

SOUND

- a) There is NO sound in vacuum; you need matter to propagate a sound wave.
 b) SOUND: Any mechanical wave whose frequency lies between 20Hz and 20,000Hz, that is, $20\text{Hz} \leq f \leq 20\text{kHz}$ (It's called SOUND b/c you can hear it!)
 c) We will work with sound in Gases only-then sound is a purely Longitudinal wave.
 d) Sound is a longitudinal displacement wave or a longitudinal pressure wave.
 e) Periodic Sound wave *properties*

DISPLACEMENT

Sine wave, $\phi = 0$

$$S = S_m \sin(kx - \omega t)$$

amplitude $S_m \parallel \hat{x}$

$$\omega = vk$$

Displacement oscillates about zero.

PRESSURE

To write corresponding pressure wave we have to realize that the variation occurs so rapidly that there is no possibility for exchange of heat (DQ) to ensure equilibrium with surroundings, so $DQ=0$, sound is an adiabatic process: *Pressure and Volume satisfy:*

$$P_0 V_0^\gamma = \text{constant.}$$

$$P_0 = \text{ambient pressure}$$

$$\gamma = \frac{C_p}{C_v}, C_p = \text{sp ht at const } P$$

$$C_v = \text{sp ht at const } V$$

$$\gamma_{\text{monoatomic}} = \frac{5}{3}$$

$$\gamma_{\text{diatomic}} = \frac{7}{5}$$

$$\rightarrow \phi = \frac{-\pi}{2}$$

$$P = P_0 - \gamma P_0 S_m k \cos(kx - \omega t)$$

Pressure oscillates about P_0

Amp of pressure wave

$$P_m = \gamma P_0 S_m k$$

$$P_m \parallel \hat{x}$$

Pressure is $\frac{\pi}{2}$ out of phase with displacement. where S is max. $(P-P_0) = 0!$

f) Speed of sound in a gas

$$v = \sqrt{\frac{\gamma P_0}{\rho_0}} \quad \rho_0 = \text{ambient density}$$

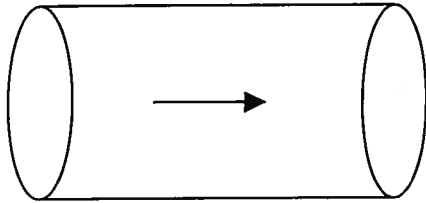
since $P_0 V_0 = N k_B T$ $[k_B = 1.383 \times 10^{-23} \text{ J/K}]$

If gas particles have mass m , we can write

$$P_0 = \frac{Nm}{V_0} \frac{k_B T}{m}, \text{ or } \frac{P_0}{\rho_0} = \frac{k_B T}{m}$$

$$v = \sqrt{\frac{\gamma k_B T}{m}} = \sqrt{\gamma} \frac{v_{rms}}{\sqrt{3}} \quad [T \text{ in Kelvin Scale}]$$

g) Intensity of sound wave: Imagine that the wave is traveling with velocity v through a tube of cross-section A .



Since $S = S_m \sin(\kappa x - \omega t)$ the particle velocity is $V_p = S_m \omega \cos(\kappa x - \omega t)$ and kinetic energy

per unit volume is $K \cdot E = \frac{1}{2} \rho_0 S_m^2 \omega^2$.

Volume of wave traveling past every cross-section will be $A v$ in one second. Energy transport per second through area

$$A = \frac{1}{2} A \rho_0 S_m^2 \omega^2 v$$

Intensity I = energy transport per second per m^2

$$= \frac{1}{2} \rho_0 S_m^2 \omega^2 v \quad \left[\rho_0 = \frac{\gamma P_0}{v^2} \right]$$

$$= \frac{1}{2} \gamma P_0 S_m^2 \frac{\omega^2}{v}$$

Please compare this with energy transport per second on wire $\langle P \rangle = \frac{1}{2} A^2 \frac{\omega^2}{v} F$

h) Smallest discernable intensity is $I_0 = 10^{-12} \text{ Watt / m}^2$

So we define decibels $db = 10 \log \frac{I}{I_0}$

[bel comes from Alexander Graham Bell]

That is 90db sound has $9 = \log \frac{I}{10^{-12}}$, $I = 10^{-3} \text{ Watt / m}^2$

i) Amplitude of displacement wave for I_0 ($\omega = 10^3 \text{ rad/s}$, $\gamma = 1.4$, $P_0 = 10^5 \text{ N/m}^2$),

$$10^{-12} = \frac{1}{2} \times 1.4 \times 10^5 \times \frac{S_m^2 \times 10^6}{340}$$

$$S_m \cong 10^{-10} \text{ m}$$

Roughly equal to diameter of hydrogen atom. REMARKABLE!!!

That is your Ear is sensitive to motion of air molecules whose displacement is ~~also~~ equal to the diameter of a Hydrogen Atom. Congratulations!!

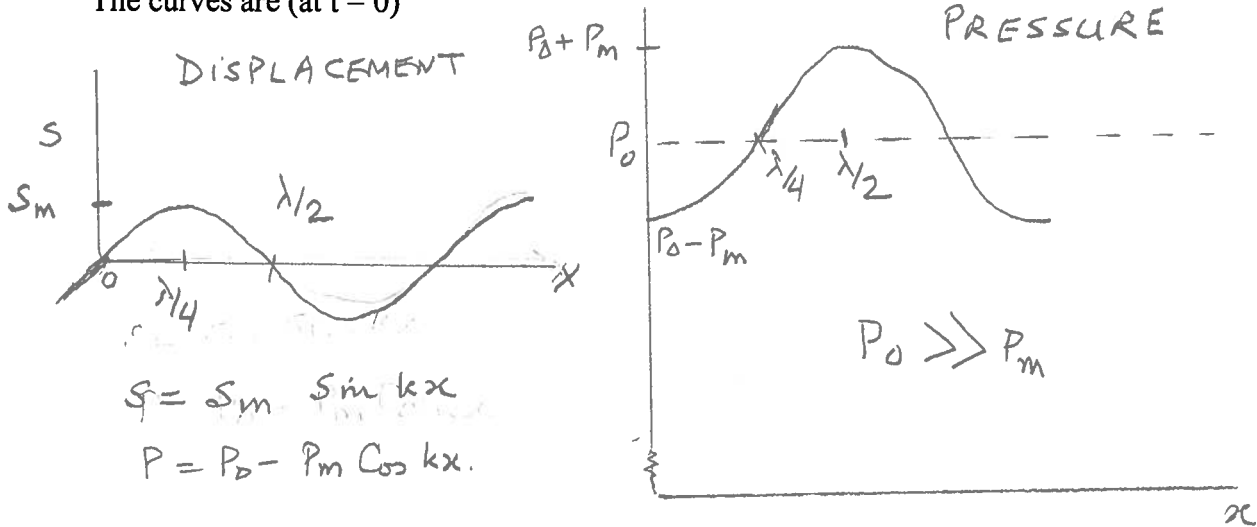
Special Note

Detailed interpretation of displacement and pressure curves in a sound wave

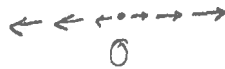
Or

Why is pressure variation $\pi/2$ out of phase with displacement as a function of position?

The curves are (at $t = 0$)



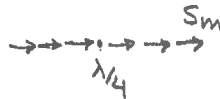
Near $x = 0$, displacements look like



All displacements away from 0.

That is displacements of particles increase rapidly as you go away from $x = 0$. Consequently, gas is in expansion. That is why pressure is at a minimum.

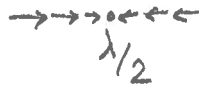
Near $x = \lambda/4$, displacements look like



That is, all the displacements are nearly equal so there is little change in volume and hence P is at its equilibrium value. Derivation from ΔV is zero. ~~Near $x = \lambda/2$~~

NEAR $x = \lambda/2$

Displacements look like



Displacements are toward $\lambda/2$ and increase as you go away from $\lambda/2$. So here gas is in contraction and that is why pressure is at a maximum.

Crucial point is that change of volume and hence change of pressure happens only if displacement everywhere is not the same.