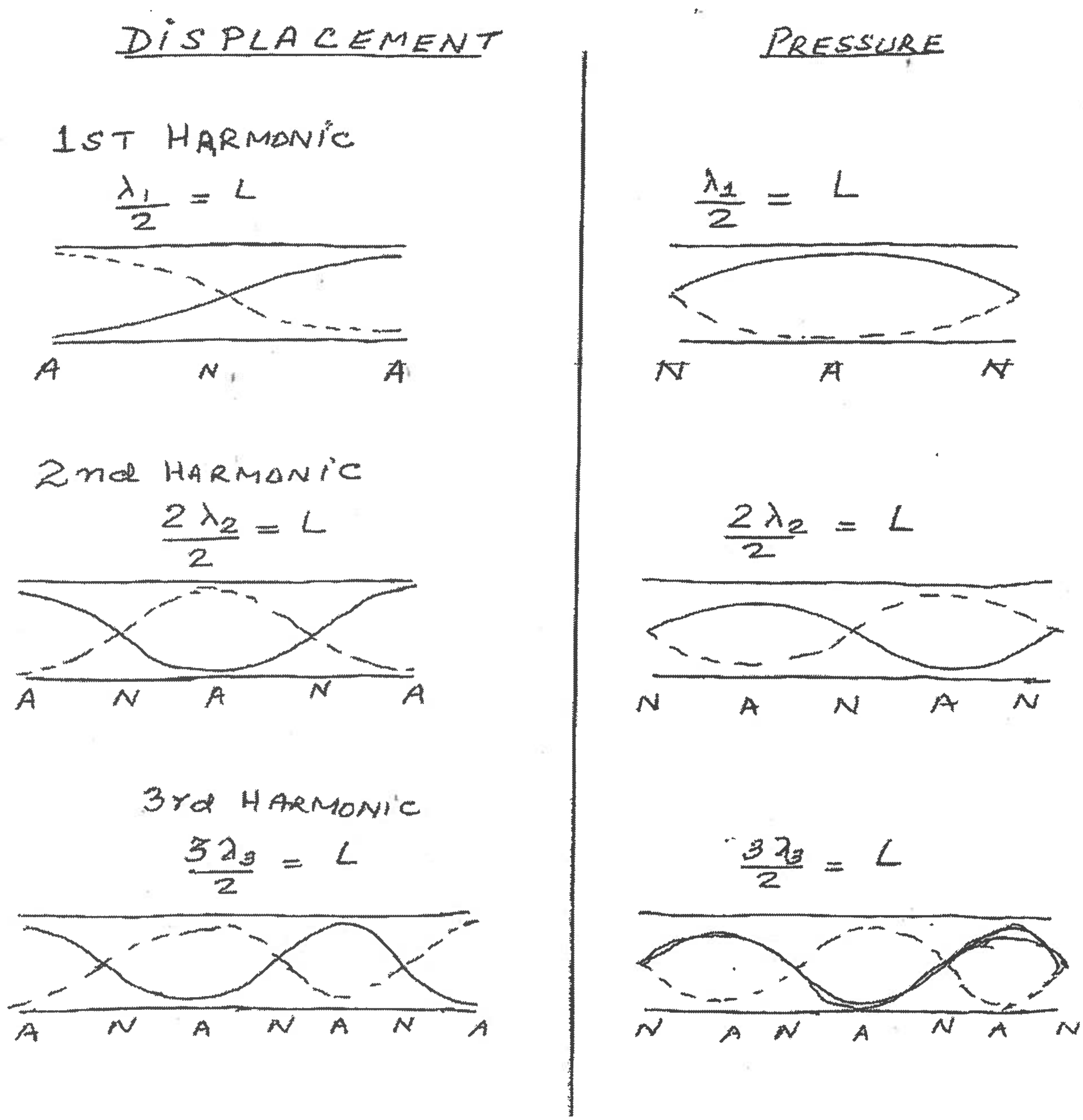
So for an "open" end we also get stationary wave but now the end is an antinode.

The above applies to displacement waves and since sound can be thought of as a displacement or pressure wave we have the rules

	Displacement ($\varnothing = 0$)	($\varnothing = -\pi/2$)
Open end	ANTINODE	NODE
Closed end	NODE	ANTINODE

Now we can draw the nodes for vibrations in a column of air in a tube of length L.

Case I: Both ends open



That is $n\left(\frac{\lambda_n}{2}\right) = L$ $\qquad$ $n = 1,2,3\ldots$

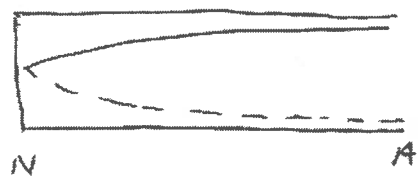
So the frequencies go as 1,2,3…

Case II One end open, one closed

DISPLACEMENT

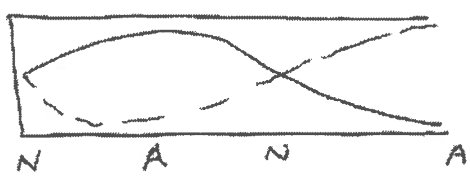
1st HARMONIC

$$\frac{\lambda_1}{4} = L$$



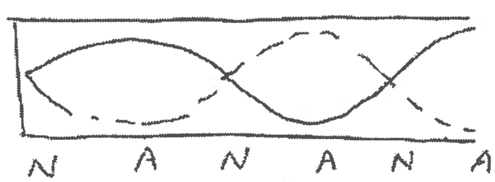
2nd HARMONIC
(ACTUALLY 3rd).

$$\frac{3\lambda_2}{4} = L.$$



3rd HARMONIC
(ACTUALLY 5th).

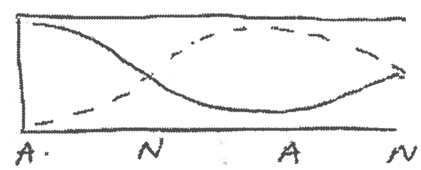
$$\frac{5\lambda_3}{4} = L$$



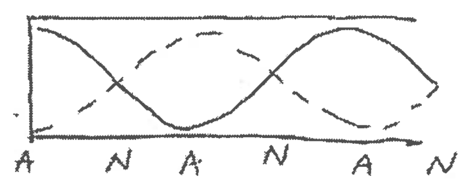
PRESSURE

$$\frac{\lambda_1}{4} = L$$

$$\frac{3\lambda_2}{4} = L$$



$$\frac{5\lambda_3}{4} = L$$



So the modes obey

$$(2n-1)\frac{\lambda_n}{4} = L$$

$$n = 1,2,3\ldots$$

$$f_n = \frac{(2n-1)}{4L}\sqrt{\frac{\gamma P_0}{\rho_0}}$$

and in this case the frequencies go as 1,3,5…(first, third, fifth harmonic)