MAXWELL'S EQUATIONS: RADIATION ⇒ LIGHT

To summarize, the field Equations derived from Experiments are:

GAUSS' LAW FOR COULOMB $\underset{\rightarrow}{E}$

$$\Sigma_C \underset{\rightarrow}{E} \bullet \Delta \underset{\rightarrow}{A} = \frac{1}{\varepsilon_0} \Sigma Q_i \qquad (1)$$

GAUSS' LAW FOR $\underset{\rightarrow}{B}$

$$\Sigma_C \underset{\rightarrow}{B} \bullet \Delta \underset{\rightarrow}{A} \equiv 0 \qquad (2)$$

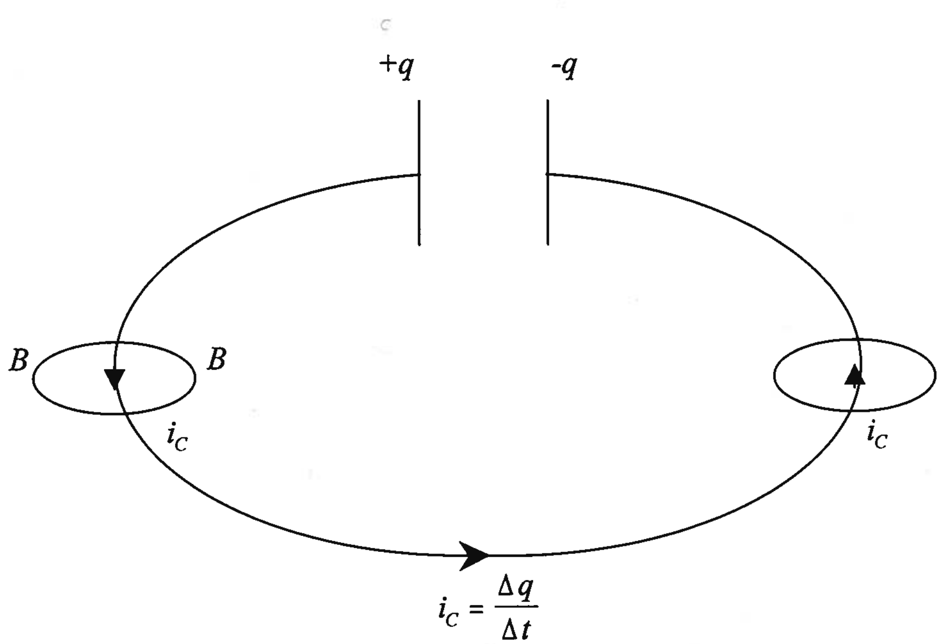
LENZ'S LAW

$$\Sigma_C \underset{\rightarrow}{E_{NC}} \bullet \Delta \underset{\rightarrow}{l} = -\frac{\Delta \Phi_B}{\Delta t} \qquad (3)$$

AMPERE'S LAW

$$\Sigma_C \underset{\rightarrow}{B} \bullet \Delta \underset{\rightarrow}{l} = \mu_0 \Sigma I_i \qquad (4')$$

When Maxwell began to study these equations, he realized that there was a serious problem. Scientists believe that at its most fundamental level nature must be symmetric. Maxwell noticed that whereas a time varying flux of $\underset{\rightarrow}{B}$ gave rise to an $\underset{\rightarrow}{E}$ -field $\left[\underset{\rightarrow}{E_{NC}} \text{ in } Eq.(3)\right]$ there was no corresponding term in *Eq. (4')*. He immediately asserted that the above field equations could not be regarded as being complete. This was a FUNDAMENTAL PROBLEM Maxwell also noted a "PRACTICAL PROBLEM" in using *Eq. (4')*. Imagine that we charge a capacitor to $\pm q$ and then connect a wire between the two plates as shown.



It is clear that a conduction current $\frac{\Delta q}{\Delta t}$ begins to flow through the wire and so [using *Eq. (4′)*] it must create a $\vec{B}$-field encircling the wire as shown. However, as soon as you cross one of the capacitor plates, both the current and $\vec{B}$ must be zero. Again, Maxwell asserted that such a discontinuity cannot be physically meaningful.

To resolve the fundamental problem Maxwell <u>postulated</u> that if the flux of $\vec{E}$ varies with time it must be equivalent to a current. He called this new type of current a displacement current and introduced the definition $i_D = \varepsilon_0 \frac{\Delta \phi_E}{\Delta t}$ (5)

Of course, *Eq. (4′)* implies that every current generates a $\vec{B}$ so Maxwell "completed" *Eq. (4′)* by writing $\Sigma_C \vec{B} \cdot \Delta \vec{l} = \mu_0 \Sigma I_C + \mu_0 \varepsilon_0 \frac{\Delta \phi_E}{\Delta t}$ (4)

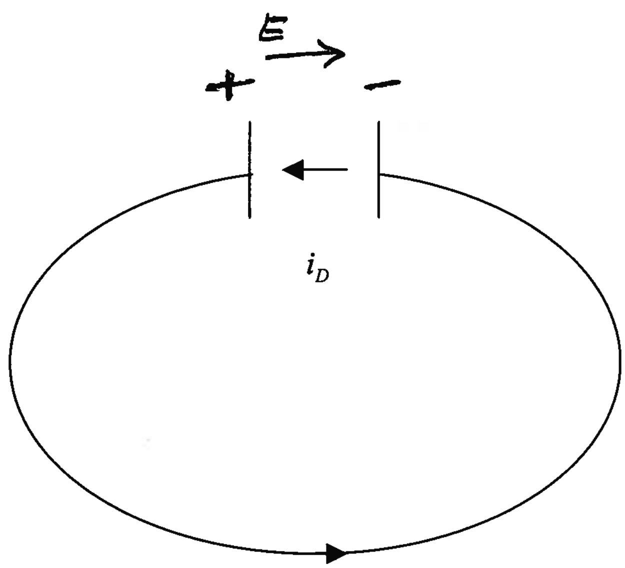
Where I_C explicitly signifies a conduction current = flow of charge in a conductor while the second term on the right comes from $i_D [Eq. (5)]$.

Let us see if introduction of i_D also solves the practical problem. If the capacitor plates have an area A the $\vec{E}$-field between them is

$$\vec{E} = \frac{q}{\varepsilon_0 A}\hat{x}, \ \vec{A} = A\hat{x}$$

so $\Phi_E = \frac{q}{\varepsilon_0}$

and $i_D = \varepsilon_0 \frac{\Delta \Phi_E}{\Delta t} = \frac{\Delta q}{\Delta t} = i_C$!



[i_D is from –ive to +ive because of $\frac{\Delta q}{\Delta t}$ is –ive]

Since $i_D = i_C$ we will have <u>no</u> discontinuity in either the current or the $\vec{B}$-field on crossing the capacitor plate.

Maxwell has solved both the fundamental and the practical problem by proposing *Eq. (5)*.

MAXWELL'S EQUATIONS

GAUSS' LAW FOR COLLOMB $\vec{E}$:

Since a stationary charge generates a Coulomb $\vec{E}$ field, the TOTAL flux of $\vec{E}_{coul}$ THROUGH a closed surface is determined solely by the charges located in the volume enclosed by that surface:

$$\Sigma_c \vec{E}_{coul} \cdot \vec{\Delta A} = \frac{1}{\varepsilon_0} \Sigma Q_i \qquad (1).$$

GAUSS' LAW FOR $\vec{B}$

Since the elementary generators of $\vec{B}$ are point magnetic dipoles the TOTAL flux of $\vec{B}$ THROUGH A closed surface is always zero:

$$\Sigma_c \vec{B} \cdot \vec{\Delta A} \equiv 0 \qquad (2).$$

FARADAY-LENZ LAW

IF THE FLUX OF $\vec{B}$ VARIES WITH TIME A NON-COULOMB $\vec{E}$ FIELD WILL APPEAR IN EVERY CLOSED "LOOP" SORROUNDING THE REGION WHERE THE FLUX OF $\vec{B}$ IS VARYING. THE SENSE OF $\vec{E}_{NC}$ IS INVARIABLY ~~TO~~ SUCH AS OPPOSE THE VARIATION IN THE FLUX OF $\vec{B}$ THAT CAUSES IT. HENCE, CIRCULATION OF

$\vec{E}_{NC}$ around a closed loop is determined by the ~~...~~ time rate of change of flux of $\vec{B}$ through the area within the loop. [Note: Crucial -ive sign]:

$$\sum \vec{E}_{NC} \cdot \vec{\Delta l} = -\frac{\Delta \phi_B}{\Delta t} \qquad (3)$$

MAXWELL- AMPERE LAW

Every current generates a $\vec{B}$ field that circulates around it. There are two types of current: i) Conduction current which involves flow of charge in a conductor and ii) displacement current which arises when flux of $\vec{E}$ field varies with time. Hence, circulation of $\vec{B}$ around a closed loop is determined by the currents threading the surface on which the loop is drawn:

$$\sum_C \vec{B} \cdot \vec{\Delta l} = \mu_0 \sum I_c + \mu_0 \varepsilon_0 \frac{\Delta \phi_E}{\Delta t} \qquad (4).$$

<u>CAUTION</u>: i_D exists in vacuum. It never involves flow of charge. No conduction current can exist inside the capacitor!!!

Maxwell's *Equations (1)* through *(4)* have profound consequences. Let us recall his work using these in outer space, where there is vacuum, q=0, $i_C = 0$ so the Equations become:

$$\Sigma_C \underset{\rightarrow}{E} \cdot \Delta \underset{\rightarrow}{A} = 0 \qquad I$$

$$\Sigma_C \underset{\rightarrow}{B} \cdot \Delta \underset{\rightarrow}{A} = 0 \qquad II$$

$$\Sigma_C \underset{\rightarrow}{E_{NC}} \cdot \Delta \underset{\rightarrow}{l} = -\frac{\Delta \phi_B}{\Delta t} \qquad III$$

$$\Sigma_C \underset{\rightarrow}{B} \cdot \Delta \underset{\rightarrow}{l} = \mu_0 \varepsilon_0 \frac{\Delta \phi_E}{\Delta t} \qquad IV$$

and now indeed there is total symmetry with respect to $\underset{\rightarrow}{E}$ and $\underset{\rightarrow}{B}$. This is what led Maxwell to propose that rather than think of $\underset{\rightarrow}{E}$ and $\underset{\rightarrow}{B}$ fields, one should think of a single entity:

<u>Electromagnetic or EM field</u>

And call *Equations I* through *IV*, EM field Equations. He next used these Equations to <u>predict</u> that in vacuum there must exist <u>EM-waves!</u> He was able to show that the structure of these Equations is such that both the $\underset{\rightarrow}{E}$ and $\underset{\rightarrow}{B}$ have the functional form (propagation along x for example) $f(x \pm ct)$. That is, they propagate as an Electromagnetic wave with the enormous speed $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \, m/s$. This was a giant step forward: Maxwell had solved the problem of the nature of Radiation or Radiant energy. $\Rightarrow$ Radiation is an Electromagnetic wave. <u>Our observable universe = Matter + Radiation</u>

Incidentally, Einstein demonstrated that matter and radiation convert into one another there by further simplifying our picture of the universe.

$\rightarrow$ *Heat*

$\rightarrow$ *Light*

$\rightarrow$ *x – rays*

$\rightarrow$ *radiowaves*

are all cases of EM waves. They are distinguished only by their frequencies (or wavelengths). We will concentrate on

FINALLY WE COME TO → LIGHT

LIGHT: is a transverse EM wave ($\underset{\rightarrow}{E}$ and $\underset{\rightarrow}{B}$ fields perpendicular to direction of propagation and also $\underset{\rightarrow}{E} \perp \underset{\rightarrow}{B}$) whose wavelength lies between 400 *nm* and 800 *nm* and whose speed in vacuum is $3 \times 10^8 \, m/s$. As always, light waves transport energy. Let us compare transport of Energy by:

Wave on a string: Power

$$P = \frac{1}{2}\mu A^2 \omega^2 v$$

$$v = \sqrt{\frac{F}{\mu}}$$

[Av. Energy Stored per unit length multiplied by velocity]

Sound: Intensity

$$I = \frac{1}{2}\rho_0 S_m^{\,2} \omega^2 v$$

$$v = \sqrt{\frac{\gamma P_0}{\rho_0}}$$

[Av. Energy Stored per unit volume multiplied by velocity].

EM-wave Light: Intensity

$$I = \frac{B_m^{\,2}}{2\mu_0} c = \frac{1}{2} \epsilon_0 E_m^{\,2} c$$

$$c = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$$

[Av. Energy Stored per unit length multiplied by velocity].