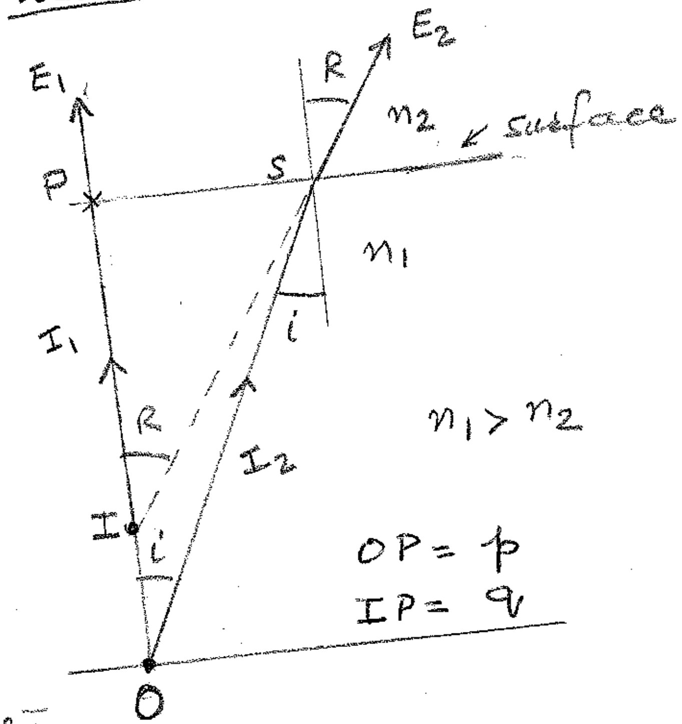
FORMATION OF IMAGES - REFRACTION AT A SINGLE SURFACE [SIGN CONVENTION: ALONG LIGHT +ive, AGAINST LIGHT -ive]

I : Apparent depth of water in a pool.

Supposing you are standing at the edge of a swimming pool and look straight down. If the actual depth of water is d meters what value do you perceive? We can solve this problem by putting a point object O at the bottom and locate its image formed by the water as the light refracts through its surface. Look at the picture from O:



OPTICAL SYSTEM so all distances are measured from P.

Take two rays starting from O:

I_1 makes angle of incidence zero and gives rise to E_1

I_2 makes angle of incidence i and causes E_2 satisfying

$$n_2 \sin R = n_1 \sin i$$

since you are looking straight down all angles are small.

The virtual IMAGE at I [q is -ive]. is located by intersection of E_1 and (extended backwards).

Next, from the picture we see

$$\tan R = \frac{SP}{IP} \qquad (1)$$

$$\tan i = \frac{SP}{OP} \qquad (2).$$

Divide (2) by (1)

$$\frac{IP}{OP} = \frac{\tan i}{\tan R}$$

$$\simeq \frac{\sin i}{\sin R} \qquad \left[\begin{matrix} i \ll 1 \\ R \ll 1 \end{matrix}\right].$$

$$= \frac{n_2}{n_1}$$

Clearly $IP =$ apparent depth

$OP =$ Rl. depth

$$\frac{d_{app}}{d} = \frac{n_2}{n_1}$$

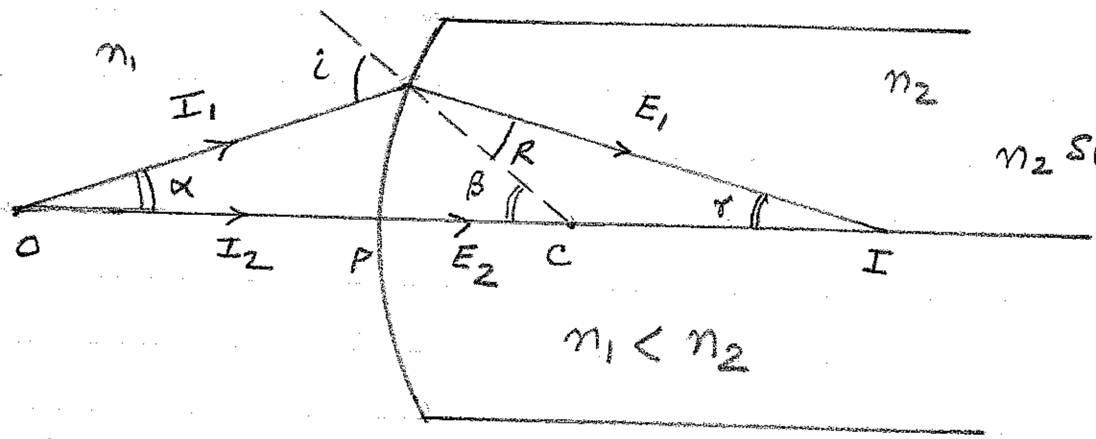
for water $n = 1.33$

for air $n = 1$.

$$\text{so} \quad \frac{d_{app}}{d} = \frac{3}{4}.$$

So if water is 80 cm deep, to a person at the edge it will appear to be only 60 cm [Small children should be warned before they jump in and suddenly find that they are too short].

II SINGLE CONVEX SURFACE (+ive r)



$n_2 \sin R = n_1 \sin i$

Here OP = object distance (p).
IP = Image distance (q).
CP = Radius of Curved surface (r)
and all ~~to~~ angles are Small
Equs. are $n_1 \sin i = n_2 \sin R$

$$\text{or } n_1 i = n_2 R. \quad -(1)$$

$$\beta = \gamma + R \quad -(2)$$

$$i = \alpha + \beta \quad -(3).$$

From (2)

$$\beta = \gamma + \frac{n_1}{n_2} i$$

$$= \gamma + \frac{n_1}{n_2}(\beta + \alpha)$$

$$\beta\left[1 - \frac{n_1}{n_2}\right] = \gamma + \frac{n_1}{n_2}\cancel{\beta}\alpha.$$

$$\text{or} \quad (n_2 - n_1)\beta = n_2\gamma + n_1\alpha.$$

$$(n_2 - n_1)\tan\beta = n_2 \tan\gamma + n_1 \tan\alpha.$$

$$\boxed{\frac{(n_2 - n_1)}{r} = \frac{n_2}{q} + \frac{n_1}{p}} \quad (4)$$

In order to access the magnification for a small object, imagine rotating OPI through a small angle, you will get

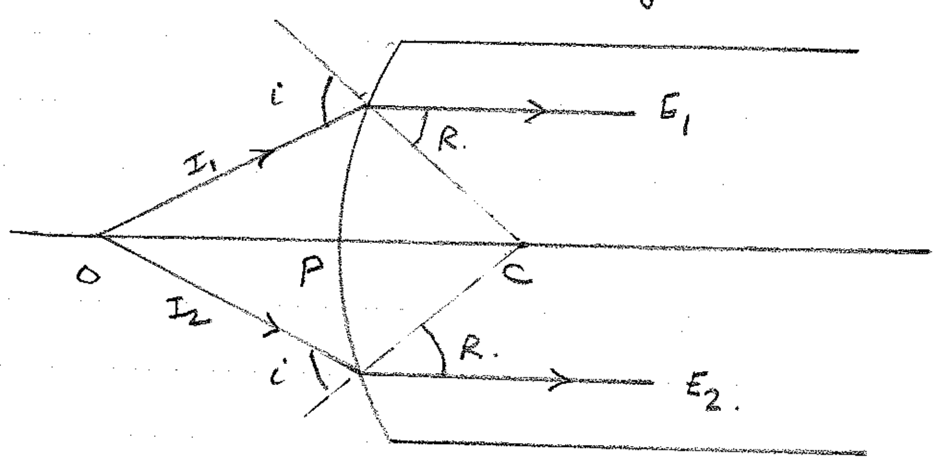
$$\tan i = \frac{OO'}{p}$$

$$\tan R = \frac{II'}{q}$$

II' is -ive!

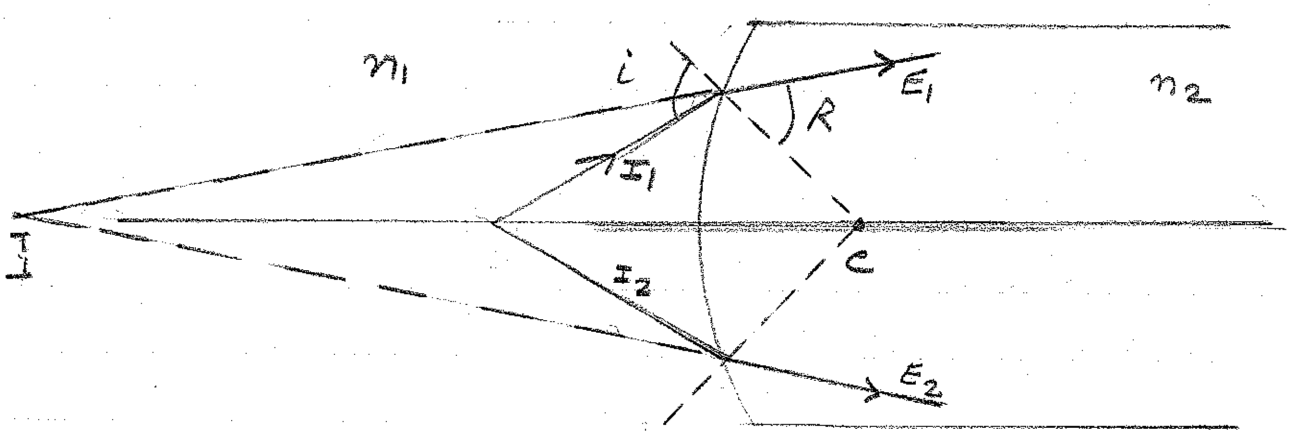
$$m = \frac{II'}{OO'} = -\frac{q \tan R}{p \tan i} = -\frac{n_1}{n_2}\frac{q}{p} \quad (5)$$

SPECIAL CASES A p is such that $q \to \infty$, That is light becomes a parallel beam on entering the surface.



$$p = \frac{n_1 R}{n_2 - n_1} \quad (6).$$

B If p becomes even smaller than that given by Eq (6); E_1 and E_2 will diverge the image will switch to the left of the surface and become virtual.



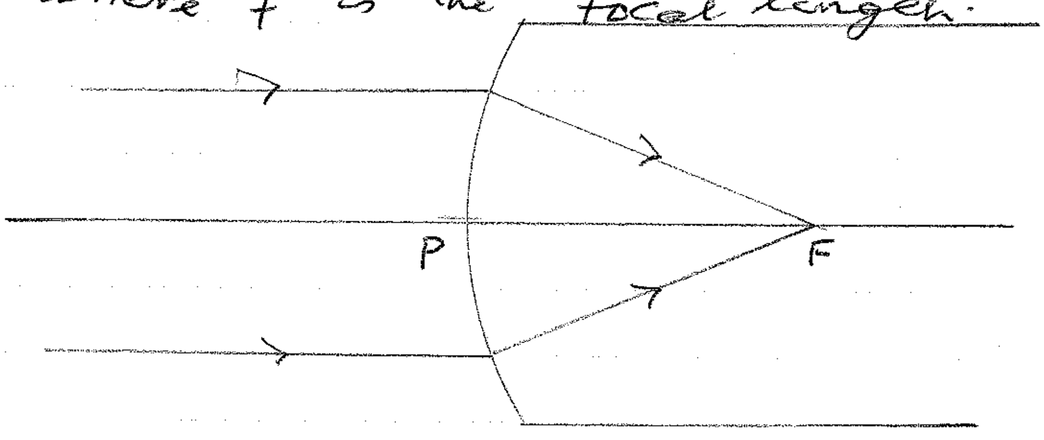
C Please Note that if you set $r \to \infty$ in Eq (4) you will recover the result of Case I

$$\frac{q}{p} = -\frac{n_2}{n_1}$$

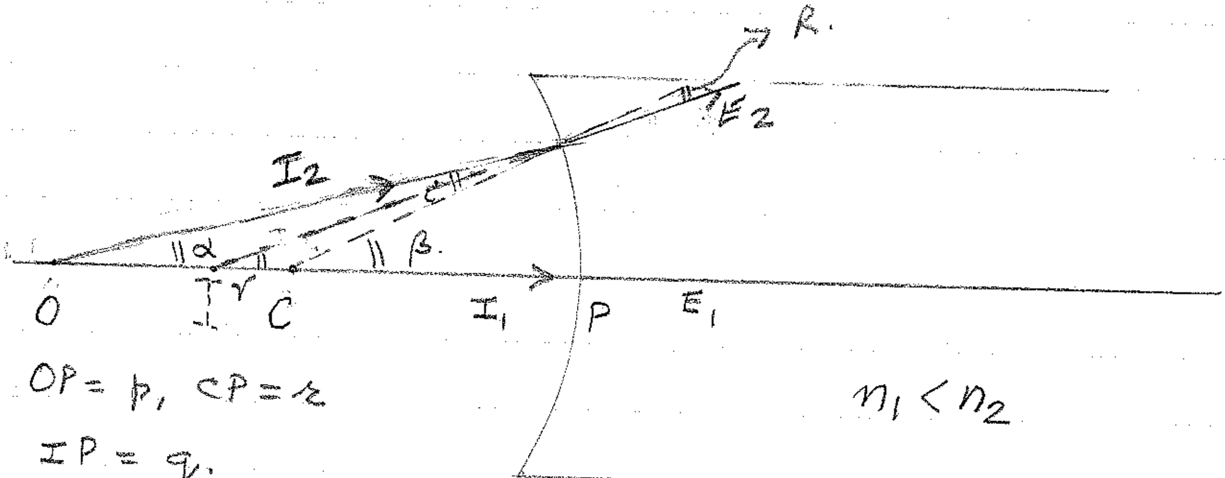
D If $p \to \infty$, INCIDENT BEAM IS Parallel

$$\frac{n_2}{f} = \frac{n_2 - n_1}{r}.$$

where f is the focal length.



III SINGLE CONCAVE SURFACE (−ive r)



$OP = p$, $CP = r$

$IP = q$.

IMAGE IS ALWAYS VIRTUAL (q is −ive)

Eqns are $\quad n_1 \sin i = n_2 \sin R$

$$\text{or} \quad n_1\, i = n_2\, R \quad \text{— (1)} \quad \text{Small angles.}$$

$$\beta = \alpha + i \quad \text{— (2)}$$

$$\beta = \gamma + R \quad \text{— (3).}$$

$$\beta = \gamma + \frac{n_1}{n_2} i = \gamma + \frac{n_1}{n_2}(\beta - \alpha)$$

$$\tan\beta = \tan\gamma + \frac{n_1}{n_2}(\tan\beta - \tan\alpha)$$

$$\frac{1}{r} = \frac{1}{q} + \frac{n_1}{n_2}\left[\frac{1}{r} - \frac{1}{p}\right]$$

$$\frac{1}{r}\left[1 - \frac{n_1}{n_2}\right] = \frac{n_2}{q} - \frac{n_1}{p}$$

$$\frac{n_1}{p} - \frac{n_2}{q} = -\frac{1}{r}(n_2 - n_1)$$

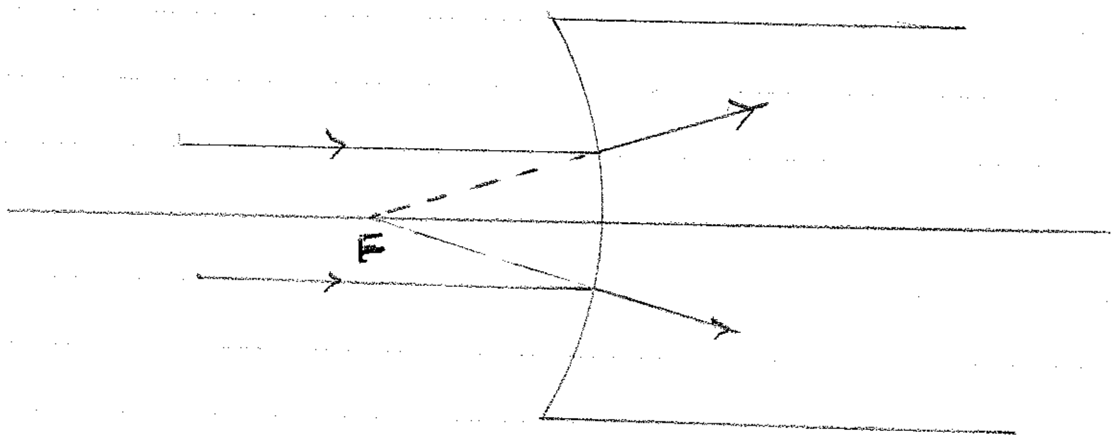
But q and r are −ive, Hence again

$$\boxed{\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{r}}$$

q −ive

r −ive

Special Case $p \to \infty$ $q \to f$



f is negative.

$$\frac{n_2}{f} = \frac{n_2 - n_1}{r}$$